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Quarter____________ Course___________________ Date_______________

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Econ 220B Final Exam
Winter 2015

DIRECTIONS: No books or notes of any kind are allowed. 250 points are possible on this exam.

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1.) (20 points total) A certain economist was interested in whether encouraging birth control could be a policy that could raise a country's standard of living. She assembled a data set on $n = 98$ different countries, where y_i is the log of GDP per capita in country i in 1985 and x_i is country i's average population growth rate over 1960-1980. Here are the results of her OLS regression, with standard errors in parentheses:

$$y_i = \underset{(0.27)}{8.82} - \underset{(11.6)}{35.8}x_i + e_i.$$

a.) (10 points) Do you see any reason to be concerned about correlation between x_i and the error term? *Briefly* explain what you see as the biggest concern and in which direction this would tend to bias the coefficient on x_i.

b.) (10 points) Can you suggest a better approach to estimating the effect that the researcher was interested in?

2.) (60 points total) Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0},\sigma^2\mathbf{I}_T)$$

where $\mathbf{y}$ is a $(T \times 1)$ vector, $\mathbf{X}$ is a $(T \times 2)$ matrix, and $\boldsymbol{\beta}$ is a (2×1) vector.

a.) (20 points) Write down expressions for the OLS estimates of $\boldsymbol{\beta}$ and σ^2.

b.) (20 points) Suppose you want to test a nonlinear hypothesis that the first element of $\boldsymbol{\beta}$ is equal to the square of the second:

$$H_0 : \beta_1 = \beta_2^2.$$

Use the delta method to suggest a t statistic you could use to test the above null hypothesis. What is the distribution of this statistic under the null hypothesis? Hint: if you're not sure how to answer this, describe the test of the null hypothesis $H_0 : \beta_1 = 2\beta_2$ for partial credit).

c.) (20 points) Suggest an alternative way you could test the null hypothesis $H_0 : \beta_1 = \beta_2^2$ based on a likelihood ratio test. You do not have to find a closed-form expression for the test statistic. Just describe in words how you would calculate it and how you would decide whether or not to reject H_0 based on what you found.

3.) (65 points total) Consider the regression model

$$y_t = \mathbf{z}_t'\boldsymbol{\beta} + u_t$$

where $\mathbf{z}_t$ is a $(k \times 1)$ vector and $\mathbf{x}_t$ is an $(r \times 1)$ vector of instruments. Suppose that the vector $(\mathbf{x}_t', \mathbf{z}_t')'$ is stationary and ergodic, that $u_t\mathbf{x}_t$ is a martingale difference sequence, and that

$$E(u_t^2\mathbf{x}_t\mathbf{x}_t') = \mathbf{S}$$

$$E\begin{bmatrix} \mathbf{x}_t\mathbf{x}_t' & \mathbf{x}_t\mathbf{z}_t' \\ \mathbf{z}_t\mathbf{x}_t' & \mathbf{z}_t\mathbf{z}_t' \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{xx}} & \boldsymbol{\Sigma}_{\mathbf{xz}} \\ \boldsymbol{\Sigma}_{\mathbf{zx}} & \boldsymbol{\Sigma}_{\mathbf{zz}} \end{bmatrix}.$$

The 2SLS estimator is given by

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{2SLS} &= \left[\left(\textstyle\sum_{t=1}^T \mathbf{z}_t\mathbf{x}_t'\right)\left(\textstyle\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(\textstyle\sum_{t=1}^T \mathbf{x}_t\mathbf{z}_t'\right)\right]^{-1} \times \\ &\quad \left[\left(\textstyle\sum_{t=1}^T \mathbf{z}_t\mathbf{x}_t'\right)\left(\textstyle\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(\textstyle\sum_{t=1}^T \mathbf{x}_t y_t\right)\right]. \end{aligned}$$

a.) (25 points) What does it mean to say that $\hat{\boldsymbol{\beta}}_{2SLS}$ is a consistent estimate of $\boldsymbol{\beta}$? What assumptions in addition to those stated above (if any) would you need to prove that $\hat{\boldsymbol{\beta}}_{2SLS}$ is consistent? Prove that under these assumptions $\hat{\boldsymbol{\beta}}_{2SLS}$ is consistent.

b.) (25 ponts) What assumptions in addition to those stated above (if any) would you need to calculate the asymptotic distribution of $\hat{\boldsymbol{\beta}}_{2SLS}$? Calculate the asymptotic distribution under these assumptions.

c.) (15 points) Suggest an estimate you might use for the variance matrix that appeared in your answer to (b).

4.) (105 points total) If $E[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1} \sum_{t=1}^{T} \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

and $\hat{\mathbf{S}}$ is a consistent estimate of

$$\mathbf{S} = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} \sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] \left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where $\mathbf{V}$ can be estimated consistently from

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}\hat{\mathbf{S}}^{-1}\hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}}.$$

In this question you are asked to use the GMM framework to explore the properties of maximum likelihood estimation for exponentially distributed random variables.

a.) (15 points) Suppose you have an i.i.d. sample of observations y_t drawn from the density $f(y_t;\lambda) = \lambda \exp(-\lambda y_t)$ for $y_t \geq 0$, which is known as an exponential distribution. Recall that the mean of an exponential variable is given by $E(y_t) = 1/\lambda$ and variance is $E(y_t - \lambda^{-1})^2 = \lambda^{-2}$. Calculate the log likelihood function.

b.) (15 points) Calculate the maximum likelihood estimate $\hat{\lambda}$.

c.) (15 points) If you wanted to view maximum likelihood estimation for this example as a special case of GMM, what corresponds to the function $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$?

d.) (20 points) Find the value of $\mathbf{S}$ and $\hat{\mathbf{D}}'$ for this example.

e.) (20 points) Use the expressions you found in part (d) to calculate an estimate of the asymptotic variance of $\hat{\lambda}$.

f.) (20 points) Suppose now that you suspect there is serial correlation of the form

$$E(y_t - \mu)(y_{t-v} - \mu) = \begin{cases} \gamma_0 & \text{for } v = 0 \\ \gamma_1 & \text{for } v = 1 \\ 0 & \text{for } v = 2, 3, ... \end{cases}$$

where you do not know the values for γ_0 and γ_1 but would have to estimate them somehow from the data. Describe how you would modify your answer to (e) for this case.