Econ 220B Final Exam
Winter 2016

DIRECTIONS: No books or notes of any kind are allowed. 250 points are possible on this exam.

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1.) (30 points possible) Consider the following regression model

$$y_t = x_t\beta + u_t \qquad t = 1, ..., T$$

where x_t is a scalar. Describe how you would implement White's R^2 test of the null hypothesis of homoskedastic residuals.

2.) (20 points possible) Comment on the following statement:

> Economists often overlook the fact that a constant term $x_t = 1$ makes a perfectly good instrument. As long as the residual u_t has mean zero and the explanatory variable z_t does not have mean zero, the instrument $x_t = 1$ is both valid and relevant.

3.) (110 points possible) This question concerns the following regression model

$$y_t = z_t\beta + u_t \qquad t = 1, 2, ..., T.$$

Here z_t is a scalar and the (2×1) vector $(z_t, u_t)'$ is stationary and ergodic with second moment matrix given by

$$\begin{bmatrix} E(z_t^2) & E(z_t u_t) \\ E(u_t z_t) & E(u_t^2) \end{bmatrix} = \begin{bmatrix} \sigma_{zz} & \sigma_{zu} \\ \sigma_{uz} & \sigma_{uu} \end{bmatrix}$$

with $\sigma_{zu} > 0$.

a.) (30 points) Write down the formula for the OLS estimate of β and calculate its plim. Describe in words the problem with using this as an estimate of β.

b.) (30 points) Suppose you have a scalar instrument x_t. Write down the definitions of what it means for x_t to be valid and relevant, and give the 2SLS estimate of β using x_t as an instrument.

c.) (15 points) Describe how you would implement the 2SLS t-test of the null hypothesis that $\beta = 0$.

d.) (20 points) Describe how you would implement the Anderson-Rubin test of the null hypothesis that $\beta = 0$. What is the main advantage of the Anderson-Rubin test over the 2SLS test?

e.) (15 points) Can you suggest a way of implementing the Anderson-Rubin test in (d) that would also be robust to conditional heteroskedasticity, that is, that would have the correct asymptotic size even if $E(u_t^2|x_t) \neq E(u_t^2)$?

4.) (90 points total) If $E[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1}\sum_{t=1}^{T}\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

and $\hat{\mathbf{S}}$ is a consistent estimate of

$$\mathbf{S} = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)]\left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where $\mathbf{V}$ can be estimated consistently from

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}\hat{\mathbf{S}}^{-1}\hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

In this question you are asked to use the GMM framework to explore the properties of maximum likelihood estimation for lognormally distributed random variables.

This question asks you to consider a sample of size T of positive random variables (denoted $x_1, x_2, ..., x_T$) that are independently and identically distributed from a standard lognormal distribution. A random variable x_t is said to have a standard lognormal distribution if its density is given by

$$f(x_t) = \begin{cases} \frac{1}{x_t\sqrt{2\pi}} \exp\left[-\frac{(\log x_t - \mu)^2}{2}\right] & \text{if } x_t > 0 \\ 0 & \text{if } x_t \leq 0. \end{cases} \qquad (R1)$$

Here are the mean, variance, and some other facts about the standard lognormal distribution that you may find useful in answering the questions below:

$$E(x_t) = \exp(\mu + 0.5) \quad (R2)$$

$$E[x_t - E(x_t)]^2 = (e-1)\exp(2\mu + 1) \quad (R3)$$

$$\log x_t \sim N(\mu, 1)$$

$$e = 2.781828...$$

a.) If you were to use the GMM principle to form an estimator of μ, one option would be to use the mean ($R2$) above as the basis for estimation, that is, to set $h(\boldsymbol{\theta}, \mathbf{w}_t) = \exp(\mu + 0.5) - x_t$.

i.) (15 points) For this specification of $h(.)$, use the GMM formulas on the previous page to find the expression for $\hat{\mu}_{GMM}$.

ii.) (15 points) Calculate the values of S and D that are implied by this specification of $h(.)$.

iii.) (10 points) Use the results from (a.ii) to find the asymptotic distribution of $\hat{\mu}_{GMM}$.

b.) An alternative way to apply GMM is to use the score of the likelihood function.

i.) (10 points) Use ($R1$) to calculate the score of the tth observation and show that it indeed has expectation zero.

ii.) (10 points) Find the values of S and D that are implied by using the score statistic as $h(.)$.

iii.) (10 points) Use the results from (b.ii) to find the asymptotic distribution of $\hat{\mu}_{MLE}$.

c.) (20 points) Use results (a.iii) and (b.iii) to show that $\hat{\mu}_{MLE}$ has a smaller asymptotic variance than $\hat{\mu}_{GMM}$. Is that a result you'd expect to find in general or is there something very special about this example that produced it?

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