Monetary policy at the zero lower bound: Theory

A. Theoretical channels

1. Conditions for complete neutrality (Eggertsson and Woodford, 2003)

2. Market frictions

3. Preferred habitat and risk-bearing (Hamilton and Wu, 2012)

B. Shadow rate (Wu and Xia, 2016)

C. Theoretical channels

1. Conditions for complete neutrality

Suppose preferences are

$$E_t\Big\{\sum_{\tau=0}^{\infty}\beta^{\tau}[U(C_{t+\tau},H_{t+\tau}/P_{t+\tau};\xi_{t+\tau}]$$
$$-\int_0^1 v(L_{t+\tau}(j,\xi_{t+\tau})dj\Big\}$$

$C_t = \left[\int_0^1 c_t(i)^{\theta/(\theta-1)}di\right]^{(\theta-1)/\theta}$ (real consumption)

$P_t = \left[\int_0^1 p_t(i)^{1-\theta}di\right]^{1/(1-\theta)}$ (Calvo sticky prices)

$H_t =$ nominal monetary base

$\xi_t =$ preference shocks

$y_t(i) = A_t f[L_t(i)]$

no investment, govt spending: $C_t = Y_t$

pricing kernel:

$$M_{t+1} = \frac{\beta U_c(Y_{t+1}, H_{t+1}/P_{t+1}, \xi_{t+1})}{U_c(Y_t, H_t/P_t, \xi_t)(1+\pi_{t+1})}$$

first-order condition:

$$\frac{U_h(Y_t, H_t/P_t, \xi_t)}{U_c(Y_t, H_t/P_t, \xi_t)} = \frac{r_t}{1+r_t}$$

r_t = risk-free nominal rate

Liquidity trap (zero lower bound):

There is a saturation level $\bar{h}(C, \xi)$

such that $U_h(C, h; \xi) = 0$ for all $h \geq \bar{h}(C, \xi)$.

Implies $r_t = 0$ whenever $H_t/P_t \geq \bar{h}(Y_t, \xi_t)$.

Define $L(Y_t, r_t; \xi_t) =$

$$\begin{cases} h_t : \dfrac{U_h(Y_t, h_t, \xi_t)}{U_c(Y_t, h_t, \xi_t)} = \dfrac{r_t}{1+r_t} & \text{if } r_t > 0 \\ \bar{h}(Y_t, \xi_t) & \text{if } r_t = 0 \end{cases}$$

Monetary policy rule in normal times:

$$r_t = \phi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$$

(Taylor type rule)

Monetary policy at the ZLB is choice for a rule $\psi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$ such that

$$H_t = P_t L(Y_t, \phi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t); \boldsymbol{\xi}_t)\psi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$$

$$\psi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t) \begin{cases} = 1 & \text{if } \phi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t) > 0 \\ \geq 1 & \text{otherwise} \end{cases}$$

where the excess reserves created by $\psi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$ are used by the central bank to purchase *any* assets.

Proposition: choice of $\psi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$ has no effect on nominal prices, inflation, interest rates, or real activity.

Reason: operations at the ZLB have no affect on the pricing kernel M_{t+1}.

Implication: all the Fed's operations in phase I and phase II were completely pointless.

How could Fed have an effect?

By changing $\phi(\pi_t, Y_t; \tilde{\boldsymbol{\xi}}_t)$, the rule it will use to set interest rates once we're away from the ZLB, that would alter behavior today.

But how do we do this in practice?

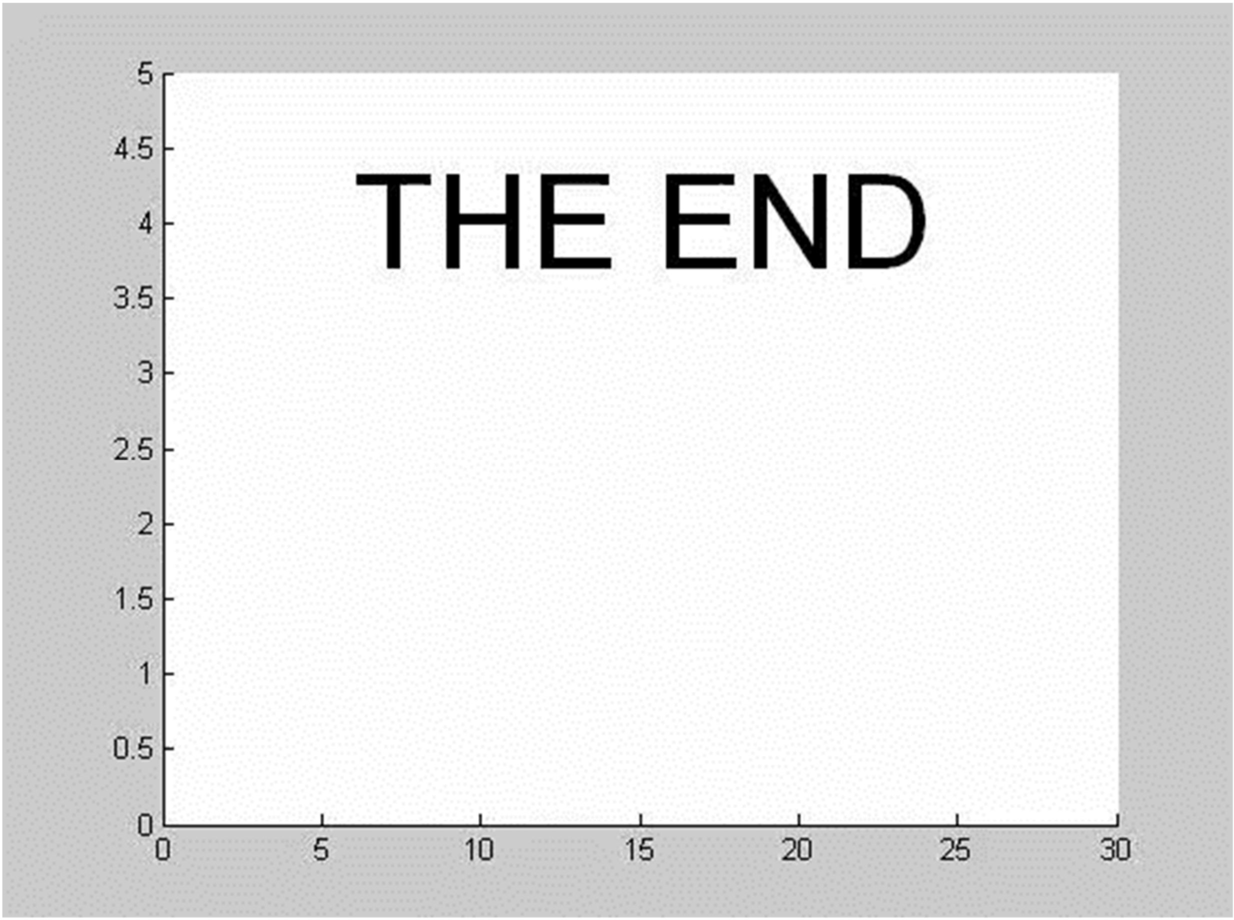
- In theoretical model, simply announce the change and it happens.
- In real world, perhaps LSAP or forward guidance persuade markets the Fed is really changing its future policy rule.
- For example, LSAP may alter state-contingent path of future tax receipts (Fed promises to monetize more of debt in some states)

C. Theoretical channels

2. Market frictions

- During financial crisis in fall of 2008, arbitrage broke down in some markets
- No pricing kernel existed

Gürkaynak and Wright, JELit, 2012, yield curve during 2008-10



Watch the movie at
http://www.econ.jhu.edu/People/Wright/loop_repealed.mpg

C. Theoretical channels

3. Preferred habitat and risk-bearing

Why does Treasury issue 10-year debt at 3% when it could borrow by rolling over 3-month debt at much lower rate?

Answer: Treasury is risk averse, and is willing to compensate government creditors for assuming this risk.

Consider representative "arbitrageur"

q_{t+1} = total return on portfolio

$\max E_t(q_{t+1}) - (\gamma/2)\text{Var}_t(q_{t+1})$

first-order condition:

$$r_{1t} = E_t(q_{n,t+1}) - \gamma\vartheta_{nt}$$

where r_{1t} = return on riskless asset

$q_{n,t+1}$ = 1-period-holding yield for asset n

ϑ_{nt} = (1/2) change in variance from one more unit of asset n

If portfolio allocates share z_{nt} to n-period pure discount bonds with maturity n and price P_{nt},

$$q_{n,t+1} = \frac{P_{n-1,t+1}}{P_{nt}} - 1$$

$$q_{t+1} = \sum_{n=1}^{N} z_{nt} q_{n,t+1}$$

Suppose we conjecture:

$$p_{nt} = \alpha_n + \boldsymbol{\beta}_n' \boldsymbol{\xi}_t$$

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

$$\mathbf{u}_{t+1} \sim \text{i.i.d. } N(\mathbf{0}, \mathbf{I}_r)$$

Then:

$$\frac{\partial E_t(q_{t+1})}{\partial z_{nt}} \simeq \alpha_{n-1} + \boldsymbol{\beta}_{n-1}'(\mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t)$$

$$- \alpha_n - \boldsymbol{\beta}_n'\boldsymbol{\xi}_t + (1/2)\boldsymbol{\beta}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\boldsymbol{\beta}_{n-1}$$

$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$$

$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}'_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$$

Let $\mathbf{q}_t = \boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$

$= (3 \times 1)$ vector of arbitrageur's risk exposure

$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}'_{n-1}\mathbf{q}_t$$

So expected excess return from increasing z_{nt} by one unit must be $\gamma\boldsymbol{\beta}'_{n-1}\mathbf{q}_t$

$\Rightarrow$ price of risk $\boldsymbol{\lambda}_t = \gamma\mathbf{q}_t = \gamma\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$

Suppose that:

- Arbitrageurs correspond to entire private sector
- U.S. Treasury debt is sole asset held by arbitageurs

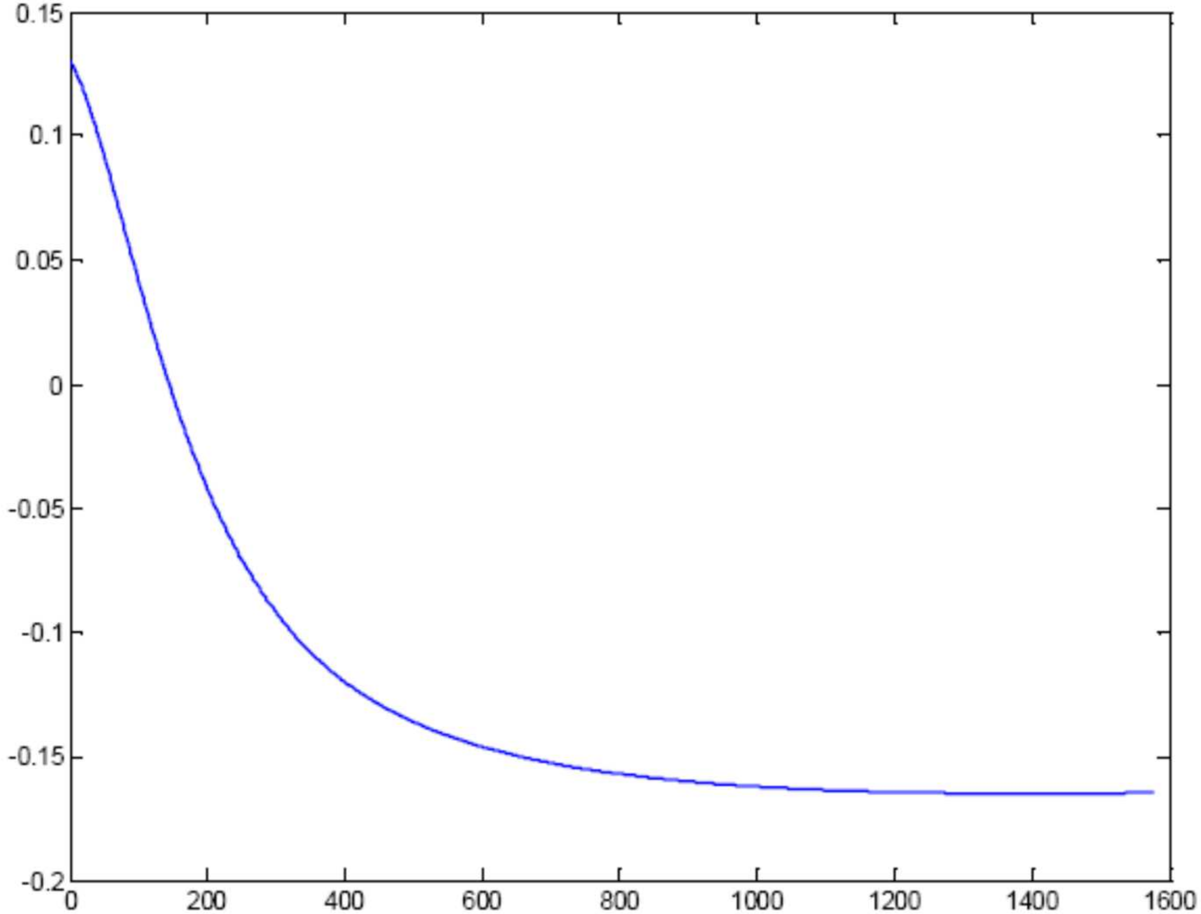
Then:

z_{nt} = share of publicly held Treasury debt of maturity n should enter as factor in risk pricing

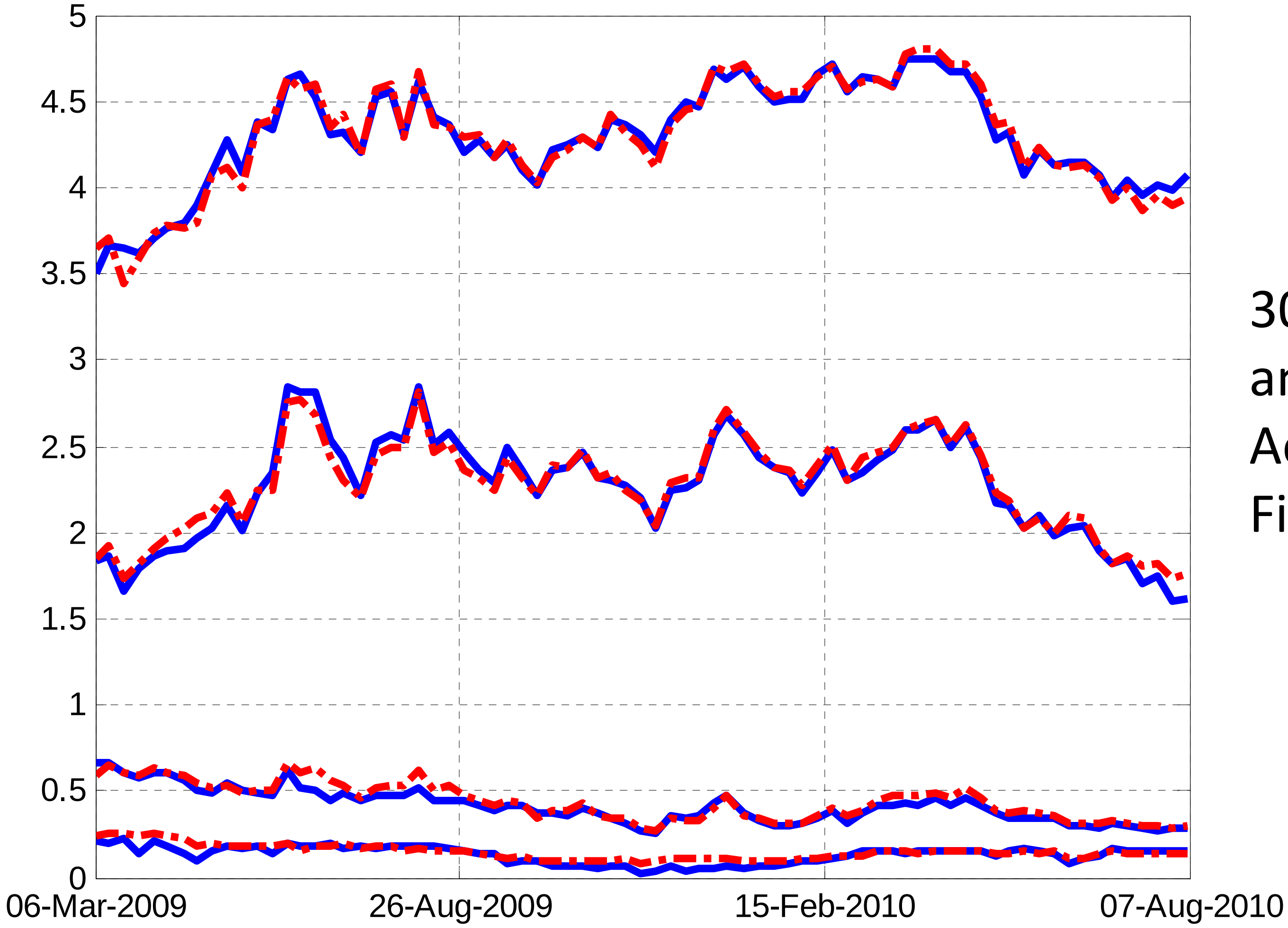
Hamilton and Wu (2012)

- Historical variations in composition of publicly held Treasury debt (as summarized by $\mathbf{q}_t$ as measured using 3-factor affine weights) were associated with detectable (but small) changes in predicted excess returns
- Total stock of Treasury securities 10 years or longer held by public in 2006 was about $400 B
- What would affine model incorporating Treasury debt holdings imply would happen to yields if supply of 3m Treasuries increased $400B and all long-term debt retired?

Hamilton-Wu estimates of effect on yield (in %) as function of maturity (in weeks)



How apply to ZLB?



30y, 5y, 1y,
and 3m yields
Actual (solid)
Fitted (dashed)

Long rates are still responding to daily news, short rates are not.

Interpretation: there are still some factors $\boldsymbol{\xi}_t$ changing daily that matter for long yields but not short.

Suppose latent factors following same dynamics as estimated historically:

$$\boldsymbol{\xi}_t = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_{t-1} + \boldsymbol{\Sigma}\mathbf{u}_t$$

Suppose once we escape from ZLB

all yields will behave same as before:

$$\tilde{p}_{nt} = \tilde{\alpha}_n + \tilde{\boldsymbol{\beta}}_n' \boldsymbol{\xi}_t$$

$$\{\tilde{\boldsymbol{\beta}}_1, \tilde{\boldsymbol{\beta}}_2, \ldots, \tilde{\boldsymbol{\beta}}_N, \tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_N\}$$

as historically estimated

But if we are at ZLB at date t,

$$y_{1t}^{*} = a_1^{*}$$

$$p_{nt}^{*} = \alpha_n^{*} + \boldsymbol{\beta}_n^{*\prime}\boldsymbol{\xi}_t$$

$$\boldsymbol{\beta}_1^{*} = \mathbf{0}$$

$\pi^Q = Q$-measure probability

that will escape ZLB next period

$$\exp(y_{1t}^*) = \pi^Q E_t^Q\left(\frac{\tilde{P}_{n-1,t+1}}{P_{nt}^*}\right) + (1-\pi^Q)E_t^Q\left(\frac{P_{n-1,t+1}^*}{P_{nt}^*}\right)$$

$$\tilde{p}_{n-1,t+1} = \tilde{\alpha}_{n-1} + \tilde{\boldsymbol{\beta}}_{n-1}'\boldsymbol{\xi}_{t+1}$$

$$p_{n-1,t+1}^* = \alpha_{n-1}^* + \boldsymbol{\beta}_{n-1}^{*\prime}\boldsymbol{\xi}_{t+1}$$

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

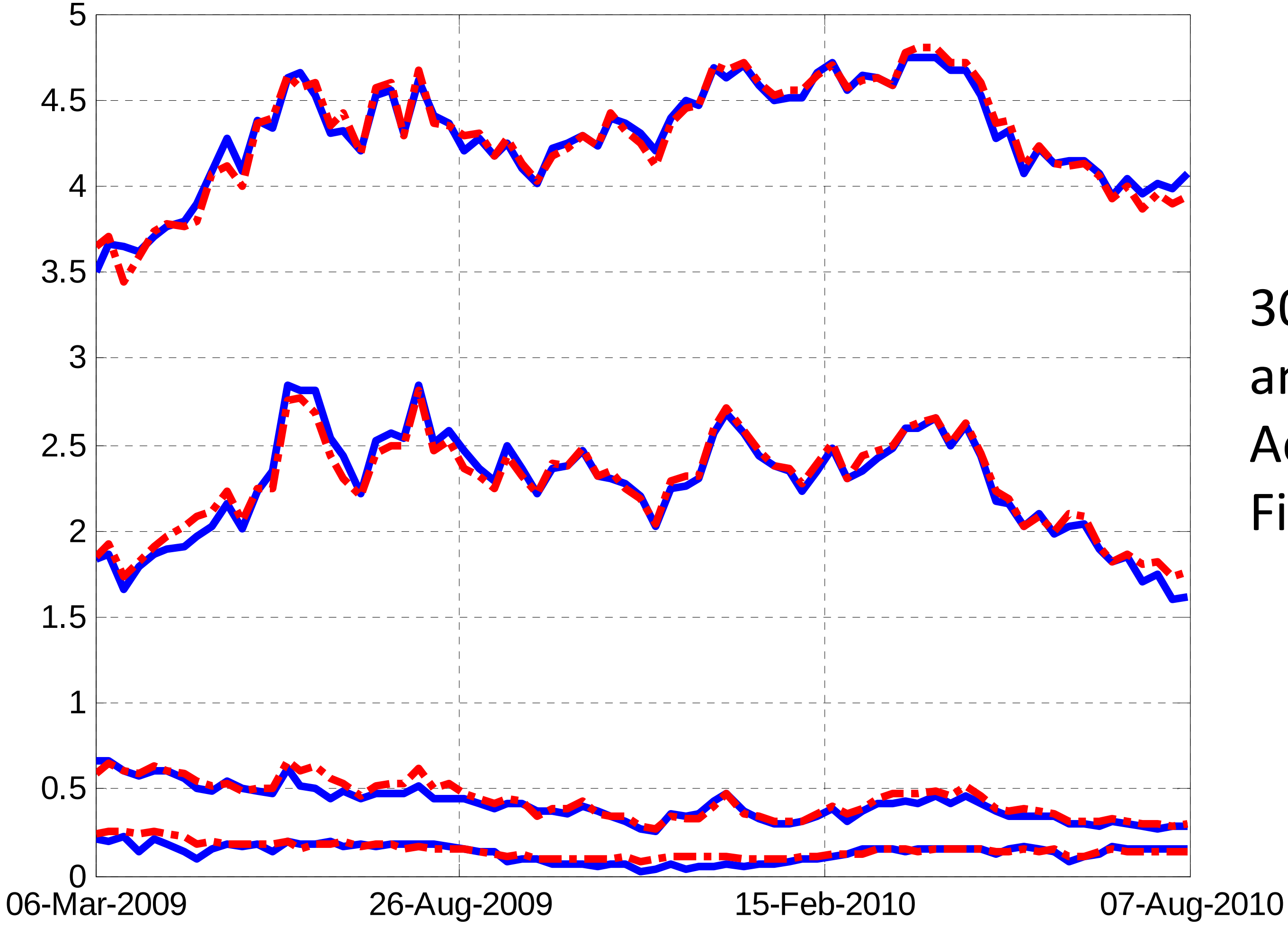
$\mathbf{c}, \boldsymbol{\rho}, \boldsymbol{\Sigma}$ and $\{\tilde{\alpha}_n, \tilde{\boldsymbol{\beta}}_n\}_{n=1}^N$ estimated

from pre-ZLB data.

Know $\boldsymbol{\beta}_1^* = \mathbf{0}$ by definition.

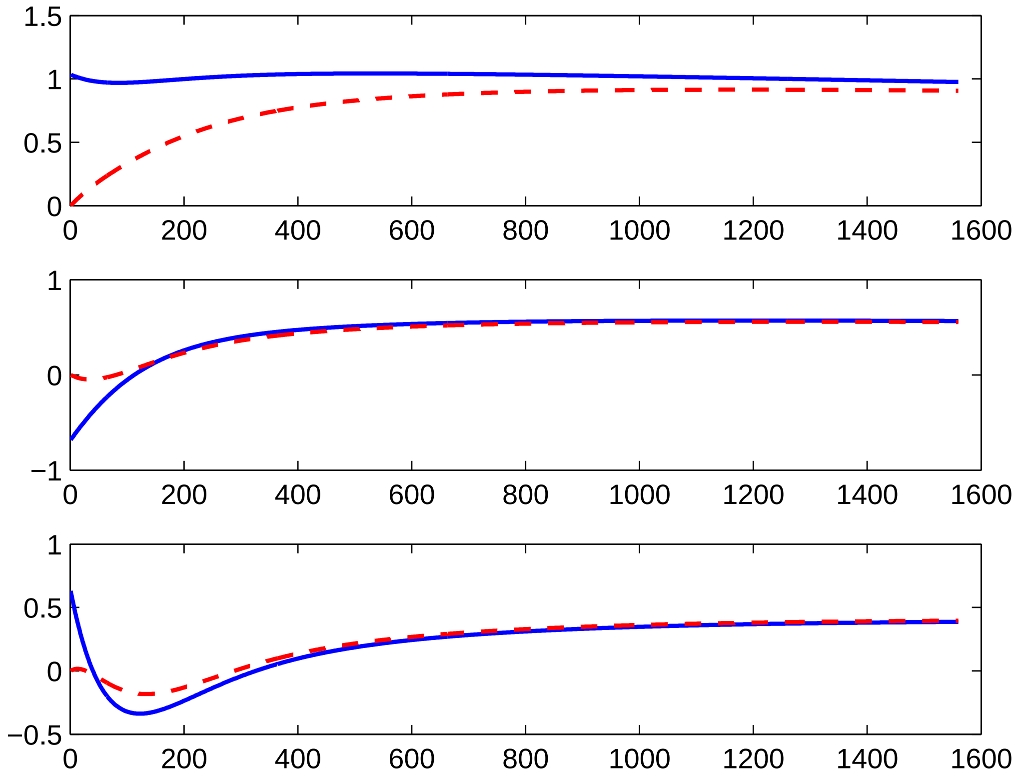
Can calculate $\{\alpha_n^*, \boldsymbol{\beta}_n^*\}_{n=2}^N$ as functions of α_1^* and π^Q.

Choose α_1^*, π^Q to fit ZLB data by MCS. Slightly better fit if also allow $\tilde{\alpha}_1$ (governs level interest rates return to once exit ZLB) to be different from historically estimated α_1.

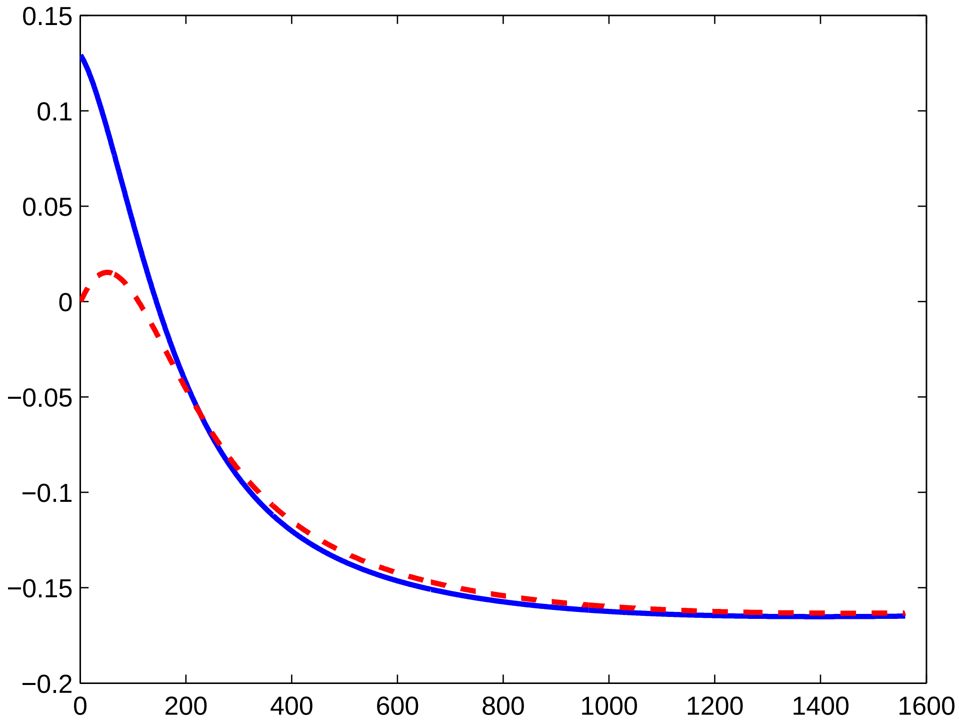


30y, 5y, 1y, and 3m yields
Actual (solid)
Fitted (dashed)

Factor loadings in normal times (solid blue) and at the ZLB (dashed red)



Effect of swap $400 B short-term for long-term in normal times (blue) and ZLB (red)



D. Shadow rate model (Wu and Xia, 2016)

Again assume there is a latent vector of factors that follow same dynamics in normal times as at ZLB:

P-measure:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

Q-measure:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c}^Q + \boldsymbol{\rho}^Q\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}^Q$$

$$\mathbf{c}^Q = \mathbf{c} - \boldsymbol{\Sigma}\boldsymbol{\lambda}$$

$$\boldsymbol{\rho}^Q = \boldsymbol{\rho} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}$$

s_t = shadow rate

$$= \delta_0 + \boldsymbol{\gamma}_0' \boldsymbol{\xi}_t$$

r_t = short-term interest rate

$$= \max\{s_t, \theta\}$$

$\theta = 0$ (or some positive lower bound)

$$P_{nt} = E_t^Q\{\exp[-(r_t + r_{t+1} + \cdots + r_{t+n-1})]\}$$

$$s_t = \delta_0 + \boldsymbol{\gamma}_0' \boldsymbol{\xi}_t$$

$$r_t = \max\{s_t, \theta\}$$

$$\Rightarrow p_{nt} = q_n(\boldsymbol{\xi}_t)$$

One approach: calculate $q_n(.)$ by simulation

Wu and Xia show an excellent approximation to the forward rate f_{nt} is given by

$$f_{nt} \simeq \theta + \sigma_n^Q g\left(\frac{\delta_n + \boldsymbol{\gamma}_n' \boldsymbol{\xi}_t - \theta}{\sigma_n^Q}\right)$$

$$g(z) = z\Phi(z) + \phi(z)$$

$\phi(z) = N(0,1)$ density

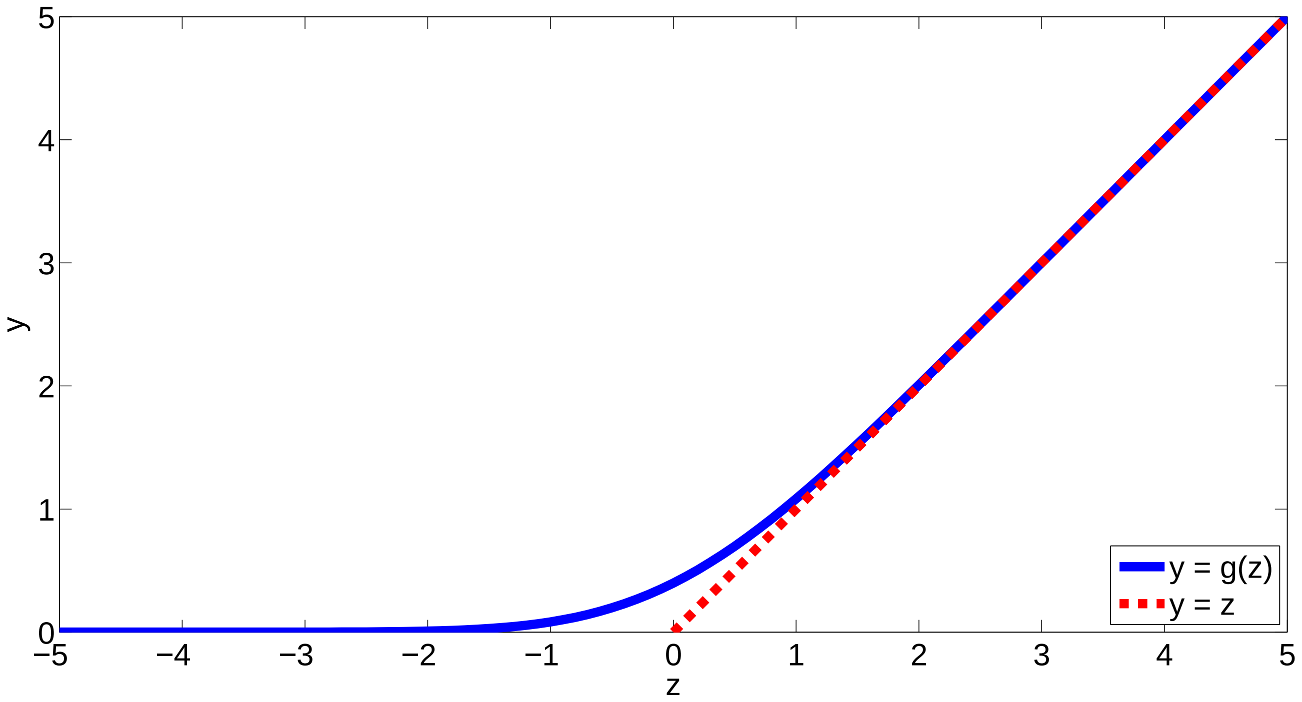
$\Phi(z) = N(0,1)$ cdf

$$\boldsymbol{\gamma}_n' = \boldsymbol{\gamma}_0' (\boldsymbol{\rho}^Q)^n$$

$$\sigma_n^2 = \sum_{j=0}^{n-1} \boldsymbol{\gamma}_0' (\boldsymbol{\rho}^Q)^j \boldsymbol{\Sigma\Sigma}' (\boldsymbol{\rho}^{Q\prime})^j \boldsymbol{\gamma}_0$$

$$\delta_n = \delta_0 + \boldsymbol{\gamma}_0' \mathbf{H}_n \mathbf{c}^Q - (1/2) \boldsymbol{\gamma}_0' \mathbf{H}_n \boldsymbol{\Sigma\Sigma}' \mathbf{H}_n' \boldsymbol{\gamma}_0$$

$$\mathbf{H}_n = \sum_{j=0}^{n-1} (\boldsymbol{\rho}^Q)^j$$



5
4
3
2
1
0
y
−5
−4
−3
−2
−1
0
1
2
3
4
5
z
y = g(z)
y = z

Estimate with nonlinear state-space model.

State equation:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

Observation equation:

$$\begin{bmatrix} f_{n_1,t} \\ \vdots \\ f_{n_N,t} \end{bmatrix} = \begin{bmatrix} \theta + \sigma_{n_1}^{Q} g\left(\frac{\delta_{n_1} + \boldsymbol{\gamma}_{n_1}' \boldsymbol{\xi}_t - \theta}{\sigma_{n_1}^{Q}}\right) \\ \vdots \\ \theta + \sigma_{n_N}^{Q} g\left(\frac{\delta_{n_N} + \boldsymbol{\gamma}_{n_N}' \boldsymbol{\xi}_t - \theta}{\sigma_{n_N}^{Q}}\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{n_1,t} \\ \vdots \\ \varepsilon_{n_N,t} \end{bmatrix}$$

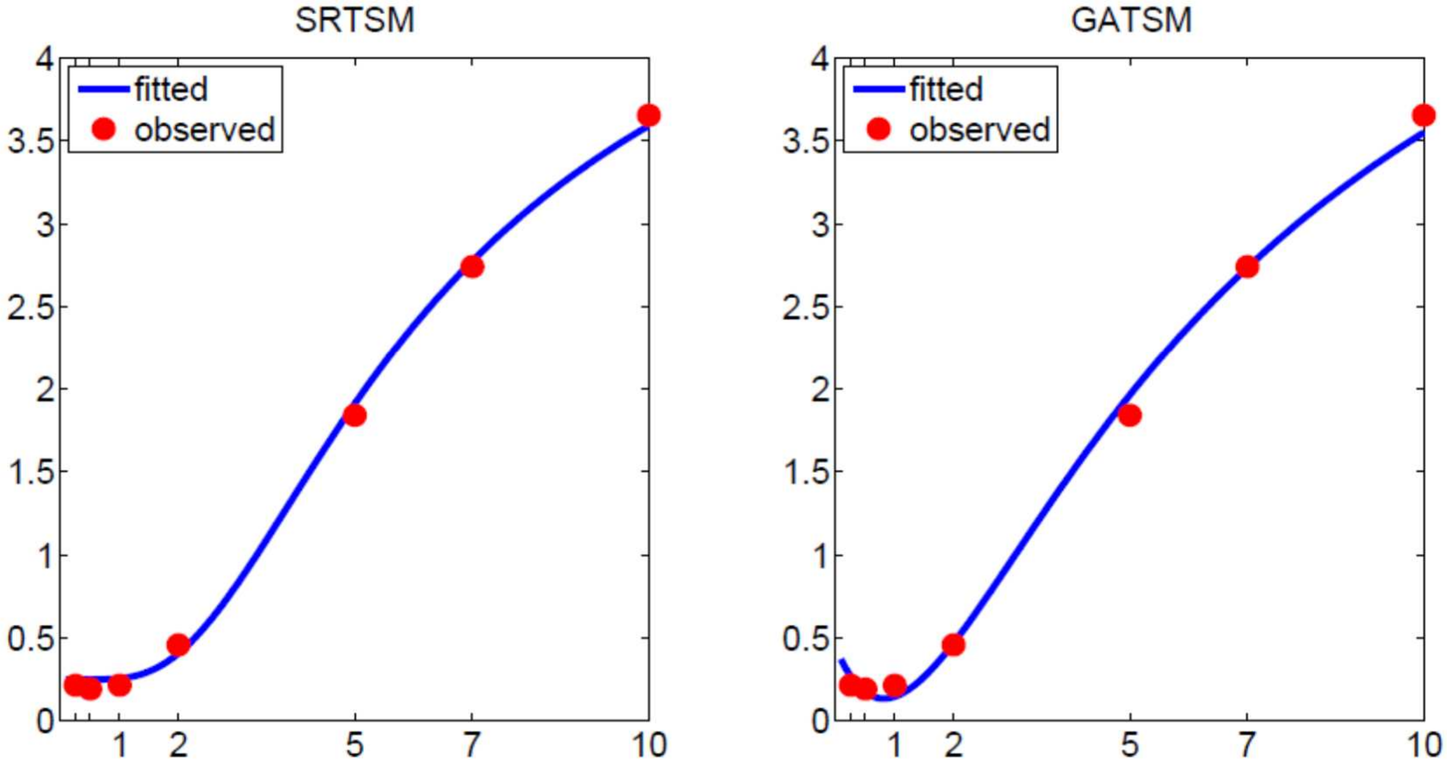
Normalization:

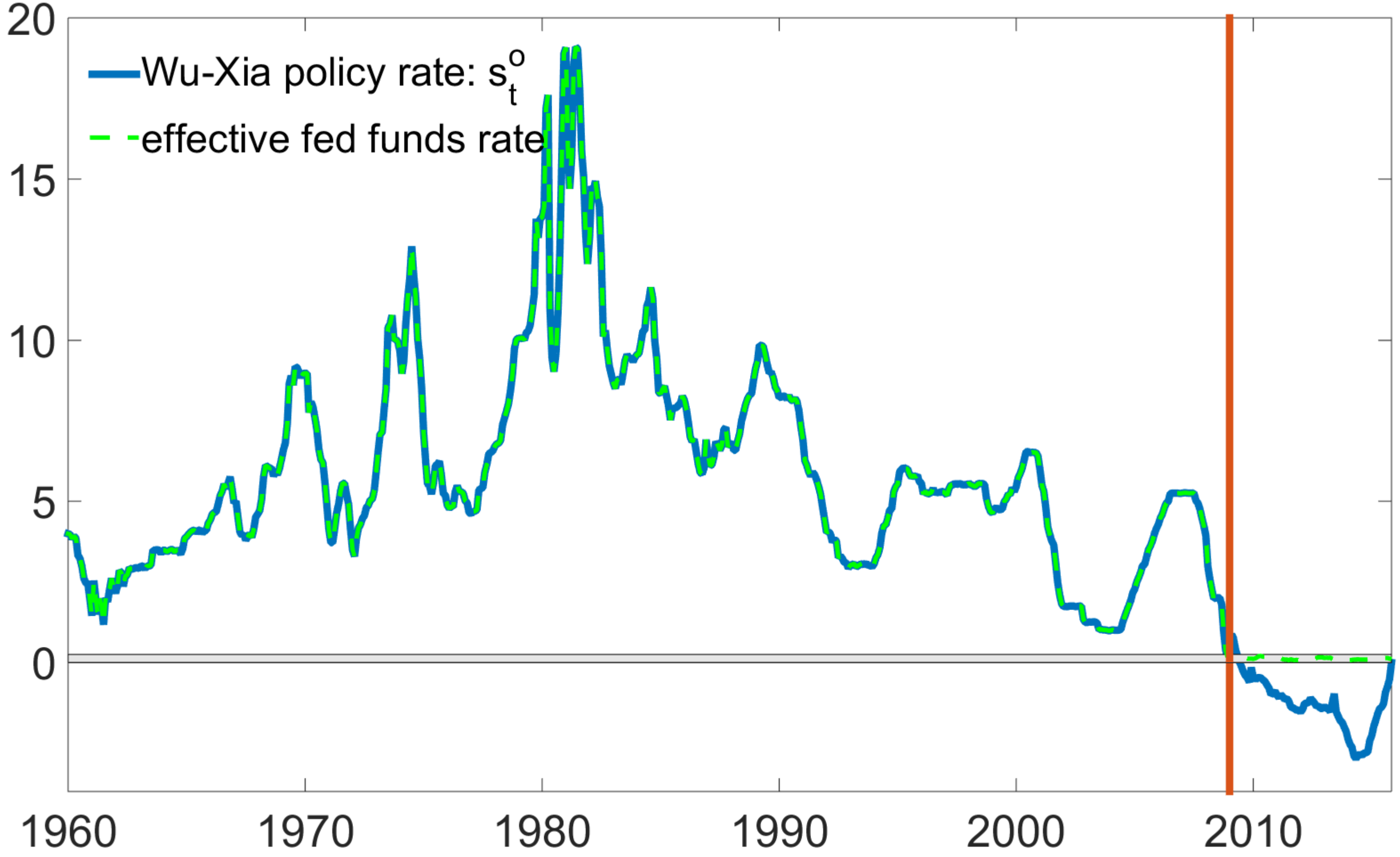
$\boldsymbol{\gamma}_0 = (1, 1, 0)', \mathbf{c}^Q = \mathbf{0}, \boldsymbol{\Sigma}$ lower triangular

$\boldsymbol{\rho}^Q$ in Jordan Normal form

Linear GATSM puts in odd curves at short end (average yield curve during 2012)

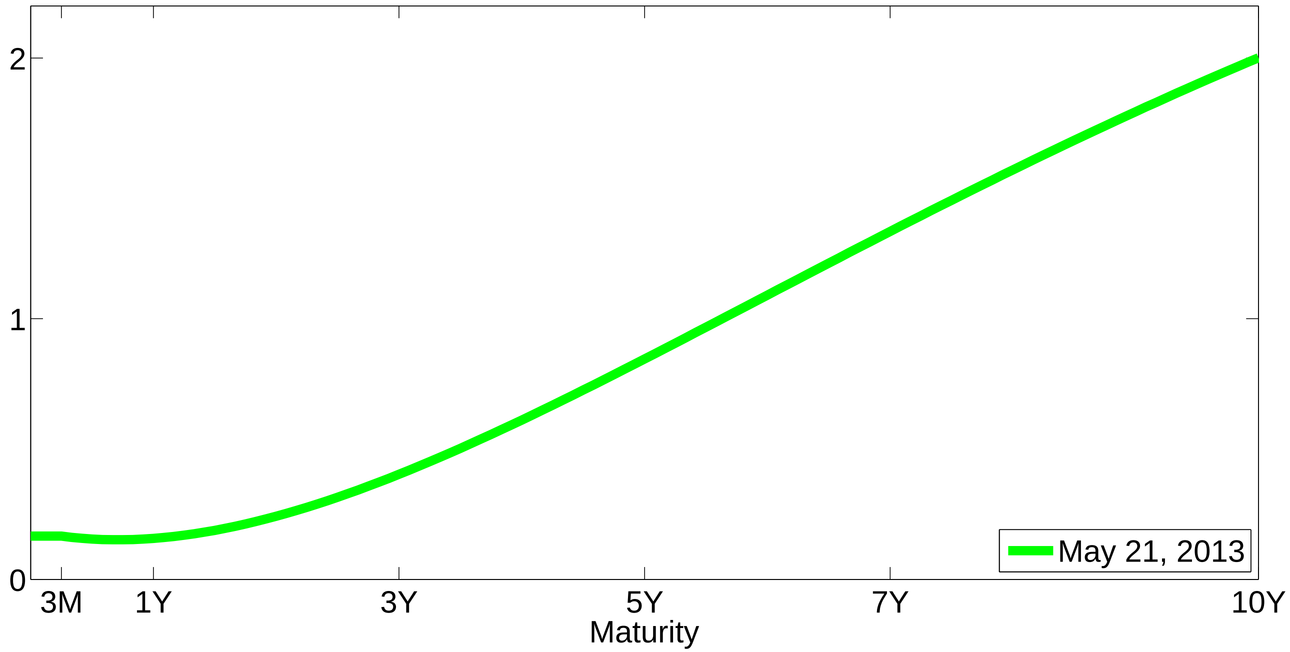
Figure 3: Observed and fitted forward curves



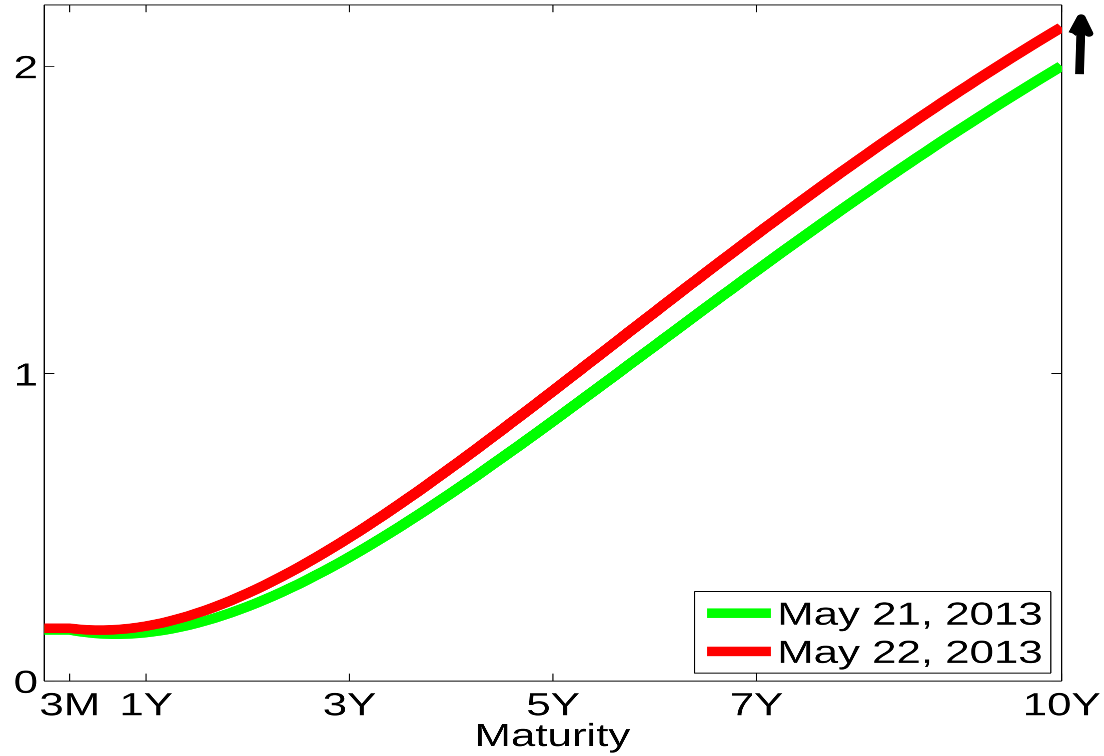


Wu-Xia policy rate: s_t^o
effective fed funds rate
20
15
10
5
0
1960
1970
1980
1990
2000
2010

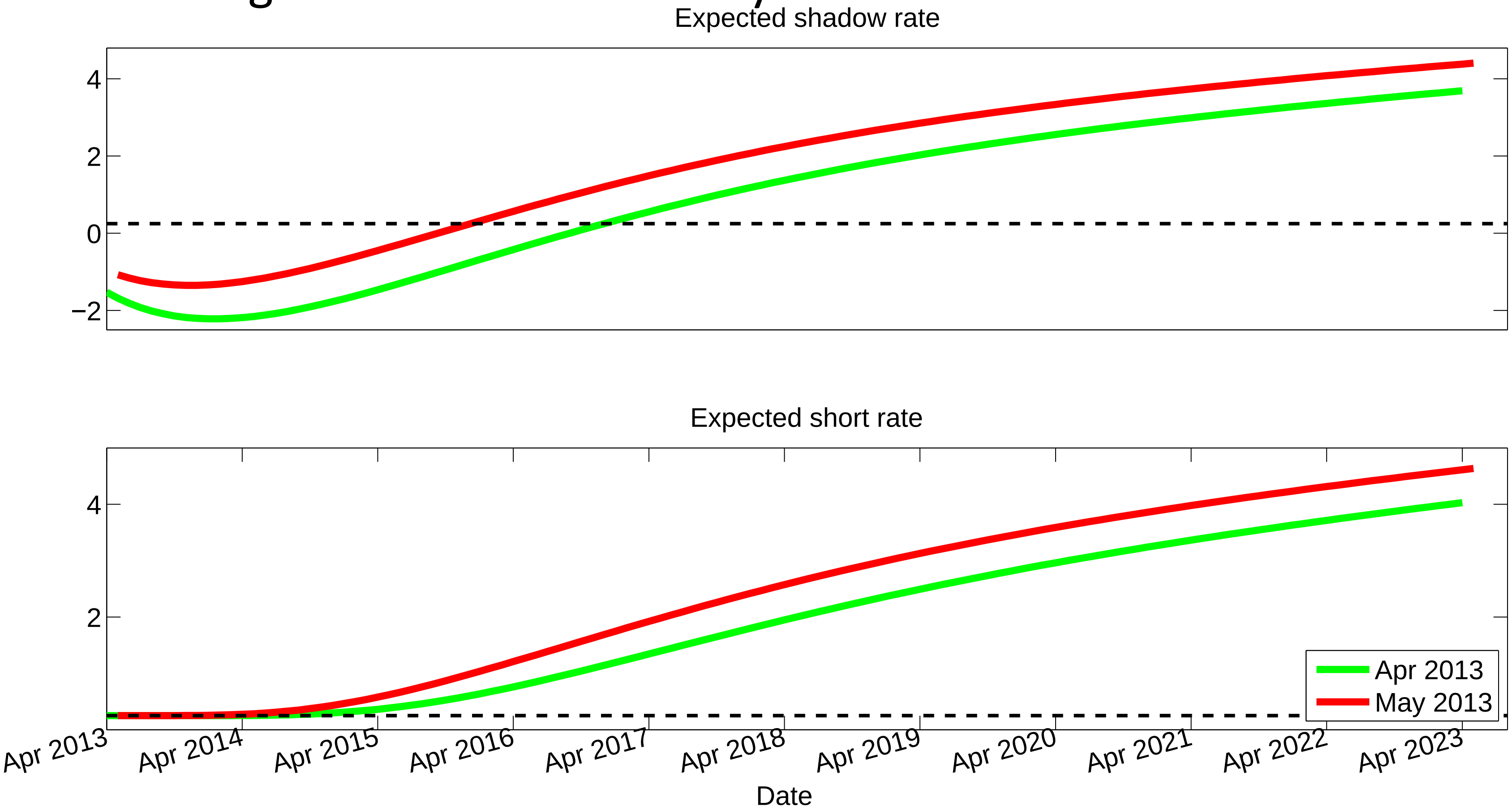
Yield curve on May 21, 2013



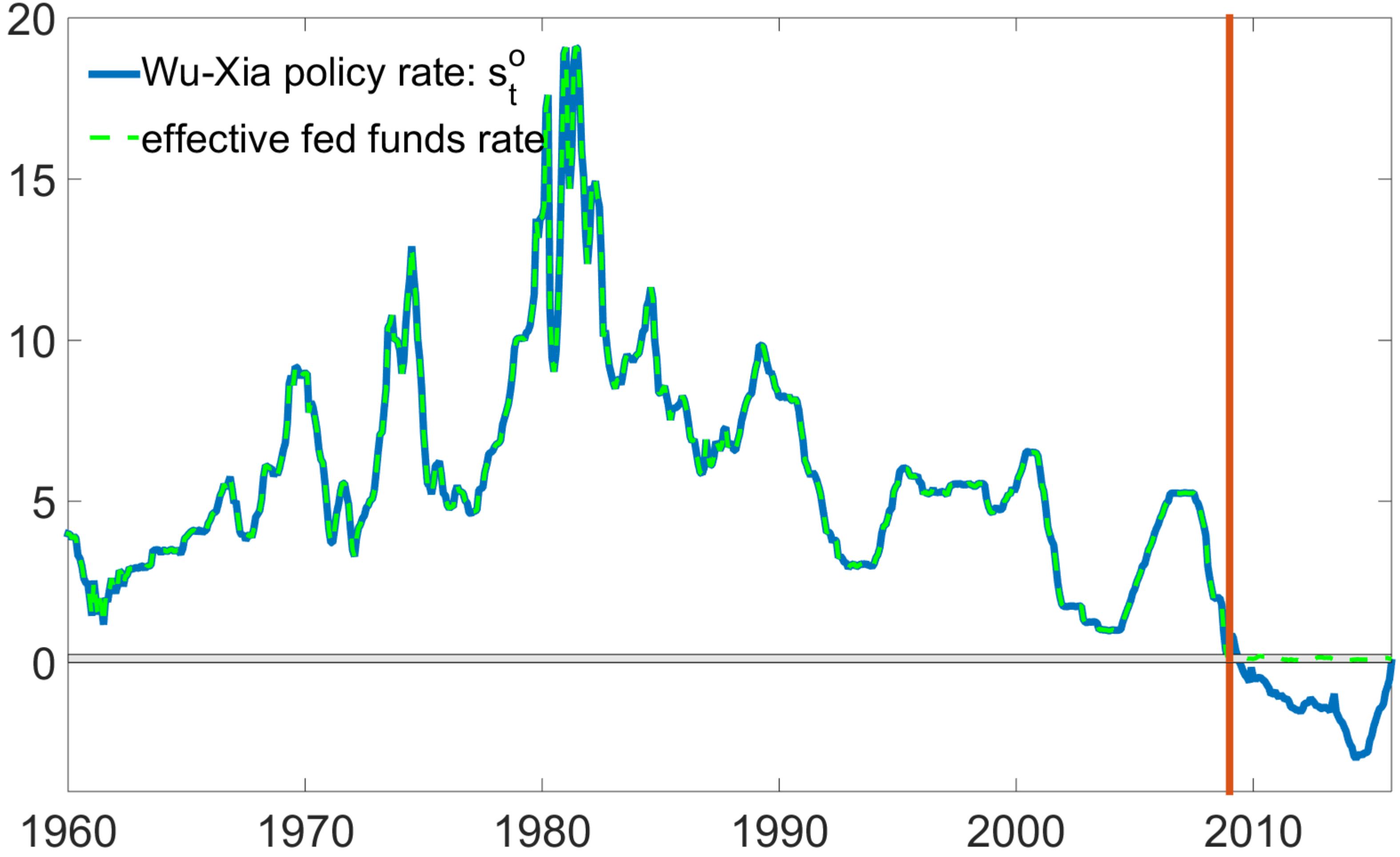
May 22: Bernanke tells Congress Fed may decrease the size of monthly large-scale asset purchases



Might summarize effect by shift in shadow rate



Question: can we use movements in shadow rate as similar summary of Fed actions as fed funds rate provided historically?



$$h_t = \begin{cases} \text{fed funds rate} & \text{before 2009} \\ \text{Wu-Xia shadow rate} & \text{since 2009} \end{cases}$$

Consider using h_t in place of fed funds rate in Bernanke, Boivin, Eliasz (2005) FAVAR:

$$\begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\xi}_t} \\ \underset{(1\times 1)}{h_t} \end{bmatrix} = \begin{bmatrix} \underset{(r\times r)}{\boldsymbol{\Phi}_{11}(L)} & \underset{(r\times 1)}{\boldsymbol{\Phi}_{12}(L)} \\ \underset{(1\times r)}{\boldsymbol{\Phi}_{21}(L)} & \underset{(1\times 1)}{\Phi_{22}(L)} \end{bmatrix} \begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\xi}_t} \\ \underset{(1\times 1)}{h_t} \end{bmatrix} + \begin{bmatrix} \underset{(r\times 1)}{\boldsymbol{\varepsilon}_{1t}} \\ \underset{(1\times 1)}{\varepsilon_{2t}} \end{bmatrix}$$

Fail to reject $H_0 : \Phi_{12}(L)$ is same before and after Great Recession

$\Rightarrow$ shadow rate helps forecast macro variables same way that fed funds rate did

Fail to reject $H_0 : \Phi_{21}(L)$ is same before and after Great Recession

$\Rightarrow$ same coefficients that used to predict fed funds rate now predict shadow rate

Implication: might use Wu-Xia shadow rate to update earlier studies that had been based on the historical fed funds rate