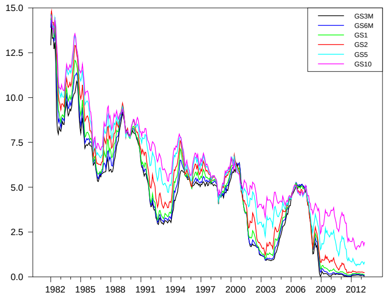
Dynamic models for large-dimensional vector systems

A. Principal components analysis

Suppose we have a large number of variables observed at date t

Goal: can we summarize most of the features of the data using just a few indicators?

Yields on U.S. Treasury securities (3 months to 10 years)



$\mathbf{y}_t = (n \times 1)$ vector of stationary observations

$\hat{\mu}_i = T^{-1} \sum_{t=1}^{T} y_{it}$ (mean of variable i)

$\hat{\sigma}_{ii} = T^{-1} \sum_{t=1}^{T} (y_{it} - \hat{\mu}_i)^2$

$\tilde{y}_{it} = \hat{\sigma}_{ii}^{-1/2} (y_{it} - \hat{\mu}_i)$

$\tilde{\mathbf{y}}_t = (\tilde{y}_{1t}, \ldots, \tilde{y}_{nt})'$

$\hat{\boldsymbol{\Omega}} = T^{-1} \sum_{t=1}^{T} \tilde{\mathbf{y}}_t \tilde{\mathbf{y}}_t'$

(sample correlation matrix)

Goal is to find a scalar ξ_t and
$(n \times 1)$ vector $\mathbf{h}$ so as to minimize

$$\sum_{t=1}^{T} (\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)$$

Note: $\mathbf{h}$ and ξ_t are not unique
($\mathbf{h}\xi_t = \mathbf{h}^*\xi_t^*$ for $\mathbf{h}^* = q\mathbf{h}$, $\xi_t^* = q^{-1}\xi_t$)
but $\mathbf{h}\xi_t$ is unique.
One normalization: $\mathbf{h}'\mathbf{h} = 1$.

$\tilde{\mathbf{y}}_t \simeq \mathbf{h}\xi_t$
Scalar ξ_t explains as much of variation of $\tilde{\mathbf{y}}_t$ as possible.
Solution ξ_t^* is called the "first principal component" of $\mathbf{y}_t$ (determined up to arbitrary scale factor).
Elements of vector $\mathbf{h}$ are called "factor loadings".

$$\min_{\{\mathbf{h},\xi_1,\ldots,\xi_T\}} \sum_{t=1}^{T} (\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)$$

Concentrate objective function:
(1) for any $\mathbf{h}$, find best $\{\xi_1, \ldots, \xi_T\}$
(2) substitute $\xi_t(\mathbf{h})$ into objective and min with respect to $\mathbf{h}$

(1) for fixed **h**:

$$\min_{\{\xi_1,\ldots,\xi_T\}} \sum_{t=1}^{T} (\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)$$

$$\min_{\xi_t} (\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)$$

OLS regression of $\tilde{\mathbf{y}}_t$ on $\mathbf{h}$

$$\xi_t(\mathbf{h}) = (\mathbf{h}'\mathbf{h})^{-1}\mathbf{h}'\tilde{\mathbf{y}}_t$$

$$(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t)$$
$$= \tilde{\mathbf{y}}_t'(\mathbf{I}_n - \mathbf{h}(\mathbf{h}'\mathbf{h})^{-1}\mathbf{h}')\tilde{\mathbf{y}}_t$$

(2) minimize over **h**:

$$\min_{\{\mathbf{h}\}} \sum_{t=1}^{T} (\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t(\mathbf{h}))'(\tilde{\mathbf{y}}_t - \mathbf{h}\xi_t(\mathbf{h}))$$
$$= \sum_{t=1}^{T} \tilde{\mathbf{y}}_t'(\mathbf{I}_n - \mathbf{h}(\mathbf{h}'\mathbf{h})^{-1}\mathbf{h}')\tilde{\mathbf{y}}_t$$
$$\Leftrightarrow \quad \max_{\{\mathbf{h}\}} \sum_{t=1}^{T} \tilde{\mathbf{y}}_t'\mathbf{h}(\mathbf{h}'\mathbf{h})^{-1}\mathbf{h}'\tilde{\mathbf{y}}_t$$

subject to $\mathbf{h}'\mathbf{h} = 1$

$$= \sum_{t=1}^{T} \mathbf{h}'\tilde{\mathbf{y}}_t\tilde{\mathbf{y}}_t'\mathbf{h}$$
$$= \mathbf{h}'\left(\sum_{t=1}^{T} \tilde{\mathbf{y}}_t\tilde{\mathbf{y}}_t'\right)\mathbf{h}$$
$$= T\mathbf{h}'\hat{\Omega}\mathbf{h}$$

$$\max_{\{\mathbf{h}\}} \mathbf{h}'\hat{\Omega}\mathbf{h}$$

subject to $\mathbf{h}'\mathbf{h} = 1$

Consider eigenvalues of $\hat{\boldsymbol{\Omega}}$

$$\hat{\boldsymbol{\Omega}}\mathbf{x}_i = \hat{\lambda}_i\mathbf{x}_i \quad \text{for } i = 1,..,n$$

$$\hat{\boldsymbol{\Lambda}} = \text{diag}(\hat{\lambda}_1,..,\hat{\lambda}_n)$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_n \end{bmatrix}$$

$$\mathbf{X}'\mathbf{X} = \mathbf{I}_n$$

$$\hat{\boldsymbol{\Omega}}\mathbf{X} = \mathbf{X}\hat{\boldsymbol{\Lambda}}$$

$$\mathbf{X}'\hat{\boldsymbol{\Omega}}\mathbf{X} = \hat{\boldsymbol{\Lambda}}$$

$\max_{\{\mathbf{h}\}} \mathbf{h}'\hat{\boldsymbol{\Omega}}\mathbf{h}$ subject to $\mathbf{h}'\mathbf{h} = 1$

Let $\mathbf{h} = \mathbf{X}\mathbf{h}^*$

where $\mathbf{X}'\mathbf{X} = \mathbf{I}_n$ and $\mathbf{X}'\hat{\boldsymbol{\Omega}}\mathbf{X} = \hat{\boldsymbol{\Lambda}}$

$\max_{\{\mathbf{h}\}} \mathbf{h}'\hat{\boldsymbol{\Omega}}\mathbf{h}$ subject to $\mathbf{h}'\mathbf{h} = 1$

$\Leftrightarrow \quad \max_{\{\mathbf{h}^*\}} \mathbf{h}^{*\prime}\mathbf{X}'\hat{\boldsymbol{\Omega}}\mathbf{X}\mathbf{h}^*$ subject to $\mathbf{h}^{*\prime}\mathbf{h}^* = 1$

$$\mathbf{h}^{*\prime}\mathbf{X}'\hat{\boldsymbol{\Omega}}\mathbf{X}\mathbf{h}^* = \mathbf{h}^{*\prime}\hat{\boldsymbol{\Lambda}}\mathbf{h}^*$$
$$= h_1^{*2}\hat{\lambda}_1 + \cdots + h_n^{*2}\hat{\lambda}_n$$

$$\max_{\{\mathbf{h}^*\}} h_1^{*2}\hat{\lambda}_1 + \cdots + h_n^{*2}\hat{\lambda}_n$$

s.t. $h_1^{*2} + \cdots + h_n^{*2} = 1$

Solution: $h_1^* = 1$

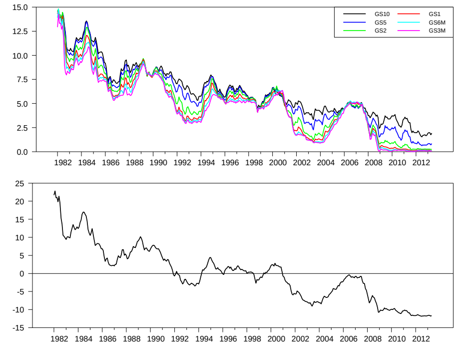
$$h_2^* = h_3^* = \cdots = h_n^* = 0$$

$\Rightarrow \mathbf{h} = \mathbf{x}_1$

Conclusion: the factor loadings are given by the eigenvector of $\hat{\mathbf{\Omega}}$ associated with largest eigenvalue. The first principal component is given by $\mathbf{h}'\tilde{\mathbf{y}}_t$, the product of this eigenvector with de-meaned data vector.

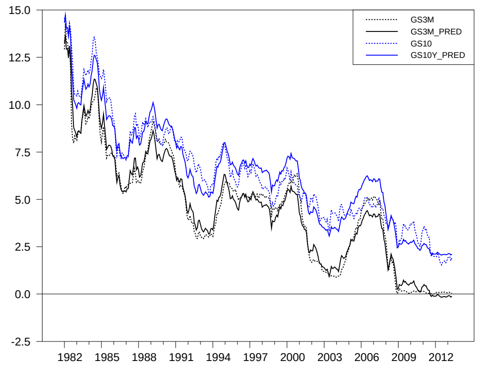
Example:
$\mathbf{y}_t$ = interest rates for month t for U.S. Treasury securities with maturities 3m, 6m, 1y, 2y, 5y, 10y
$n = 6$
t = 1982:M1 - 2013:M5
$\hat{\mathbf{\Omega}} = T^{-1}\sum_{t=1}^{T}(\mathbf{y}_t - \hat{\boldsymbol{\mu}})(\mathbf{y}_t - \hat{\boldsymbol{\mu}})'$

Eigenvector of $\hat{\mathbf{\Omega}}$ associated with largest eigenvalue:
(0.3999, 0.4153, 0.4244, 0.4344, 0.4061, 0.3659)$'$
$1/\sqrt{6} = 0.4082$
Conclusion: first principal component is essentially the average of the 6 yields.



Fitted value for yield i:

$$y_{it} \simeq \hat{\mu}_i + h_i \hat{\xi}_t$$



Could also ask: suppose I could use 2 variables to summarize the 6 yields. Choose (2×1) vector $\boldsymbol{\xi}_t$ for $t = 1, \ldots, T$ and $(n \times 2)$ matrix $\mathbf{H}$ to minimize $\sum_{t=1}^{T} (\tilde{\mathbf{y}}_t - \mathbf{H}\boldsymbol{\xi}_t)'(\tilde{\mathbf{y}}_t - \mathbf{H}\boldsymbol{\xi}_t)$.

Again not unique:

$\mathbf{Q}$ nonsingular (2×2) matrix

$\mathbf{H}^* = \mathbf{HQ}$

$\boldsymbol{\xi}_t^* = \mathbf{Q}^{-1}\boldsymbol{\xi}_t$

$\mathbf{H}\boldsymbol{\xi}_t = \mathbf{H}^*\boldsymbol{\xi}_t^*$.

Normalize $\mathbf{H}'\mathbf{H} = \mathbf{I}_2$.

$$\min_{\{\boldsymbol{\xi}_t\}} \ (\tilde{\mathbf{y}}_t - \mathbf{H}\boldsymbol{\xi}_t)'(\tilde{\mathbf{y}}_t - \mathbf{H}\boldsymbol{\xi}_t)$$
$$= \tilde{\mathbf{y}}_t'(\mathbf{I}_n - \mathbf{H}(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}')\tilde{\mathbf{y}}_t$$
$$\max_{\{\mathbf{H}\}} \ \sum_{t=1}^{T} \tilde{\mathbf{y}}_t'\mathbf{H}(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'\tilde{\mathbf{y}}_t$$
$$= \sum_{t=1}^{T} \text{trace}[(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'\tilde{\mathbf{y}}_t\tilde{\mathbf{y}}_t'\mathbf{H}]$$
$$= T(\mathbf{H}'\mathbf{H})^{-1}\text{trace}\left[\mathbf{H}'\hat{\boldsymbol{\Omega}}\mathbf{H}\right]$$

Solution: $\mathbf{H}$ is in the linear space spanned by the eigenvectors of $\hat{\boldsymbol{\Omega}}$ associated with the two largest eigenvalues.

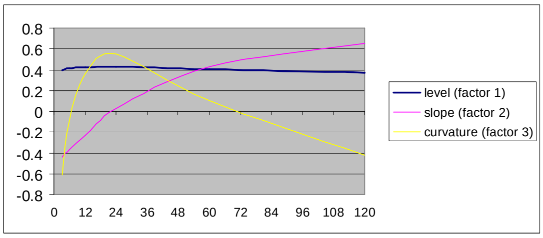
Second principal component refers to $\mathbf{h}_2'\tilde{\mathbf{y}}_t$ for $\mathbf{h}_2$ the eigenvector of $\hat{\boldsymbol{\Omega}}$ associated with the second largest eigenvalue. Note second PC is orthogonal to the first:

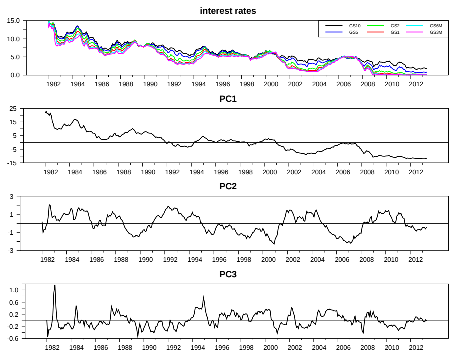
$$\sum_{t=1}^{T}(\mathbf{h}_1'\tilde{\mathbf{y}}_t)(\tilde{\mathbf{y}}_t'\mathbf{h}_2) = T\mathbf{h}_1'\hat{\boldsymbol{\Omega}}\mathbf{h}_2$$
$$= T\hat{\lambda}_2\mathbf{h}_1'\mathbf{h}_2 = 0$$

Interest rates: eigenvalues of $\boldsymbol{\Omega}$

	Eigenvalue	Percent
1	56.0309	0.980548
2	1.0423	0.998789
3	0.0519	0.999697
4	0.013	0.999924
5	3.05E-03	0.999978
6	1.28E-03	1

Factor loadings associated with first three principal components





Dynamic models for large-dimensional vector systems

A. Principal components analysis

B. Dynamic factor models

$\mathbf{y}_t$ = observed variables

ξ_t = unobserved factors

$$\underset{(n\times 1)}{\mathbf{y}_t} = \underset{(n\times r)}{\mathbf{H}}\ \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{\mathbf{u}_t}$$

$$\underset{(r\times 1)}{\xi_{t+1}} = \underset{(r\times r)}{\mathbf{\Phi}}\ \underset{(r\times 1)}{\xi_t} + \underset{(r\times 1)}{\mathbf{v}_t}$$

$$\underset{(n\times 1)}{\mathbf{u}_{t+1}} = \underset{(n\times n)}{\mathbf{D}}\ \underset{(n\times 1)}{\mathbf{u}_t} + \underset{(n\times 1)}{\boldsymbol{\varepsilon}_t}$$

$\mathbf{D}$ = diag$(d_1, d_2, \ldots, d_n)$

$$\underset{(n\times 1)}{\mathbf{y}_t} = \underset{(n\times r)}{\mathbf{H}}\ \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{\mathbf{u}_t}$$

Assumption 1:

$$n^{-1}\mathbf{H}'\mathbf{H} \underset{n\to\infty}{\to} \underset{(r\times r)}{\mathbf{Q}_{\mathbf{H}}}$$

with rank($\mathbf{Q}_{\mathbf{H}}$) $= r$.

Means factors matter for more than just finite subset of $\mathbf{y}_t$ and are different from each other (columns of $\mathbf{H}$ not too similar).

Assumption 2:

maximum eigenvalue of $E(\mathbf{u}_t\mathbf{u}_t')$ is $\leq c$ for all n.

Means $\mathbf{u}_t$ does not have its own factor structure.

(e.g., for $E(\mathbf{u}_t\mathbf{u}_t') = \sigma^2\mathbf{1}\mathbf{1}'$ then $\mathbf{1}$ is eigenvector with eigenvalue $\sigma^2\mathbf{1}'\mathbf{1} = \sigma^2 n$)

Suppose these assumptions held and there was an $(n \times r)$ matrix $\mathbf{W}$ such that:

(i) $n^{-1}\mathbf{W}'\mathbf{W} \to \mathbf{I}_r$

(ii) $n^{-1}\mathbf{W}'\mathbf{H} \to \underset{(r\times r)}{\mathbf{\Lambda}}$

(iii) rank$(\mathbf{\Lambda}) = r$

For yields example and $r = 1$,

$\mathbf{W}' = (1, 1, \ldots, 1)$

$$\underset{(n\times 1)}{\mathbf{y}_t} = \underset{(n\times r)}{\mathbf{H}}\ \underset{(r\times 1)}{\boldsymbol{\xi}_t} + \underset{(n\times 1)}{\mathbf{u}_t}$$

$$n^{-1}\ \underset{(r\times n)}{\mathbf{W}'}\ \underset{(n\times 1)}{\mathbf{y}_t} = n^{-1}\ \underset{(r\times n)}{\mathbf{W}'}\ \underset{(n\times r)}{\mathbf{H}}\ \underset{(r\times 1)}{\boldsymbol{\xi}_t}$$

$$+\ n^{-1}\ \underset{(r\times n)}{\mathbf{W}'}\ \underset{(n\times 1)}{\mathbf{u}_t}$$

$$n^{-1}\mathbf{W}'\mathbf{u}_t \xrightarrow{p} \mathbf{0}$$

(e.g., $n^{-1}\sum_{i=1}^{n} u_{it} \xrightarrow{p} 0$)

Stock and Watson (JASA, 2002) showed that under related assumptions, the first r principal components of $\mathbf{y}_t$ provide a consistent estimate of $\mathbf{\Lambda}\boldsymbol{\xi}_t$ for some nonsingular $(r \times r)$ matrix $\mathbf{\Lambda}$.

However, literature on term structure of interest rates suggests that first 3 PC of interest rates do not span set of linear combinations most useful for forecasting (e.g., Gregory Duffee, "Information in (and not in) the term structure," Rev Financial Studies, 2011)

Selecting the number of factors r

$$V_r(\underset{(n\times r)}{\mathbf{H}^{(r)*}}, \underset{(r\times 1)}{\xi_t^{(r)*}}) = (\tilde{\mathbf{y}}_t - \mathbf{H}^*\xi_t^*)'(\tilde{\mathbf{y}}_t - \mathbf{H}^*\xi_t^*)$$

$$\{\mathbf{H}^*, \hat{\xi}_t^*\} =$$

$$\arg \min_{\{\mathbf{H},\xi_1,\ldots,\xi_T\}} (\tilde{\mathbf{y}}_t - \mathbf{H}\xi_t)'(\tilde{\mathbf{y}}_t - \mathbf{H}\xi_t)$$

subject to $\mathbf{H}'\mathbf{H} = \mathbf{I}_r$

Bai and Ng, Econometrica (2002):
Choose r to minimize

$$\log V_r(\mathbf{H}^{(r)*}, \xi_t^{(r)*}) + r\frac{(n+T)\log[\min(n,T)]}{nT}$$

Ahn and Horenstein,
Econometrica (2013):

$\hat{\mathbf{\Omega}} = T^{-1} \sum_{t=1}^{T} \tilde{\mathbf{y}}_t \tilde{\mathbf{y}}_t'$

$\hat{\lambda}_1$ = largest eigenvalue of $\hat{\mathbf{\Omega}}$

$\vdots$

$\hat{\lambda}_n$ = smallest eigenvalue of $\hat{\mathbf{\Omega}}$

Choose r to be value for which $\hat{\lambda}_r / \hat{\lambda}_{r+1}$ is largest.

Dynamic models for large-dimensional vector systems

A. Principal components analysis

B. Dynamic factor models

C. Nowcasting with large "jagged-edge" real-time dataset

Giannone, Reichlin, and Small, JME, 2008

Suppose we have potentially hundreds of different monthly indicators, only some of which are currently available.
What is the optimal estimate of what this quarter's GDP growth will turn out to be?

$\mathbf{y}_t = (n \times 1)$ vector of stationary indicators that will eventually be available for month t

$\tilde{y}_{it} = (y_{it} - \bar{y}_i)/\hat{\sigma}_i$.

Calculate first r PC using largest T for which full sample is available

$\underset{(r\times1)}{\boldsymbol{\xi}_t}$

$y_{it} = \mu_i + \mathbf{h}_i'\boldsymbol{\xi}_t + w_{it}$

$E(w_{it}^2) = r_i^2$

Could calculate analytically or with OLS regressions for $i = 1, \ldots, n$

$\underset{(n\times1)}{\mathbf{y}_t} = \underset{(n\times r)}{\mathbf{H}'}\ \underset{(r\times1)}{\boldsymbol{\xi}_t} + \underset{(n\times1)}{\mathbf{w}_t}$

$E(\mathbf{w}_t\mathbf{w}_t') = \mathbf{R} = \text{diag}(r_1^2, \ldots, r_n^2)$

$=$ obs eq for state-space model

$\underset{(r\times1)}{\boldsymbol{\xi}_{t+1}} = \underset{(r\times r)}{\mathbf{F}}\ \underset{(r\times1)}{\boldsymbol{\xi}_t} + \mathbf{v}_t$

$E(\mathbf{v}_t\mathbf{v}_t') = \mathbf{Q}$

$=$ state eq (could estimate by OLS)

Conclusion: we could use Kalman filter to obtain optimal estimate of factors for last date T for which we have complete data,
$\boldsymbol{\xi}_T | \mathbf{y}_T, \ldots, \mathbf{y}_1 \sim N(\hat{\boldsymbol{\xi}}_{T|T}, \mathbf{P}_{T|T})$,
and forecast for $T+1$:
$\boldsymbol{\xi}_{T+1} | \mathbf{y}_T, \ldots, \mathbf{y}_1 \sim N(\hat{\boldsymbol{\xi}}_{T+1|T}, \mathbf{P}_{T+1|T})$.

If we had full observation of $\mathbf{y}_{T+1}$, we would update inference with
$$\hat{\boldsymbol{\xi}}_{T+1|T+1} = \hat{\boldsymbol{\xi}}_{T+1|T} +$$
$$\mathbf{P}_{T+1|T}\mathbf{H}(\mathbf{H}'\mathbf{P}_{T+1|T}\mathbf{H} + \mathbf{R})^{-1}\hat{\boldsymbol{\varepsilon}}_{T+1}$$
$$\hat{\boldsymbol{\varepsilon}}_{T+1} = \mathbf{y}_{T+1} - \boldsymbol{\mu} - \mathbf{H}'\hat{\boldsymbol{\xi}}_{T+1|T}$$

If instead for some day v_{T+1} we only have some subset of $\mathbf{y}_{T+1}$, just set rows of $\mathbf{H}'$ and $\hat{\boldsymbol{\varepsilon}}_{T+1}$ corresponding to missing obs equal to 0:
$$\hat{\boldsymbol{\xi}}_{T+1|v_{T+1}} = \hat{\boldsymbol{\xi}}_{T+1|T} +$$
$$\mathbf{P}_{T+1|T}\mathbf{H}_{v_{T+1}}(\mathbf{H}'_{v_{T+1}}\mathbf{P}_{T+1|T}\mathbf{H}_{v_{T+1}} + \mathbf{R})^{-1}\hat{\boldsymbol{\varepsilon}}_{T+1|v_{T+1}}$$
$$\hat{\varepsilon}_{i,T+1|v_{T+1}} = y_{i,T+1} - \mu_i - \mathbf{h}_i'\hat{\boldsymbol{\xi}}_{T+1|T}$$
if i is observed
$$\hat{\varepsilon}_{i,T+1|v_{T+1}} = 0 \text{ if } i \text{ is not observed}$$

If our goal is to estimate GDP growth for quarter q, regress it on factor value in third month of quarter ($t(q)$) using full set of observed data:

$$y_q = \alpha + \boldsymbol{\beta}'\boldsymbol{\xi}_{t(q)} + e_q$$

$$q = 1, 2, \ldots, Q, \; t(Q) = T$$

$$\hat{y}_{q|\nu_{T+k}} = \alpha + \boldsymbol{\beta}'\hat{\boldsymbol{\xi}}_{T+3|\nu_{T+k}}$$

where for example

$$\hat{\boldsymbol{\xi}}_{T+3|\nu_{T+1}} = \mathbf{F}^2\hat{\boldsymbol{\xi}}_{T+1|\nu_{T+1}}$$

D. Giannone et al. / Journal of Monetary Economics 55 (2008) 665–676

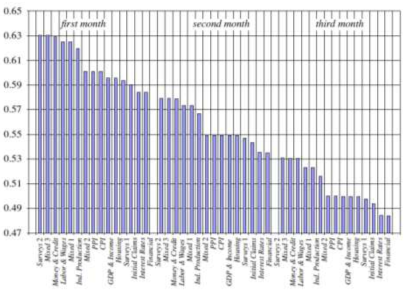


Fig. 3. MSFE across data blocks (in sample).

D. Giannone et al. / Journal of Monetary Economics 55 (2008) 665–676

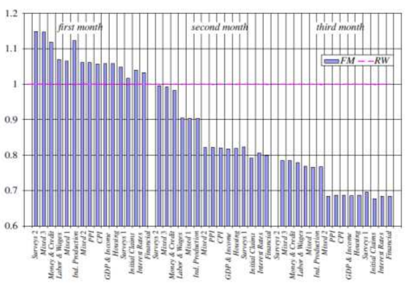


Fig. 4. MSFE across data blocks (out-of-sample).

Sample of news releases during week of Oct 21, 2013

Date	Time (ET)	Statistic	For	Actual	Briefing Forecast	Market Expects	Prior	Revised From
21-Oct	10:00 AM	Existing Home Sales	Sep	5.29M	5.15M	5.30M	5.39M	5.48M
22-Oct	8:30 AM	Nonfarm Payrolls	Sep	148K	165K	183K	193K	169K
22-Oct	8:30 AM	Unemployment Rate	Sep	7.20%	7.30%	7.30%	7.30%	-
22-Oct	8:30 AM	Hourly Earnings	Sep	0.10%	0.20%	0.20%	0.30%	0.20%
24-Oct	8:30 AM	Initial Claims	Oct	350K	330K	341K	362K	358K
25-Oct	8:30 AM	Durable Orders	Sep	3.70%	4.20%	3.50%	0.20%	0.10%
25-Oct	8:30 AM	Durable Goods -ex transportation	Sep	-0.10%	0.30%	0.30%	-0.40%	-0.10%
25-Oct	9:55 AM	Michigan Sentiment - Final	Oct	73.2	73	74.5	75.2	-

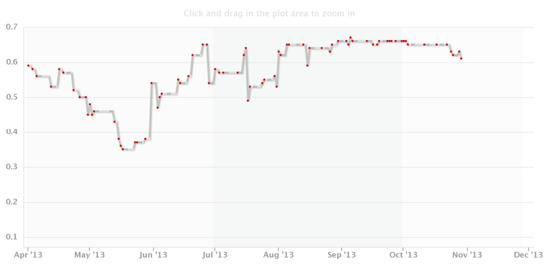
Source: http://biz.yahoo.com/c/e.html

Nowcasts of 2013:Q4 U.S. real GDP growth (quarterly rate)

Oct 22: 0.81
Oct 24: 0.76
Oct 25: 0.71

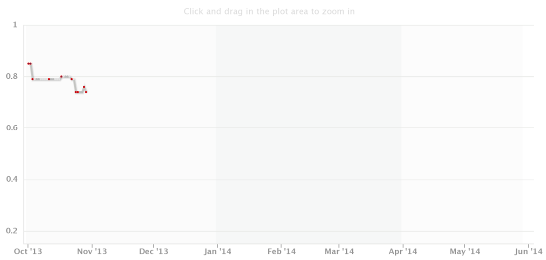
Source: www.now-casting.com

Nowcasts of 2013:Q3 U.S. real GDP growth rate (quarterly rate)



Source: www.now-casting.com

Forecasts of 2014:Q1 U.S. real GDP growth rate (quarterly rate)



Source: www.now-casting.com

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D. Factor-Augmented Vector Autoregressions (FAVAR)

Bernanke, Boivin and Eliasz, QJE, 2005

$\mathbf{y}_t = (n \times 1)$ vector of observed variables $(n = 120)$

$\mathbf{x}_t = (m \times 1)$ subset of $\mathbf{y}_t$ of special interest or importance.

BBE take $\mathbf{x}_t = r_t$ (the fed funds rate) or $\mathbf{x}_t =$ fed funds rate, industrial production, and inflation, in deviations from their means.

Factor-Augmented VAR:

$$\begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\xi}_t} \\ \underset{(m\times 1)}{\mathbf{x}_t} \end{bmatrix} = \begin{bmatrix} \underset{(r\times r)}{\boldsymbol{\Phi}_{11}(L)} & \underset{(r\times m)}{\boldsymbol{\Phi}_{12}(L)} \\ \underset{(m\times r)}{\boldsymbol{\Phi}_{21}(L)} & \underset{(m\times m)}{\boldsymbol{\Phi}_{22}(L)} \end{bmatrix} \begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\xi}_t} \\ \underset{(m\times 1)}{\mathbf{x}_t} \end{bmatrix} + \begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\varepsilon}_{1t}} \\ \underset{(m\times 1)}{\boldsymbol{\varepsilon}_{2t}} \end{bmatrix}$$

$$\boldsymbol{\Phi}_{ij}(L) = \boldsymbol{\Phi}_{ij}^{(1)}L^1 + \boldsymbol{\Phi}_{ij}^{(2)}L^2 + \cdots + \boldsymbol{\Phi}_{ij}^{(p)}L^p$$

Could estimate space spanned by $\boldsymbol{\xi}_t$ by that spanned by $\hat{\boldsymbol{\xi}}_t$, the first r principal components of $\mathbf{y}_t$.
Question: how to identify monetary policy shock?

Note: since r_t is included in $\mathbf{y}_t$, each element of $\hat{\boldsymbol{\xi}}_t = \mathbf{H}\mathbf{y}_t$ is linear function of r_t.
Claim: a monetary policy shock does not affect "slow-moving variables" (wages, prices) in the current month.

$\mathbf{y}_t^*$ = subset of $\mathbf{y}_t$ that is
"slow-moving"
$\hat{\boldsymbol{\xi}}_t$ = first r PC of $\mathbf{y}_t$
$\hat{\boldsymbol{\xi}}_t^*$ = first r PC of $\mathbf{y}_t^*$

(1) regress $\hat{\xi}_{it} = \boldsymbol{\beta}_i'\hat{\boldsymbol{\xi}}_t^* + \alpha_i r_t + e_{it}$
for $i = 1, \ldots, r$
(2) Calculate $\tilde{\xi}_{it} = \hat{\xi}_{it} - \hat{\alpha}_i r_t$
(3) Estimate VAR for $\tilde{\mathbf{x}}_t = (\tilde{\boldsymbol{\xi}}_t', r_t)'$
$\tilde{\mathbf{x}}_t = \boldsymbol{\Phi}(L)\tilde{\mathbf{x}}_t + \boldsymbol{\varepsilon}_t$

(4) Calculate nonorthogonalized impulse-response function
$\boldsymbol{\Psi}(L) = [\mathbf{I}_{r+1} - \boldsymbol{\Phi}(L)]^{-1}$
$\boldsymbol{\Psi}_s = \frac{\partial \tilde{\mathbf{x}}_{t+s}}{\partial \boldsymbol{\varepsilon}_t'}$
and Cholesky factorization
$\hat{\boldsymbol{\Omega}} = T^{-1}\sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t' = \hat{\mathbf{P}}\hat{\mathbf{P}}'$

(5) Effect of monetary policy shock (u_t^M) on $\tilde{\mathbf{x}}_{t+x}$ is

$$\frac{\partial \tilde{\mathbf{x}}_{t+s}}{\partial u_t^M} = \hat{\boldsymbol{\Psi}}_s \hat{\mathbf{p}}_{r+1}$$

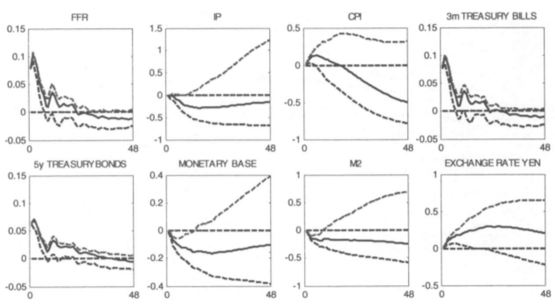
for $\hat{\mathbf{p}}_{r+1}$ the last column of $\hat{\mathbf{P}}$

(6) Since $\underset{(n\times 1)}{\mathbf{y}_t} = \underset{(n\times r)}{\mathbf{H}_\xi} \underset{(r\times 1)}{\tilde{\boldsymbol{\xi}}_t} + \underset{(n\times 1)}{\mathbf{h}_r} r_t,$

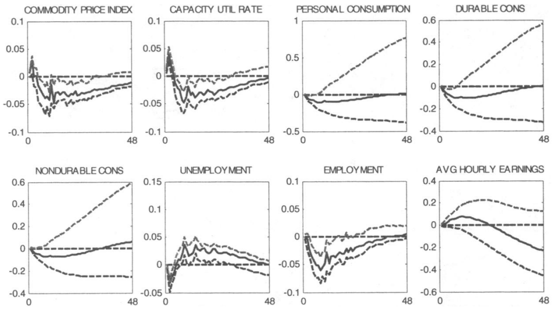
effect of monetary policy on any variable is

$$\frac{\partial \mathbf{y}_{t+s}}{\partial u_t^M} = \begin{bmatrix} \mathbf{H}_\xi & \mathbf{h}_r \end{bmatrix} \hat{\boldsymbol{\Psi}}_s \hat{\mathbf{p}}_{r+1}$$

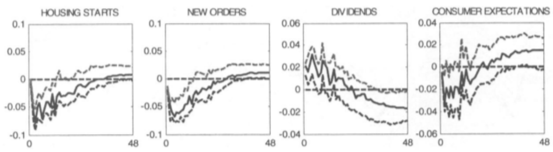
Effects of monetary policy shock



Effects of monetary policy shock



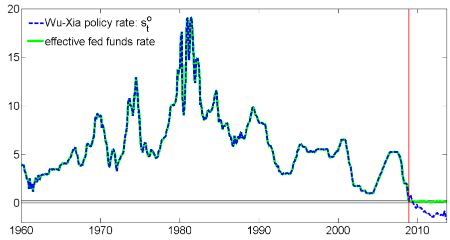
Effects of monetary policy shock



But how do we update this approach now that fed funds rate is stuck at zero?

- Fischer Black, Journal of Finance, 1995: Can think of latent or shadow short rate that is allowed to be negative
- Actual short rate is maximum of this and (say) 0.25
- Wu and Xia (UCSD, 2013) develop convenient algorithm to calculate shadow rate

Wu and Xia fail to reject hypothesis that FAVAR coefficients since 2009 are same as those using fed funds for historical data



Data available at http://econweb.ucsd.edu/~faxia/PolicyRate.html