Econ 220B Final Exam
Winter 2011

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam.

1.) (85 points total) Consider the following regression model:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \mathbf{V})$$

$$\mathbf{V} = \begin{bmatrix} \exp(\gamma x_1) & 0 & \cdots & 0 \\ 0 & \exp(\gamma x_2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \exp(\gamma x_T) \end{bmatrix}.$$

Here γ is a known scalar and α and β must be estimated from the data.

a.) (20 points) Give an expression for the OLS estimate of α and β. Is this estimate unbiased under the conditions stated? (Note: you do not have to prove whether it is unbiased, just answer yes or no).

b.) (20 points) Give an expression for the GLS estimate of α and β. Is this estimate unbiased under the conditions stated? (Note: you do not have to prove whether it is unbiased, just answer yes or no).

c.) (10 points) What are the main advantages of GLS relative to OLS for this particular example?

d.) (15 points) Suggest a statistic you could use to test the hypothesis that $\beta = 0$ and state its distribution. Is that distribution exact or only an approximation? (Again, you do not have to derive the distribution, only state what it is)

e.) (20 points) Suppose now that you don't know the true value of γ but would have to estimate it along with α and β. Give specific details of how you would go about estimating α, β, and γ in this case. Suggest the statistic that you could use to test the hypothesis that $\beta = 0$ in this case and state its distribution. Is that distribution exact or only an approximation?

2.) (65 points total) Consider the following regression model. There is a true relation of interest between y_t and a variable z_t^*:

$$y_t = \alpha + \beta z_t^* + \varepsilon_t.$$

However, z_t^* is not observed directly. Instead we have observations on z_t, which differs from z_t^* by a measurement error term u_t:

$$z_t = z_t^* + u_t.$$

We will assume that the vector $(z_t^*, \varepsilon_t, u_t)'$ is a stationary ergodic martingale difference sequence with

$$E\left(\begin{bmatrix} z_t^* \\ \varepsilon_t \\ u_t \end{bmatrix}\begin{bmatrix} z_t^* & \varepsilon_t & u_t \end{bmatrix}\right) = \begin{bmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & \sigma_u^2 \end{bmatrix}.$$

a.) (25 points) Suppose you did an OLS regression of y_t on the observed variable z_t:

$$y_t = \alpha + \beta z_t + v_t. \tag{1}$$

Write down the formula for the OLS estimates α and β and calculate their plims. (Hint: use the facts that $z_t = z_t^* + u_t$ and $y_t = \alpha + \beta z_t^* + \varepsilon_t$.)

b.) (20 points) Let x_t be another observed variable where $(x_t, z_t^*, \varepsilon_t, u_t)'$ is a stationary ergodic martingale difference sequence with

$$E\left(\begin{bmatrix} x_t \\ z_t^* \\ \varepsilon_t \\ u_t \end{bmatrix}\begin{bmatrix} x_t & z_t^* & \varepsilon_t & u_t \end{bmatrix}\right) = \begin{bmatrix} \sigma_x^2 & \sigma_{xz} & 0 & 0 \\ \sigma_{xz} & \sigma_z^2 & 0 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 & \sigma_u^2 \end{bmatrix}.$$

Show that x_t would be a valid instrument for purposes of estimating α and β on the basis of the regression (1). Under what conditions would x_t be a relevant instrument?

c.) (20 points) Show that 2SLS estimation of (1) using $(1, x_t)'$ as instruments leads to consistent estimates of $(\alpha, \beta)'$.

3.) (25 points total) An economist is studying a cross-section of different countries by using the following regression:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$$

$$y_i = \text{ average real GDP growth rate for country } i \text{ over 1980-2000}$$

$$x_{i1} = \text{ average investment spending as a fraction of GDP for country } i$$

$$x_{i2} = \text{measure of political stability for country } i$$

$$x_{i3} = \text{ measure of how democratic is country } i.$$

The economist is particularly interested in the value of β_3, hoping to find the answer to the question, if a country adopts more democratic institutions, how would that affect its growth rate?

a.) (15 points) Describe the most important reason, in your opinion, why the OLS regression estimate of β_3 might not give the answer to the question of interest. Which direction do you think the coefficient might be biased?

b.) (10 points) Even if the regression might not answering the question the economist is interested in, it is nevertheless answering some question. Explain what you think would be an appropriate interpretation of the plim of the regression as run.

4.) (75 points total) If $E[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T)]'\mathbf{S}^{-1}[\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T) = T^{-1}\sum_{t=1}^{T}\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

$$\mathbf{S} = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)]\left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The first-order conditions for this minimization are

$$\left[\frac{\partial \mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right]' \mathbf{S}^{-1}\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T) = \mathbf{0}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where

$$\mathbf{V} = (\mathbf{D}\mathbf{S}^{-1}\mathbf{D}')^{-1}$$

$$\mathbf{D}' = E\left\{\left.\frac{\partial \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}\right\}.$$

Hansen's J-statistic is given by

$$J = T[\mathbf{g}(\hat{\boldsymbol{\theta}}, \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\hat{\boldsymbol{\theta}}, \mathbf{Y}_T)]$$

for $\hat{\mathbf{S}}$ a consistent estimate of $\mathbf{S}$.

In this question you are asked to apply these results to a regression model of the following structure:

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$$

$$E(\mathbf{x}_t\varepsilon_t) = \mathbf{0}$$

$$E(\varepsilon_t\mathbf{x}_t\varepsilon_{t-v}\mathbf{x}_{t-v}') = \begin{cases} \boldsymbol{\Gamma}_0 & \text{if } v = 0 \\ \boldsymbol{\Gamma}_1 & \text{if } v = 1 \\ \mathbf{0} & \text{if } v = 2, 3, ... \end{cases}$$

Here $\boldsymbol{\beta}$ is an unknown $(k \times 1)$ vector that is to be estimated from the observed sample $\{y_t, \mathbf{x}_t\}_{t=1}^T$ and $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are known $(k \times k)$ matrices.

a.) (20 points) What are $\boldsymbol{\theta}$, $\mathbf{w}_t$ and $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$ for this example? What is the dimension of $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$?

b.) (15 points) Find a closed-form expression for the GMM estimate $\hat{\boldsymbol{\theta}}$.

c.) (15 points) Propose a consistent estimate of $\mathbf{V}$.

d.) (15 points) Use your results from (a) and (b) to suggest a statistic you could use to test the null hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ where $\mathbf{R}$ is a known $(m \times k)$ matrix and $\mathbf{r}$ is a known $(m \times 1)$ vector. What would be the asymptotic distribution of this test statistic? (Note: you do not need to derive the asymptotic distribution, just state what you expect it to be).

e.) (10 points) What would be the asymptotic distribution of Hansen's J statistic for this example? (Again, you do not need to derive the distribution, just state what you expect it to be).