Monetary policy at the zero lower bound: Theory

A. Theoretical channels

1. Conditions for complete neutrality (Eggertsson and Woodford, 2003)
2. Market frictions
3. Preferred habitat and risk-bearing (Hamilton and Wu, 2012)

B. Shadow rate (Wu and Xia, 2016)

C. Theoretical channels

1. Conditions for complete neutrality

Suppose preferences are

$$E_t\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}[U(C_{t+\tau},H_{t+\tau}/P_{t+\tau};\xi_{t+\tau}] - \int_0^1 v(L_{t+\tau}(j,\xi_{t+\tau})dj\right\}$$

$C_t = \left[\int_0^1 c_t(i)^{\theta/(\theta-1)}di\right]^{(\theta-1)/\theta}$ (real consumption)

$P_t = \left[\int_0^1 p_t(i)^{1-\theta}di\right]^{1/(1-\theta)}$ (Calvo sticky prices)

H_t = nominal monetary base

ξ_t = preference shocks

$y_t(i) = A_t f[L_t(i)]$

no investment, govt spending: $C_t = Y_t$

pricing kernel:

$$M_{t+1} = \frac{\beta U_c(Y_{t+1},H_{t+1}/P_{t+1},\xi_{t+1})}{U_c(Y_t,H_t/P_t,\xi_t)(1+\pi_{t+1})}$$

first-order condition:

$$\frac{U_h(Y_t,H_t/P_t,\xi_t)}{U_c(Y_t,H_t/P_t,\xi_t)} = \frac{r_t}{1+r_t}$$

r_t = risk-free nominal rate

Liquidity trap (zero lower bound):

There is a saturation level $\bar{h}(C,\xi)$ such that $U_h(C,h;\xi) = 0$ for all $h \geq \bar{h}(C,\xi)$.

Implies $r_t = 0$ whenever $H_t/P_t \geq \bar{h}(Y_t,\xi_t)$.

Define $L(Y_t,r_t;\xi_t) =$

$$\begin{cases} h_t : \frac{U_h(Y_t,h_t,\xi_t)}{U_c(Y_t,h_t,\xi_t)} = \frac{r_t}{1+r_t} & \text{if } r_t > 0 \\ \bar{h}(Y_t,\xi_t) & \text{if } r_t = 0 \end{cases}$$

Monetary policy rule in normal times:

$r_t = \phi(\pi_t, Y_t;\tilde{\xi}_t)$

(Taylor type rule)

Monetary policy at the ZLB is choice for a rule $\psi(\pi_t, Y_t; \tilde{\xi}_t)$ such that

$$H_t = P_t L(Y_t, \phi(\pi_t, Y_t; \tilde{\xi}_t); \xi_t)\psi(\pi_t, Y_t; \tilde{\xi}_t)$$

$$\psi(\pi_t, Y_t; \tilde{\xi}_t) \begin{cases} = 1 & \text{if } \phi(\pi_t, Y_t; \tilde{\xi}_t) > 0 \\ \geq 1 & \text{otherwise} \end{cases}$$

where the excess reserves created by $\psi(\pi_t, Y_t; \tilde{\xi}_t)$ are used by the central bank to purchase *any* assets.

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Proposition: choice of $\psi(\pi_t, Y_t; \tilde{\xi}_t)$ has no effect on nominal prices, inflation, interest rates, or real activity.

Reason: operations at the ZLB have no affect on the pricing kernel M_{t+1}.

Implication: all the Fed's operations in phase I and phase II were completely pointless.

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How could Fed have an effect?

By changing $\phi(\pi_t, Y_t; \tilde{\xi}_t)$, the rule it will use to set interest rates once we're away from the ZLB, that would alter behavior today.

But how do we do this in practice?

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- In theoretical model, simply announce the change and it happens.
- In real world, perhaps LSAP or forward guidance persuade markets the Fed is really changing its future policy rule.
- For example, LSAP may alter state-contingent path of future tax receipts (Fed promises to monetize more of debt in some states)

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C. Theoretical channels

2. Market frictions

- During financial crisis in fall of 2008, arbitrage broke down in some markets
- No pricing kernel existed

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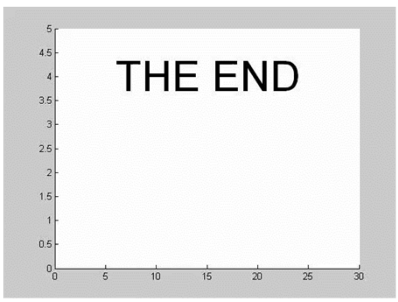
Gürkaynak and Wright, JELit, 2012, yield curve during 2008-10



Watch the movie at http://www.econ.jhu.edu/People/Wright/loop_repealed.mpg

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C. Theoretical channels

3. Preferred habitat and risk-bearing

Why does Treasury issue 10-year debt at 3% when it could borrow by rolling over 3-month debt at much lower rate?

Answer: Treasury is risk averse, and is willing to compensate government creditors for assuming this risk.

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Consider representative "arbitrageur"

q_{t+1} = total return on portfolio

$\max E_t(q_{t+1}) - (\gamma/2)\text{Var}_t(q_{t+1})$

first-order condition:

$r_{1t} = E_t(q_{n,t+1}) - \gamma\vartheta_{nt}$

where r_{1t} = return on riskless asset

$q_{n,t+1}$ = 1-period-holding yield for asset n

ϑ_{nt} = (1/2) change in variance from one more unit of asset n

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If portfolio allocates share z_{nt} to n-period pure discount bonds with maturity n and price P_{nt},

$$q_{n,t+1} = \frac{P_{n-1,t+1}}{P_{nt}} - 1$$

$$q_{t+1} = \sum_{n=1}^{N} z_{nt} q_{n,t+1}$$

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Suppose we conjecture:

$$p_{nt} = \alpha_n + \boldsymbol{\beta}_n' \boldsymbol{\xi}_t$$

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

$$\mathbf{u}_{t+1} \sim \text{i.i.d. } N(\mathbf{0}, \mathbf{I}_r)$$

Then:

$$\frac{\partial E_t(q_{t+1})}{\partial z_{nt}} \simeq \alpha_{n-1} + \boldsymbol{\beta}_{n-1}'(\mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t) - \alpha_n - \boldsymbol{\beta}_n'\boldsymbol{\xi}_t + (1/2)\boldsymbol{\beta}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\boldsymbol{\beta}_{n-1}$$

$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$$

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$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$$

Let $\mathbf{q}_t = \boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$

$= (3 \times 1)$ vector of arbitrageur's risk exposure

$$\frac{\partial \text{Var}_t(q_{t+1})}{\partial z_{nt}} \simeq 2\boldsymbol{\beta}_{n-1}'\mathbf{q}_t$$

So expected excess return from increasing z_{nt} by one unit must be $\gamma\boldsymbol{\beta}_{n-1}'\mathbf{q}_t$

$\Rightarrow$ price of risk $\boldsymbol{\lambda}_t = \gamma\mathbf{q}_t = \gamma\boldsymbol{\Sigma}\boldsymbol{\Sigma}' \sum_{\ell=2}^{N} z_{\ell t}\boldsymbol{\beta}_{\ell-1}$

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Suppose that:

- Arbitrageurs correspond to entire private sector
- U.S. Treasury debt is sole asset held by arbitageurs

Then:

z_{nt} = share of publicly held Treasury debt of maturity n should enter as factor in risk pricing

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Hamilton and Wu (2012)

- Historical variations in composition of publicly held Treasury debt (as summarized by $\mathbf{q}_t$ as measured using 3-factor affine weights) were associated with detectable (but small) changes in predicted excess returns
- Total stock of Treasury securities 10 years or longer held by public in 2006 was about $400 B
- What would affine model incorporating Treasury debt holdings imply would happen to yields if supply of 3m Treasuries increased $400B and all long-term debt retired?

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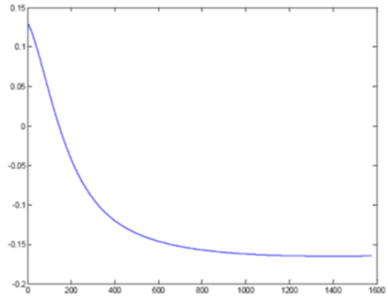
Hamilton-Wu estimates of effect on yield (in %) as function of maturity (in weeks)



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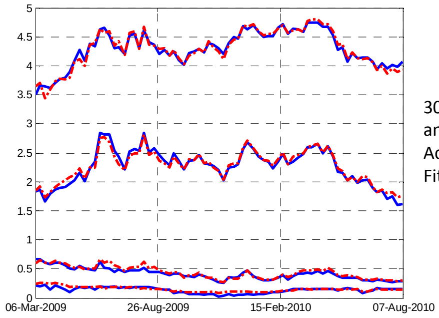
How apply to ZLB?



30y, 5y, 1y, and 3m yields
Actual (solid)
Fitted (dashed)

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Long rates are still responding to daily news, short rates are not.

Interpretation: there are still some factors ξ_t changing daily that matter for long yields but not short.

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Suppose latent factors following same dynamics as estimated historically:

$$\xi_t = \mathbf{c} + \rho\xi_{t-1} + \Sigma\mathbf{u}_t$$

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Suppose once we escape from ZLB all yields will behave same as before:

$$\tilde{p}_{nt} = \tilde{\alpha}_n + \tilde{\beta}_n' \xi_t$$

$$\{\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_N, \tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_N\}$$

as historically estimated

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But if we are at ZLB at date t,

$$y^*_{1t} = a^*_1$$
$$p^*_{nt} = \alpha^*_n + \boldsymbol{\beta}^{*\prime}_n \boldsymbol{\xi}_t$$
$$\boldsymbol{\beta}^*_1 = \mathbf{0}$$

π^Q = Q-measure probability
that will escape ZLB next period

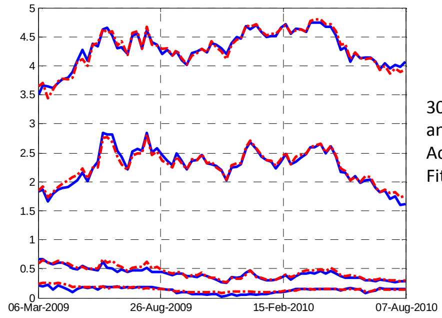
$$\exp(y^*_{1t}) = \pi^Q E^Q_t\left(\frac{\tilde{P}_{n-1,t+1}}{P^*_{nt}}\right) + (1-\pi^Q)E^Q_t\left(\frac{P^*_{n-1,t+1}}{P^*_{nt}}\right)$$
$$\tilde{p}_{n-1,t+1} = \tilde{\alpha}_{n-1} + \tilde{\boldsymbol{\beta}}'_{n-1}\boldsymbol{\xi}_{t+1}$$
$$p^*_{n-1,t+1} = \alpha^*_{n-1} + \boldsymbol{\beta}^{*\prime}_{n-1}\boldsymbol{\xi}_{t+1}$$
$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

$\mathbf{c}, \boldsymbol{\rho}, \boldsymbol{\Sigma}$ and $\{\tilde{\alpha}_n, \tilde{\boldsymbol{\beta}}_n\}_{n=1}^N$ estimated
from pre-ZLB data.

Know $\boldsymbol{\beta}^*_1 = \mathbf{0}$ by definition.

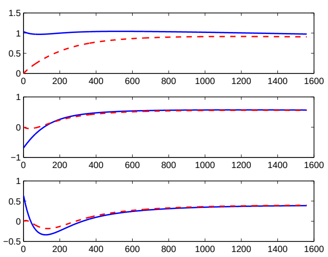
Can calculate $\{\alpha^*_n, \boldsymbol{\beta}^*_n\}_{n=2}^N$ as functions of α^*_1 and π^Q.

Choose α^*_1, π^Q to fit ZLB data by MCS. Slightly better fit if also allow $\tilde{\alpha}_1$ (governs level interest rates return to once exit ZLB) to be different from historically estimated α_1.

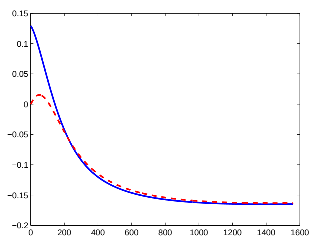


30y, 5y, 1y,
and 3m yields
Actual (solid)
Fitted (dashed)

Factor loadings in normal times (solid blue) and at the ZLB (dashed red)



Effect of swap $400 B short-term for long-term in normal times (blue) and ZLB (red)



D. Shadow rate model (Wu and Xia, 2016)

Again assume there is a latent vector of factors that follow same dynamics in normal times as at ZLB:

P-measure:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

Q-measure:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c}^Q + \boldsymbol{\rho}^Q\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}^Q$$

$$\mathbf{c}^Q = \mathbf{c} - \boldsymbol{\Sigma}\boldsymbol{\lambda}$$

$$\boldsymbol{\rho}^Q = \boldsymbol{\rho} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}$$

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s_t = shadow rate

$$= \delta_0 + \boldsymbol{\gamma}_0'\boldsymbol{\xi}_t$$

r_t = short-term interest rate

$$= \max\{s_t, \theta\}$$

$\theta = 0$ (or some positive lower bound)

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$$P_{nt} = E_t^Q\{\exp[-(r_t + r_{t+1} + \cdots + r_{t+n-1})]\}$$

$$s_t = \delta_0 + \boldsymbol{\gamma}_0'\boldsymbol{\xi}_t$$

$$r_t = \max\{s_t, \theta\}$$

$$\Rightarrow p_{nt} = q_n(\boldsymbol{\xi}_t)$$

One approach: calculate $q_n(.)$ by simulation

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Wu and Xia show an excellent approximation to the forward rate f_{nt} is given by

$$f_{nt} \simeq \theta + \sigma_n^Q g\left(\frac{\delta_n + \boldsymbol{\gamma}_n'\boldsymbol{\xi}_t - \theta}{\sigma_n^Q}\right)$$

$$g(z) = z\Phi(z) + \phi(z)$$

$\phi(z) = N(0,1)$ density

$\Phi(z) = N(0,1)$ cdf

$$\boldsymbol{\gamma}_n' = \boldsymbol{\gamma}_0'(\boldsymbol{\rho}^Q)^n$$

$$\sigma_n^2 = \sum_{j=0}^{n-1} \boldsymbol{\gamma}_0'(\boldsymbol{\rho}^Q)^j \boldsymbol{\Sigma}\boldsymbol{\Sigma}'(\boldsymbol{\rho}^{Q\prime})^j \boldsymbol{\gamma}_0$$

$$\delta_n = \delta_0 + \boldsymbol{\gamma}_0'\mathbf{H}_n\mathbf{c}^Q - (1/2)\boldsymbol{\gamma}_0'\mathbf{H}_n\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{H}_n'\boldsymbol{\gamma}_0$$

$$\mathbf{H}_n = \sum_{j=0}^{n-1}(\boldsymbol{\rho}^Q)^j$$

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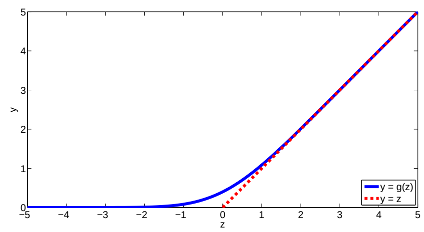

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Estimate with nonlinear state-space model.

State equation:

$$\boldsymbol{\xi}_{t+1} = \mathbf{c} + \boldsymbol{\rho}\boldsymbol{\xi}_t + \boldsymbol{\Sigma}\mathbf{u}_{t+1}$$

Observation equation:

$$\begin{bmatrix} f_{n_1,t} \\ \vdots \\ f_{n_N,t} \end{bmatrix} = \begin{bmatrix} \theta + \sigma_{n_1}^Q g\left(\frac{\delta_{n_1} + \boldsymbol{\gamma}_{n_1}'\boldsymbol{\xi}_t - \theta}{\sigma_{n_1}^Q}\right) \\ \vdots \\ \theta + \sigma_{n_N}^Q g\left(\frac{\delta_{n_N} + \boldsymbol{\gamma}_{n_N}'\boldsymbol{\xi}_t - \theta}{\sigma_{n_N}^Q}\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{n_1,t} \\ \vdots \\ \varepsilon_{n_N,t} \end{bmatrix}$$

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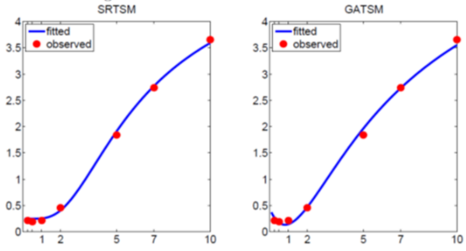
Normalization:

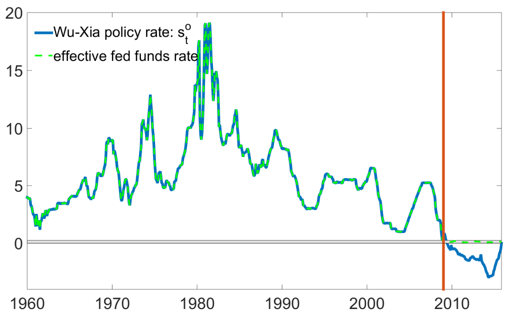
$\boldsymbol{\gamma}_0 = (1,1,0)', \mathbf{c}^Q = \mathbf{0}, \boldsymbol{\Sigma}$ lower triangular

$\boldsymbol{\rho}^Q$ in Jordan Normal form

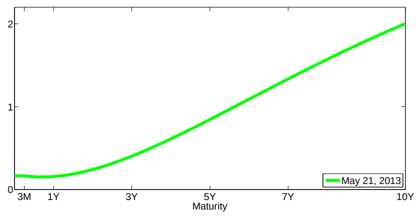
Linear GATSM puts in odd curves at short end (average yield curve during 2012)

Figure 3: Observed and fitted forward curves

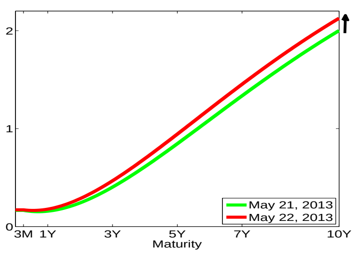




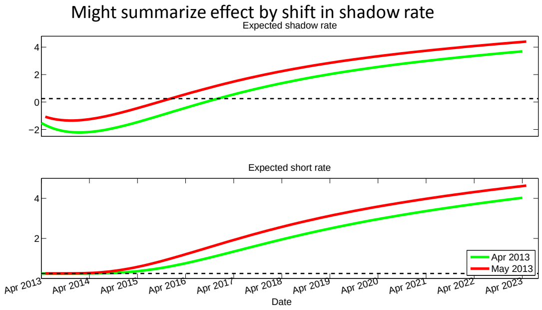
Yield curve on May 21, 2013



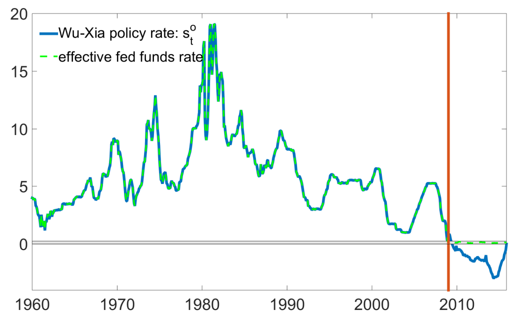
May 22: Bernanke tells Congress Fed may decrease the size of monthly large-scale asset purchases



Might summarize effect by shift in shadow rate



Question: can we use movements in shadow rate as similar summary of Fed actions as fed funds rate provided historically?



$$h_t = \begin{cases} \text{fed funds rate} & \text{before 2009} \\ \text{Wu-Xia shadow rate} & \text{since 2009} \end{cases}$$

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Consider using h_t in place of fed funds rate in Bernanke, Boivin, Eliasz (2005) FAVAR:

$$\begin{bmatrix} \underset{(r\times 1)}{\xi_t} \\ \underset{(1\times 1)}{h_t} \end{bmatrix} = \begin{bmatrix} \underset{(r\times r)}{\Phi_{11}(L)} & \underset{(r\times 1)}{\Phi_{12}(L)} \\ \underset{(1\times r)}{\Phi_{21}(L)} & \underset{(1\times 1)}{\Phi_{22}(L)} \end{bmatrix} \begin{bmatrix} \underset{(r\times 1)}{\xi_t} \\ \underset{(1\times 1)}{h_t} \end{bmatrix} + \begin{bmatrix} \underset{(r\times 1)}{\varepsilon_{1t}} \\ \underset{(1\times 1)}{\varepsilon_{2t}} \end{bmatrix}$$

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Fail to reject H_0 : $\Phi_{12}(L)$ is same before and after Great Recession

$\Rightarrow$ shadow rate helps forecast macro variables same way that fed funds rate did

Fail to reject H_0 : $\Phi_{21}(L)$ is same before and after Great Recession

$\Rightarrow$ same coefficients that used to predict fed funds rate now predict shadow rate

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Implication: might use Wu-Xia shadow rate to update earlier studies that had been based on the historical fed funds rate