Econ 220B Final Exam
Winter 2018

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam.

1.) (10 points) Consider the regression model $y_t = \mathbf{x}_t'\boldsymbol{\beta} + u_t$. Let $\mathbf{b}$ denote the $(k \times 1)$ OLS coefficient. Under what conditions would $\sum_{t=1}^{T}(y_t - \mathbf{x}_t'\mathbf{b})\mathbf{x}_t = \mathbf{0}$?

2..) (70 points total) This question concerns the following model of supply and demand:

$$q_t = \beta p_t + u_t^d \qquad \text{(demand equation)}$$

$$q_t = \gamma p_t + u_t^s \qquad \text{(supply equation).}$$

You can assume that the vector $u_t = (u_t^d, u_t^s)'$ is independent and identically distributed over time and has mean 0.

a.) (10 points) In equilibrium the value of q_t in the first equation must equal the value of q_t in the second equation. Use that fact to solve for q_t and p_t as functions of u_t.

b.) (20 points) A researcher claims to know before seeing the data that the value of the supply elasticity γ is exactly equal to γ_0. The researcher further claims that this means that the variable $q_t - \gamma_0 p_t = u_t^s$ can be used as an instrument for p_t to estimate the demand elasticity β. Assuming that the researcher is correct that γ_0 is a known value, calculate the additional conditions under which the proposed instrument would be valid and relevant. Note: do not give general statements of the definition of instrument validity and relevance. Instead explain exactly what instrument validity and relevance require in terms of this particular proposed instrument in this particular model.

c.) (20 points) Under the assumptions you stated in (b), find the asymptotic distribution of the instrumental-variable estimate of β. Did you need any additional assumptions beyond these conditions and the assumptions already given to derive this distribution?

d.) (20 points) Describe how you could test the null hypothesis $H_0 : \beta = 0$ using the Anderson-Rubin test and the proposed instrument u_t^s.

3.) (150 points total) Consider the regression model

$$y_t = x_t\beta + u_t$$

in which a scalar $x_t \sim N(0, Q)$ determines the mean of y_t and a separate Bernouli variable z_t determines the variance. The variable $z_t = 1$ with probability π and $z_t = 0$ with probability $1 - \pi$, and

$$E(u_t^2|z_t = 1) = \sigma_1^2$$
$$E(u_t^2|z_t = 0) = \sigma_0^2.$$

Note these assumptions imply that u_t has unconditional variance

$$E(u_t^2) = \pi\sigma_1^2 + (1 - \pi)\sigma_0^2.$$

The variables x_t and u_s are i.i.d. and independent of each other for all t and s. The model is conditionally Normal,

$$y_t|x_t, z_t \sim \begin{array}{ll} N(x_t\beta, \sigma_1^2) & \text{when } z_t = 1 \\ N(x_t\beta, \sigma_0^2) & \text{when } z_t = 0 \end{array},$$

with conditional log likelihood

$$\begin{aligned} \ell &= -(T/2)\log(2\pi) - (T_0/2)\log\sigma_0^2 - (T_1/2)\log\sigma_1^2 \\ &\quad -\frac{\sum_{t=1}^T (y_t - x_t\beta)^2 z_t}{2\sigma_1^2} - \frac{\sum_{t=1}^T (y_t - x_t\beta)^2(1 - z_t)}{2\sigma_0^2} \end{aligned}$$

where $T_1 = \sum_{t=1}^T z_t$ is the number of observations for which $z_t = 1$ and $T_0 = \sum_{t=1}^T (1 - z_t)$ is the number of observations for which $z_t = 0$, where $T = T_0 + T_1$.

If you can't answer (a), try (b) and come back to (a) later if you have time. You may be able to answer (e) even if you can't get anywhere with (a)-(d) and you may be able to answer (g) even if you can't get anywhere with (a)-(f).

a.) (20 points) The OLS estimate is given by $b = (\sum_{t=1}^T x_t^2)^{-1}\left(\sum_{t=1}^T x_t y_t\right)$. Calculate the asymptotic distribution of b under the assumptions stated.

b.) (30 points) Suppose you know the values of σ_0 and σ_1 and do not need to estimate them from the data. Find the value of β that maximizes ℓ given σ_0 and σ_1. By what name (other than MLE) is this particular estimator commonly referred to?

c.) (20 points) Calculate the asymptotic distribution of the estimate $\hat{\beta}_{MLE}$ that you derived in (b).

d.) (10 points) Compare your answers to (a) and (c) and give the intuition behind that finding. Hint: Jensen's Inequality implies that

$$\frac{\pi}{\sigma_1^2} + \frac{1-\pi}{\sigma_0^2} > \frac{1}{\pi\sigma_1^2 + (1-\pi)\sigma_0^2}$$

from which

$$\left[\frac{\pi}{\sigma_1^2} + \frac{1-\pi}{\sigma_0^2}\right][\pi\sigma_1^2 + (1-\pi)\sigma_0^2] > 1.$$

e.) (20 points) Calculate the second derivative

$$H = \left.\frac{\partial^2 \ell}{\partial \beta^2}\right|_{\beta=\hat{\beta}_{MLE}}$$

and find the plim of $T^{-1}H$. Compare your answers to (c) and (e) and explain the intuition behind that finding.

f.) (30 points) Now suppose that you do not know that value of σ_0 and σ_1 but have to estimate them along with β by maximizing ℓ jointly with respect to β, σ_1^2, and σ_0^2. Write an equation charaterizing the MLE of σ_1^2 and show that $\hat{\sigma}_1^2 \xrightarrow{p} \sigma_1^2$ under the assumptions stated.

g.) (20 points) You realize that you might not have to bother trying to estimate σ_1 in step (e) if the null hypothesis $H_0 : \sigma_0^2 = \sigma_1^2$ is true, and that there is a simple test of H_0 based on looking at T times the R^2 of a certain regression. Describe what the regression is whose R^2 you would look at and exactly how you would decide from the value of TR^2 whether to reject H_0.

4.) (20 points total) Let $\mathbf{y}_t$ be an $(n \times 1)$ vector of covariance-stationary variables observed at date t with $(n \times 1)$ population mean vector $\boldsymbol{\mu} = E(\mathbf{y}_t)$ and $(n \times 1)$ sample mean $\bar{\mathbf{y}} = T^{-1}\sum_{t=1}^{T} \mathbf{y}_t$ for a sample of size T. You believe that the value of $\mathbf{y}_t$ may be serially correlated. Write the equations for a) an estimate you might use of the variance of $\mathbf{y}_t$ and b) an estimate you might use for the variance of $\bar{\mathbf{y}}$.

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