Econ 220B, Winter 2020

Problem Set 3
Due Thursday January 30

Let $\{y_t\}_{t=1}^T$ be an i.i.d. $N(0, \sigma^2)$ sequence. Consider S_T, the average squared value of $y : S_T = T^{-1} \sum_{t=1}^T y_t^2$.

1.) Show that $\sqrt{T}(S_T - \sigma^2) \xrightarrow{L} N(0, 2\sigma^4)$. State the particular form of the central limit theorem that you used to get this result, and show that the assumptions of this form of the theorem are satisfied by this example. (Hint: note that y_t^2 is also i.i.d. with $E(y_t^2) = \sigma^2$ and $E(y_t^4) = 3\sigma^4$).

2.) Suppose you have a sample of size $T = 100$ observations and want to test the null hypothesis that $\sigma = 2$. Based on the asymptotic approximation in question (1), what is the probability of observing a value greater than or equal to $S_T = 5$ if the true value of $\sigma = 2$? If you observe that $S_T = 5$, should you reject the null hypothesis?

3.) Calculate the asymptotic distribution of $\ln(S_T)$.

4.) Suppose you used the result from question (3) to test the null hypothesis that $\ln(\sigma^2) = \ln(4)$, and observed a value of $\ln(S_T) = \ln(5)$ with $T = 100$. Based on the asymptotic approximation in question (3), what is the probablity of observing a value this large or lager under the null hypothesis? Is this the same answer as in question (2)? Why or why not?

5.) Calculate the asymptotic distribution of $T(S_T - \sigma^2)^2$.

6.) Suppose you used the result from question (5) to test the null hypothesis that $\sigma^2 = 4$, given again an observation of $S_T = 5$ in a sample of size $T = 100$. What is the probability of observing a value this large or larger under the null hypothesis? Is this the same answer as in questions (2) and (4)? Why or why not?

7.) Can you suggest an exact small-sample test of the hypothesis that $\sigma = 2$? Does it accept or reject the null hypothesis when $T = 100$ and $S_T = 5$?