

Answers to midterm exam

1a.)  $\mathbf{x}_{t-1} = (\mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \dots, \mathbf{y}'_{t-p}, 1)'$ ,  $k = np + 1$

b.)  $\mathbf{\Omega}$  is  $(n \times n)$ .  $\varepsilon_{1t}$  is the error we would make in forecasting  $y_{1t}$  on the basis of a linear function of  $\mathbf{x}_{t-1}$  and the (1,1) element of  $\mathbf{\Omega}$  is the variance of this forecast error.

c.) The prior for the first element of  $\mathbf{b}_i$  has variance  $\omega_{ii}m_{11}$ . Bigger values of  $m_{11}$  correspond to less confidence in this prior. If we had an earlier sample  $m_{11}$  would represent the (1,1) element of  $\left(\sum_{\tau=1}^{\tilde{T}} \mathbf{x}_{\tau-1}\mathbf{x}'_{\tau-1}\right)^{-1}$  for this sample.

d.)  $\mathbf{m}_1 = (1, 0, 0, \dots, 0)'$

e.) Advantages: (1) if based on an earlier sample would have this form; (2) if prior is in this class, then so is the posterior; (3) if prior is in this class, then posterior is known analytically. Disadvantages: Assumes that the ratio of the variance of my priors for the first to the second elements of  $\mathbf{b}_1$  is proportional to the ratio of the variance for the first and second elements of  $\mathbf{b}_2$ . This rules out for example the variances recommended by the Minnesota prior.

2a.)  $h(\boldsymbol{\theta}) = p(\mathbf{Y}|\boldsymbol{\theta})p(\boldsymbol{\theta})$

b.)

$$\hat{E}(\boldsymbol{\theta}|\mathbf{Y}) = \frac{\sum_{m=1}^M \boldsymbol{\theta}^{(m)} \omega^{(m)}}{\sum_{m=1}^M \omega^{(m)}}$$

$$\omega^{(m)} = \frac{h(\boldsymbol{\theta}^{(m)})}{g(\boldsymbol{\theta}^{(m)})}$$

c.)  $E(\boldsymbol{\theta}|\mathbf{Y}) = \int_{\mathbb{R}} \boldsymbol{\theta} p(\boldsymbol{\theta}|\mathbf{Y}) d\boldsymbol{\theta}$  exists,  $p(\boldsymbol{\theta}|\mathbf{Y}) = k p(\mathbf{Y}|\boldsymbol{\theta}) p(\boldsymbol{\theta})$ , and support of  $g(\boldsymbol{\theta})$  includes  $\mathbb{R}$

d.) No, there are no initial conditions or serial dependence of this procedure

e.) (1) Make sure get the same answer when  $M$  increases; (2) make sure get the same answer when use different  $g(\cdot)$ ; (3) Try on special case where answer is known analytically.