IV Estimation

A. Using instruments by augmenting VAR
B. Using instruments external to VAR (Stock and Watson, 2012)
C. Using IV for mixed-frequency inference: Gertler and Karadi (2015)
D. Augmented VAR versus IV estimation
E. Natural experiments in macro (Fuchs-Schuendeln and Hassan, 2015)

A. Using instruments by augmenting VAR

Example: supply and demand

$$q_t = \lambda^d + \beta^d p_t + b_{11}^d p_{t-1} + b_{12}^d q_{t-1} + b_{21}^d p_{t-2}$$
$$+ b_{22}^d q_{t-2} + \cdots + b_{m1}^d p_{t-m} + b_{m2}^d q_{t-m} + u_t^d$$

$$q_t = \lambda^s + \alpha^s p_t + b_{11}^s p_{t-1} + b_{12}^s q_{t-1} + b_{21}^s p_{t-2}$$
$$+ b_{22}^s q_{t-2} + \cdots + b_{m1}^s p_{t-m} + b_{m2}^s q_{t-m} + u_t^s$$

Textbook solution: find instrument (weather w_t) that shifts supply but not demand.

$$\mathbf{y}_t = (q_t, p_t, w_t)'$$

$$q_t = \lambda^d + \beta^d p_t + \boldsymbol{\gamma}_1^{d\prime}\mathbf{y}_{t-1} + \cdots + \boldsymbol{\gamma}_m^{d\prime}\mathbf{y}_{t-m} + u_t^d$$

$$q_t = \lambda^s + \alpha^s p_t + h^s w_t + \boldsymbol{\gamma}_1^{s\prime}\mathbf{y}_{t-1} + \cdots + \boldsymbol{\gamma}_m^{s\prime}\mathbf{y}_{t-m} + u_t^s$$

$$w_t = \lambda^w + \boldsymbol{\gamma}_1^{w\prime}\mathbf{y}_{t-1} + \cdots + \boldsymbol{\gamma}_m^{w\prime}\mathbf{y}_{t-m} + u_t^w$$

Could impose additional restrictions on $\boldsymbol{\gamma}_j^i$

$$\mathbf{A}\mathbf{y}_t = \boldsymbol{\lambda} + \mathbf{B}_1\mathbf{y}_{t-1} + \cdots + \mathbf{B}_m\mathbf{y}_{t-m} + \mathbf{u}_t$$

$$E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{D} \text{ (diagonal)}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\beta^d & 0 \\ 1 & -\alpha^s & -h^s \\ 0 & 0 & 1 \end{bmatrix}$$

Algorithm 1: Find $\hat{\beta}^{d}_{MLE}, \hat{\alpha}^{s}_{MLE}, \hat{h}^{s}_{MLE}$ by maximizing log likelihood numerically

$$(T/2)\log|\mathbf{A}|^{2} - (T/2)\log|\mathbf{D}|$$

$$-(T/2)\text{trace}\{(\mathbf{A}'\mathbf{D}^{-1}\mathbf{A})\hat{\boldsymbol{\Omega}}\}$$

Estimates will satisfy

$$\hat{\mathbf{D}} = \hat{\mathbf{A}}\hat{\boldsymbol{\Omega}}\hat{\mathbf{A}}' \text{ (diagonal)}$$

Algorithm 2: Find $\hat{\beta}_{IV}^{d}$ by IV regression of $\hat{\varepsilon}_{qt}$ on $\hat{\varepsilon}_{pt}$ using $\hat{\varepsilon}_{wt}$ as instrument:

$$\hat{\beta}_{IV}^{d} = \frac{\sum_{t=1}^{T} \hat{\varepsilon}_{wt}\hat{\varepsilon}_{qt}}{\sum_{t=1}^{T} \hat{\varepsilon}_{wt}\hat{\varepsilon}_{pt}} = \frac{\hat{\sigma}_{wq}}{\hat{\sigma}_{wp}}$$

Then find $\hat{\alpha}^s_{IV}, \hat{h}^s_{IV}$ by IV regression of $\hat{\varepsilon}_{qt}$ on $\hat{\varepsilon}_{pt}, \hat{\varepsilon}_{wt}$ using $\hat{\varepsilon}_{wt}$ and $\hat{u}^d_t = \hat{\varepsilon}_{qt} - \hat{\beta}^d_{IV}\hat{\varepsilon}_{pt}$ as instruments:

$$\begin{bmatrix} \hat{\alpha}^s_{IV} \\ \hat{h}^s_{IV} \end{bmatrix} = \begin{bmatrix} \sum \hat{u}^d_t \hat{\varepsilon}_{pt} & \sum \hat{u}^d_t \hat{\varepsilon}_{wt} \\ \sum \hat{\varepsilon}_{wt}\hat{\varepsilon}_{pt} & \sum \hat{\varepsilon}^2_{wt} \end{bmatrix}^{-1} \times \begin{bmatrix} \sum \hat{u}^d_t \hat{\varepsilon}_{qt} \\ \sum \hat{\varepsilon}_{wt}\hat{\varepsilon}_{qt} \end{bmatrix}$$

Proposition: the estimates of the two algorithms are numerically identical.

Proof:

$\sum \hat{u}_t^d \hat{\varepsilon}_{wt} = 0$ by definition of $\hat{\beta}_{IV}^d$

$\sum \hat{u}_t^s \hat{u}_t^d = \sum \hat{u}_t^s \hat{\varepsilon}_{wt} = 0$ by definition of $\hat{\alpha}_{IV}^s, \hat{h}_{IV}^s$

$\hat{\mathbf{A}}_{IV} \hat{\mathbf{\Omega}} \hat{\mathbf{A}}_{IV}'$ is diagonal

B. Using instruments external to VAR (Stock and Watson, 2012)

Structural model:

$$\mathbf{A}\mathbf{y}_t = \boldsymbol{\lambda} + \mathbf{B}_1\mathbf{y}_{t-1} + \cdots + \mathbf{B}_m\mathbf{y}_{t-m} + \mathbf{u}_t$$

$$E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{D} \text{ (diagonal)}$$

Reduced form:

$$\mathbf{y}_t = \mathbf{c} + \boldsymbol{\Phi}_1\mathbf{y}_{t-1} + \cdots + \boldsymbol{\Phi}_m\mathbf{y}_{t-m} + \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1}\mathbf{u}_t$$

Suppose we have instrument z_{it} that is

relevant: $E(z_{it}u_{it}) = \alpha_i \neq 0$

valid: $E(z_{it}u_{jt}) = 0$ for $i \neq j$

Under the above assumptions,

$E(\boldsymbol{\varepsilon}_t z_{it}) = \mathbf{A}^{-1} E(\mathbf{u}_t z_{it}) = \mathbf{A}^{-1} \alpha_i \mathbf{e}_i$

$\mathbf{e}_i =$ col i of $\mathbf{I}_n$

so can estimate ith column of

$\mathbf{A}^{-1}$ (up to unknown constant) by

$\tilde{\mathbf{a}}^{(i)} = T^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t z_{it}$

Can normalize by defining shock u_{it} to be something that increases y_{it} by one unit: $\hat{\mathbf{a}}^{(i)} = \tilde{\mathbf{a}}^{(i)}/\tilde{a}_i^{(i)}$

$$\widehat{\frac{\partial \mathbf{y}_{t+s}}{\partial u_{it}}} = \hat{\mathbf{\Psi}}_s \hat{\mathbf{a}}^{(i)}$$

Can also estimate $\hat{u}_{it}$ as follows.

Suppose we observed $\mathbf{u}_t$ and regressed z_{it} on $\mathbf{u}_t$:

$$z_{it} = \boldsymbol{\pi}_i'\mathbf{u}_t + v_{it}$$

plim $\hat{\boldsymbol{\pi}}_i = (\alpha_i/d_{ii})\mathbf{e}_i$

If instead we regressed z_{it} on $\boldsymbol{\varepsilon}_t$,

$z_{it} = \boldsymbol{\lambda}_i' \boldsymbol{\varepsilon}_t + v_{it}$

this would just be rotation of

above regression since $\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1} \mathbf{u}_t$

Hence fitted values from regression

of z_{it} on $\hat{\boldsymbol{\varepsilon}}_t$ give consistent estimate

of $(\alpha_i / d_{ii}) u_{it}$

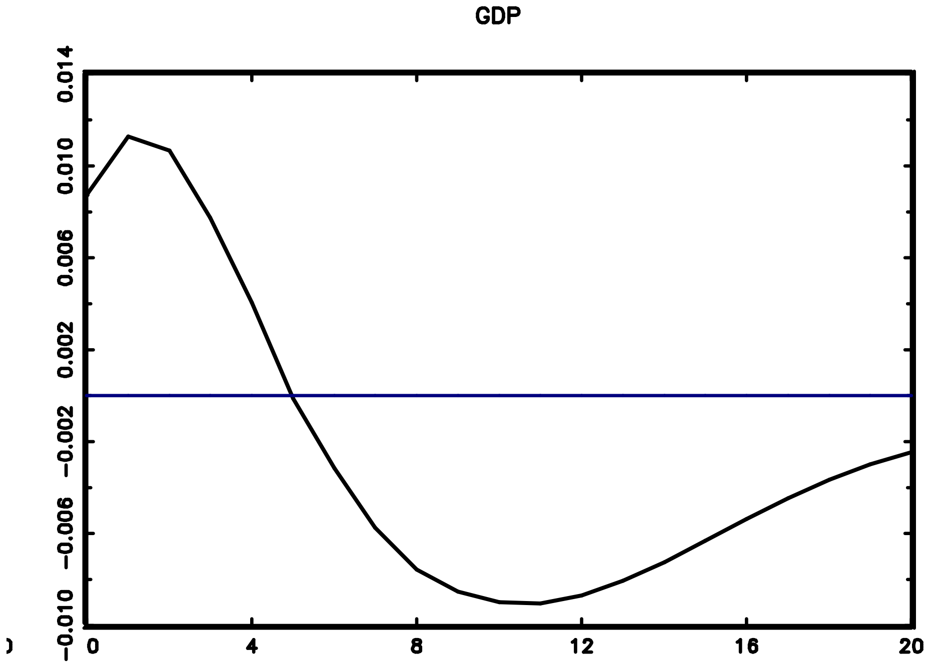
Stock-Watson examined several different proposed measures of monetary policy shocks, including

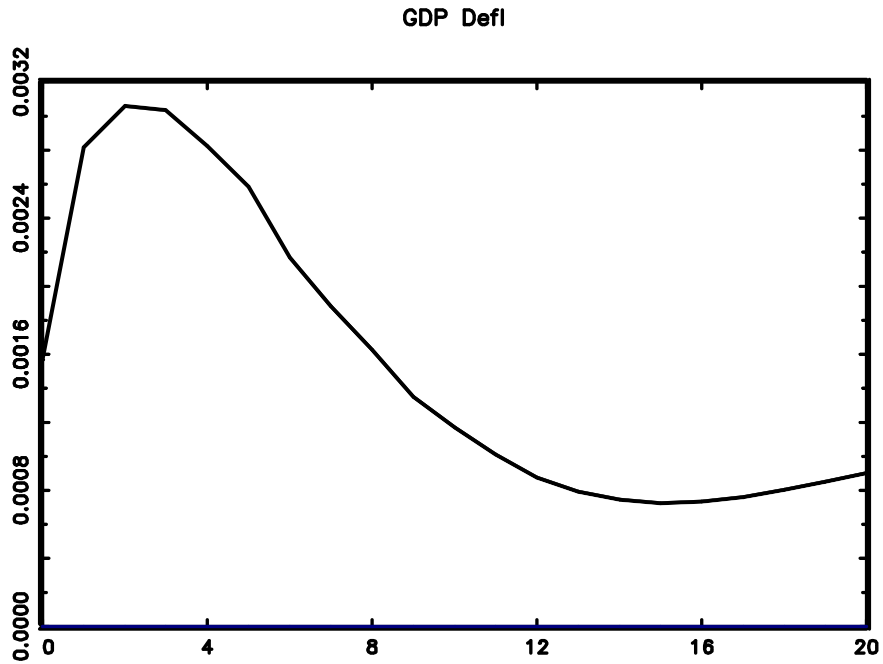
(1) Romer-Romer shocks

(2) Monetary policy shocks inferred from Smets-Wouters empirical DGSE

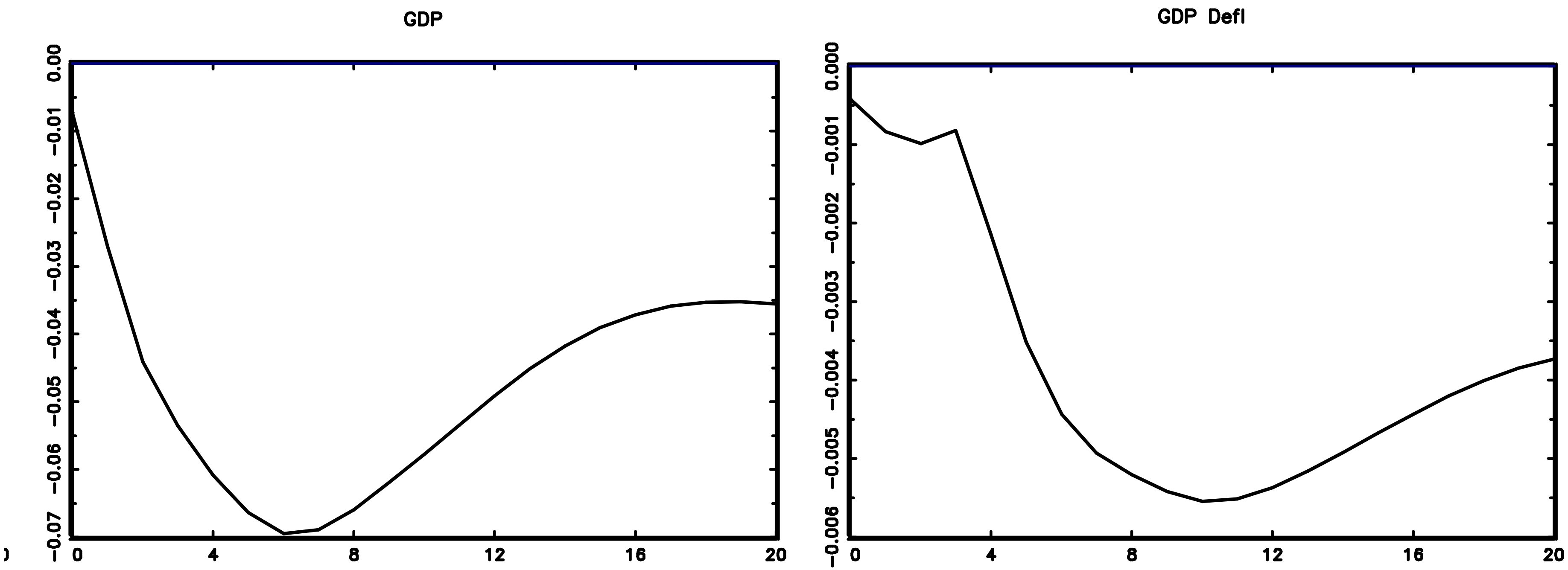
(3) Gürkaynak-Sack-Swanson (2005) Fed target shock

Structural IRF using Romer-Romer monetary shocks



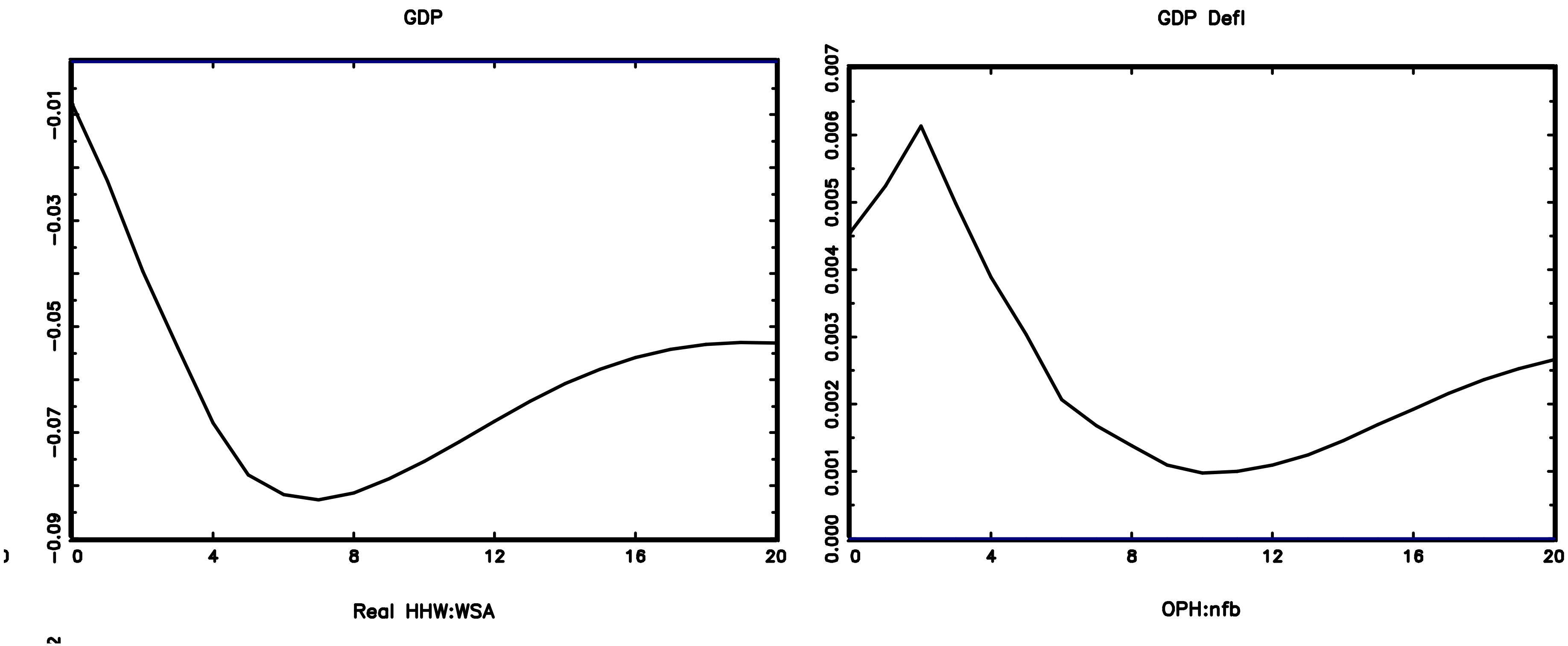


Structural IRF using Smets-Wouters monetary shocks



Correlation between RR and SW shock = 0.09

Structural IRF using Gürkaynak-Sack-Swanson monetary shocks



Stock-Watson considered 17 different instruments for 6 structural shocks

Oil shock
Hamilton
Kilian
Ramey-Vine

Monetary policy shock
Romer-Romer
Smets-Wouters
Sims-Zha
Gürkaynak-Sack-Swanson

Productivity shock
Fernald TFP
Galí long-run output per hour
Smets-Wouters

Uncertainty shock
Bloom financial uncertainty (VIX)
Baker-Bloom-Davis policy uncertainty

Liquidity-financial risk shock
Gilchrist-Zakrajšek spread
TED spread
Bassett and others bank loan supply

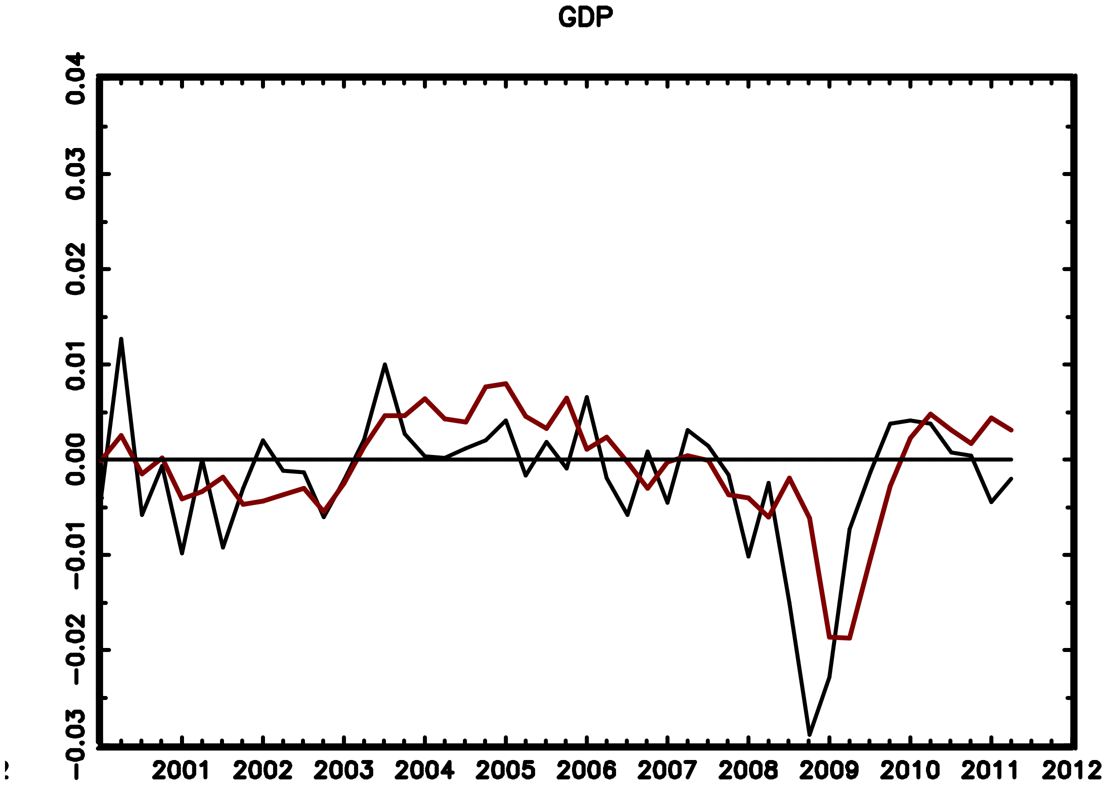
Fiscal policy shock
Ramey spending
Fisher-Peters spending
Romer-Romer tax

Instruments should be correlated within group and not across groups

	O_H	O_K	O_{RV}	M_{RR}	M_{SW}	M_{SZ}	M_{GSS}	P_F	P_G	P_{SW}	U_B	U_{BBD}	L_{GZ}	L_{TED}	L_{BCDZ}	F_R	F_{FP}	F_{RR}
O_H	1.00																	
O_K	0.42	1.00																
O_{RV}	0.15	0.60	1.00															
M_{RR}	0.37	*0.65*	*0.77*	1.00														
M_{SW}	0.09	0.11	0.39	0.09	1.00													
M_{SZ}	0.33	0.35	*0.68*	0.93	0.16	1.00												
M_{GSS}	0.44	–0.12	–0.08	0.24	0.43	0.39	1.00											
P_F	*–0.64*	0.30	0.24	0.20	–0.09	0.06	–0.57	1.00										
P_G	–0.40	0.34	0.01	–0.30	0.35	–0.53	–0.37	0.52	1.00									
P_{SW}	*–0.91*	–0.03	0.00	–0.24	–0.07	–0.36	–0.59	0.82	0.68	1.00								
U_B	–0.37	–0.37	–0.58	–0.39	0.30	–0.29	0.37	0.19	0.34	0.27	1.00							
U_{BBD}	0.10	0.11	–0.37	–0.17	0.45	–0.22	0.57	–0.06	0.45	–0.01	0.78	1.00						
L_{GZ}	–0.20	–0.42	–0.51	–0.41	0.44	–0.24	0.34	0.07	0.24	0.08	*0.92*	*0.66*	1.00					
L_{TED}	–0.09	0.01	–0.05	0.03	*0.73*	0.10	0.48	0.21	0.37	0.09	*0.80*	*0.76*	0.84	1.00				
L_{BCDZ}	0.04	0.22	*0.79*	0.56	0.13	0.55	0.04	–0.09	–0.28	–0.06	*–0.69*	–0.54	–0.73	–0.40	1.00			
F_R	–0.17	*–0.64*	*–0.77*	*–0.84*	–0.32	*–0.72*	–0.34	–0.17	–0.01	0.01	0.26	–0.08	0.40	–0.13	–0.13	1.00		
F_{FP}	0.04	–0.21	–0.35	*–0.72*	0.20	*–0.78*	–0.03	–0.49	0.40	–0.02	0.03	0.25	0.03	–0.12	–0.12	0.38	1.00	
F_{RR}	0.20	0.15	0.30	*0.77*	–0.10	*0.88*	0.37	0.18	–0.59	–0.28	0.01	–0.10	0.02	0.19	0.19	–0.45	–0.93	1.00

- Most important shocks in Great Recession seemed to be financial shocks
- TED spread = 3-month LIBOR rate (an average of interest rates offered in the London interbank market for 3-month dollar-denominated loans) and the 3-month Treasury bill rate
- Gilchrist-Zakrajšek spread = average gap between corporate and risk-free yields

Historical decomposition: contribution of financial shocks (TED)



C. Using IV for mixed-frequency inference: Gertler and Karadi (2015)

- Monthly 1979:M7 – 2012:M6
- interest rate on 1-year U.S. Treasury (takes place of fed funds rate in older regressions)
- log of CPI
- log of industrial production
- Gilchrist-Zakrajšek spread

Instruments for monetary policy shock

(1) Kuttner's surprise component of change in current-month fed funds futures contract in 30-minute window around FOMC announcement in month t

- = 0 if no announcement
- Only estimate over $\mathbb{Q}$ = {[1991:M1 – 2008:M6] U [2009:M7 – 2012:M6]}
- Identifies linear combination of reduced-form VAR residuals that is to be designated "monetary policy shock"

Instruments for monetary policy shock

(2) Change in 3-month ahead fed funds futures contract in 30-minute window around FOMC announcement in month t

(3)-(5) Change in 6, 9, and 12-month ahead 3-month Eurodollar futures in 30 minute window in month t

$\boldsymbol{\varepsilon}_t$ = reduced-form VAR residuals

(ε_{1t} = error forecasting 1-year interest rate)

$\mathbf{u}_t$ = structural shocks

(u_{1t} = monetary policy shock)

$\mathbf{z}_t = (5 \times 1)$ vector of instruments

$$\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1}\mathbf{u}_t$$

$$\frac{\partial \boldsymbol{\varepsilon}_t}{\partial u_{1t}} = \mathbf{a}^{(1)} \quad \text{(col 1 of } \mathbf{A}^{-1}\text{)}$$

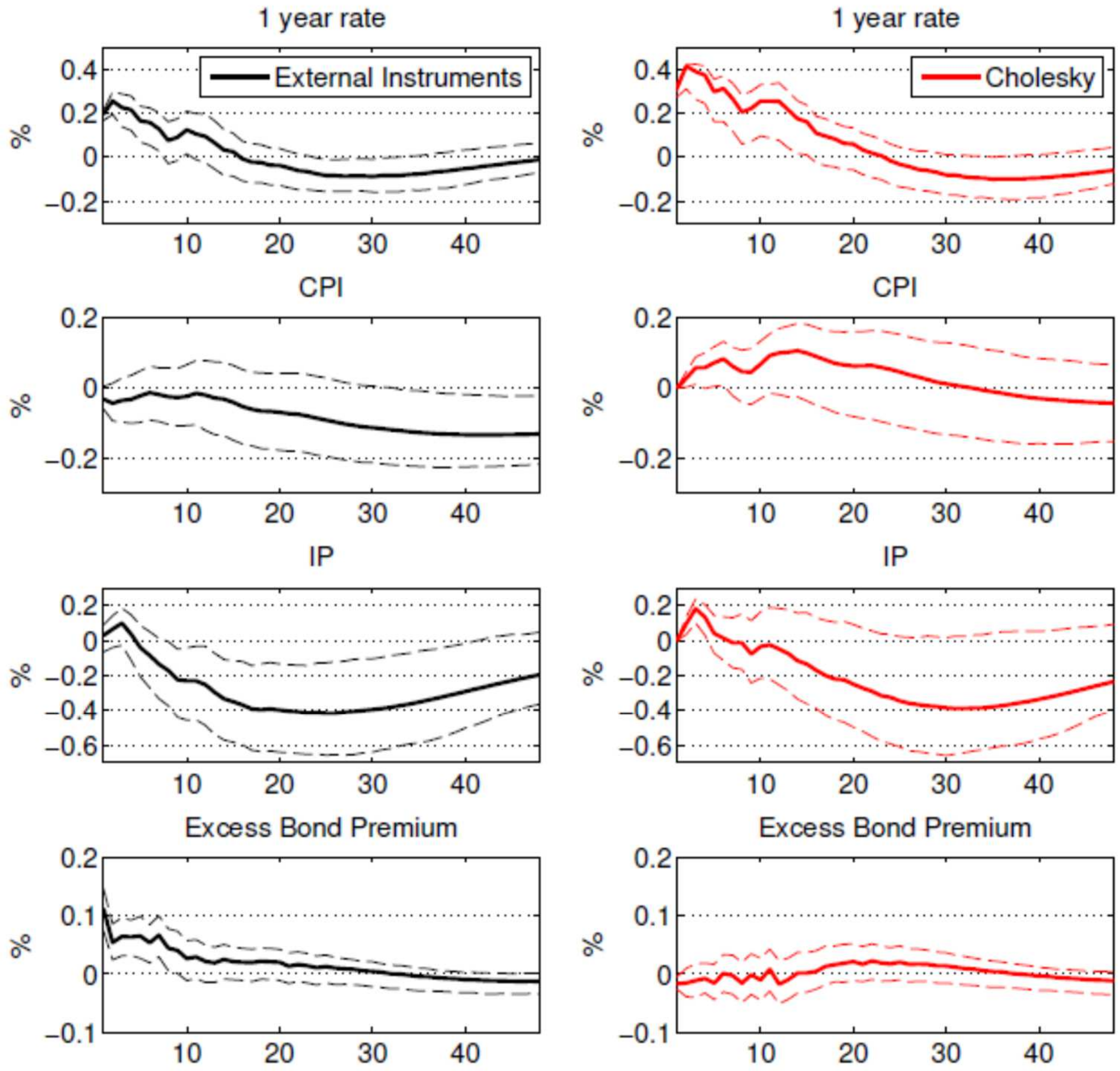
Estimate jth element of $\mathbf{a}^{(1)}$ by 2SLS regression of $\hat{\varepsilon}_{jt}$ on $\hat{\varepsilon}_{1t}$ using $\mathbf{z}_t$ as inst

$$a_j^{(1)} = \frac{\sum_{t\in\mathbb{Q}} \hat{\varepsilon}_{jt}\tilde{\varepsilon}_{1t}}{\sum_{t\in\mathbb{Q}} \tilde{\varepsilon}_{1t}^2}$$

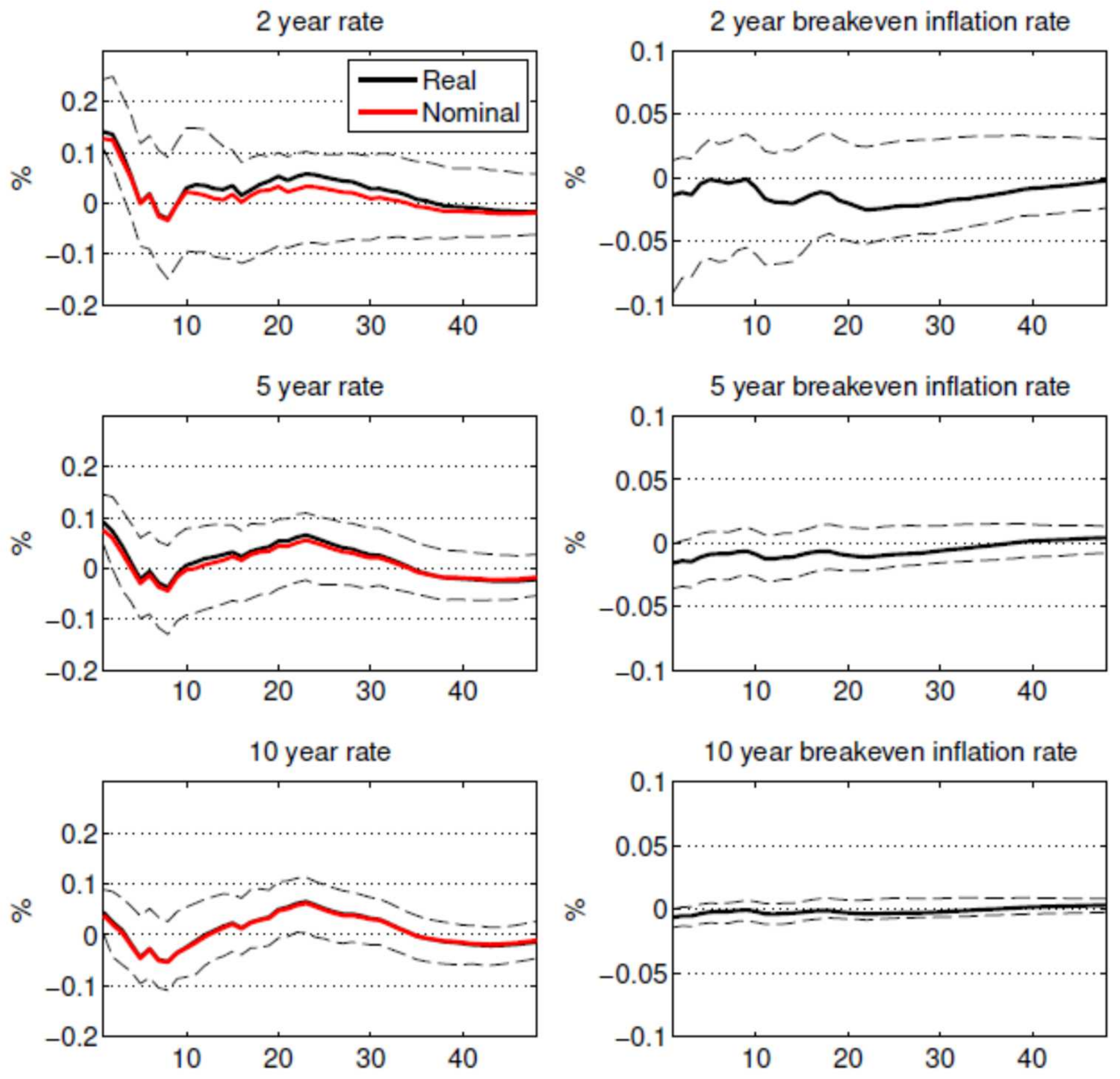
$$\tilde{\varepsilon}_{1t} = \hat{\boldsymbol{\phi}}'\mathbf{z}_t$$

$$\hat{\boldsymbol{\phi}} = \left(\sum_{t\in\mathbb{Q}} \mathbf{z}_t\mathbf{z}_t'\right)\left(\sum_{t\in\mathbb{Q}} \mathbf{z}_t\hat{\varepsilon}_{1t}\right)$$

$$\frac{\partial \mathbf{y}_{t+s}}{\partial u_{1t}} = \boldsymbol{\Psi}_s\mathbf{a}^{(1)}$$


1 year rate
External Instruments
Cholesky
CPI
IP
Excess Bond Premium
%

- Next consider 5-variable VARs, where alternative interest rate measures are added, one at a time


2 year rate
Real
Nominal
%
2 year breakeven inflation rate
5 year rate
5 year breakeven inflation rate
10 year rate
10 year breakeven inflation rate

D. Augmented VAR versus IV estimation

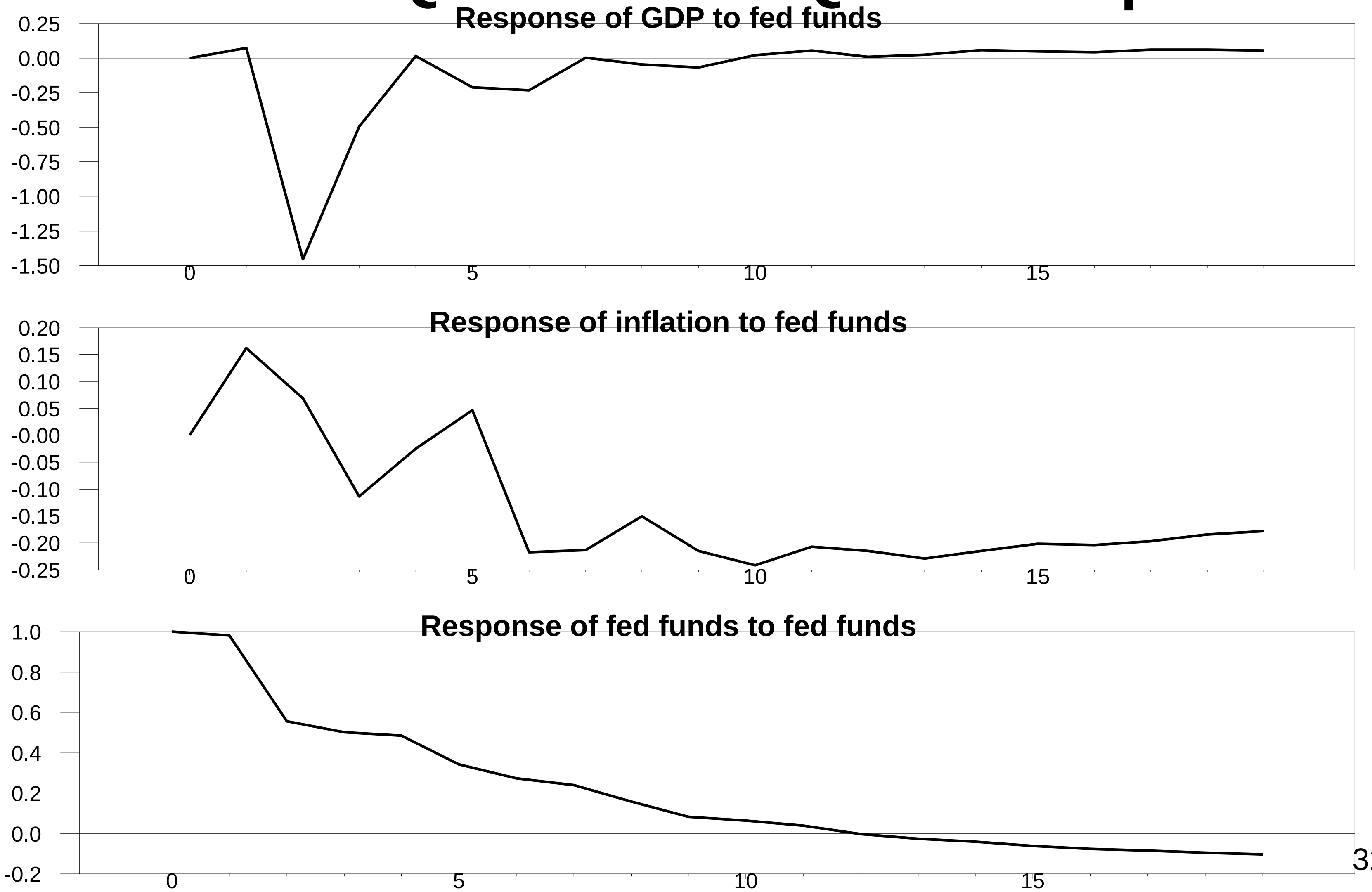
1) advantage of augmented VAR:

structural shock can be linear combination of innovations to $\mathbf{y}_t$ and $z_t,$ not just innovations to $\mathbf{y}_t$

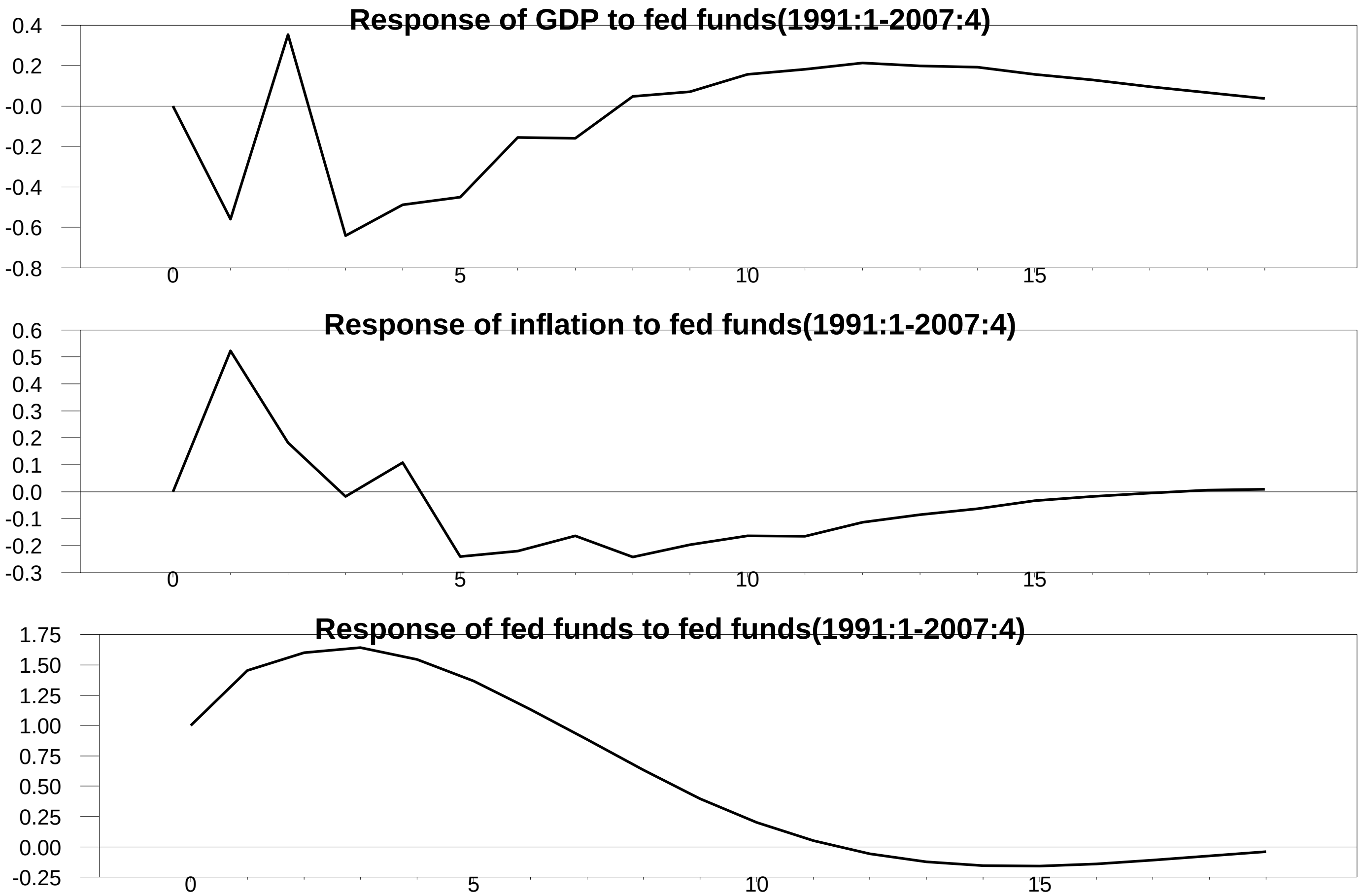
2) advantage of using Stock-Watson IV:

can use longer sample to estimate nonorthogonalized IRF $\mathbf{\Psi}_s$ than for $\mathbf{a}^{(i)}$

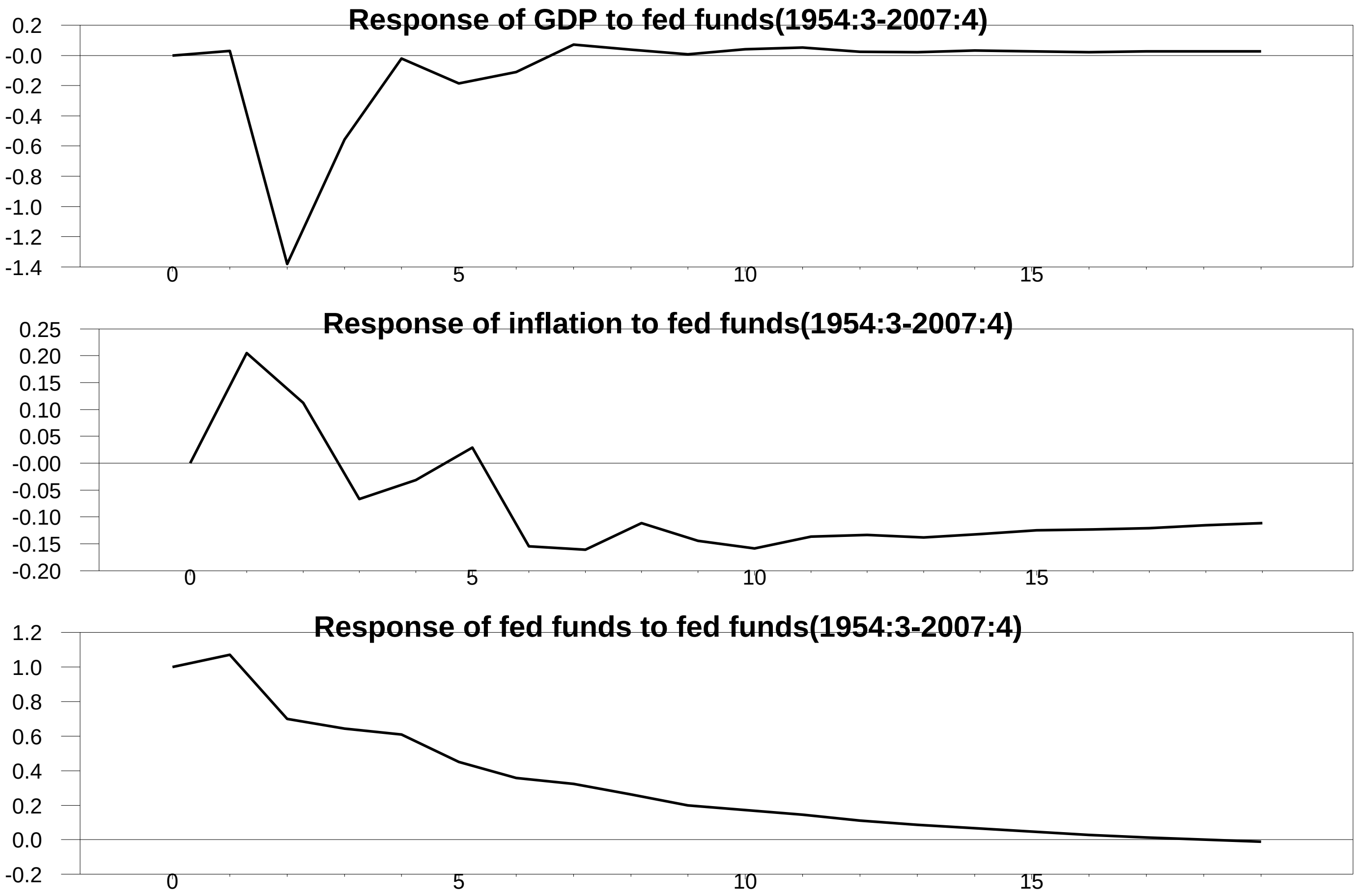
Nonorthogonalized IRF for 1960:Q1-1990:Q4 sample



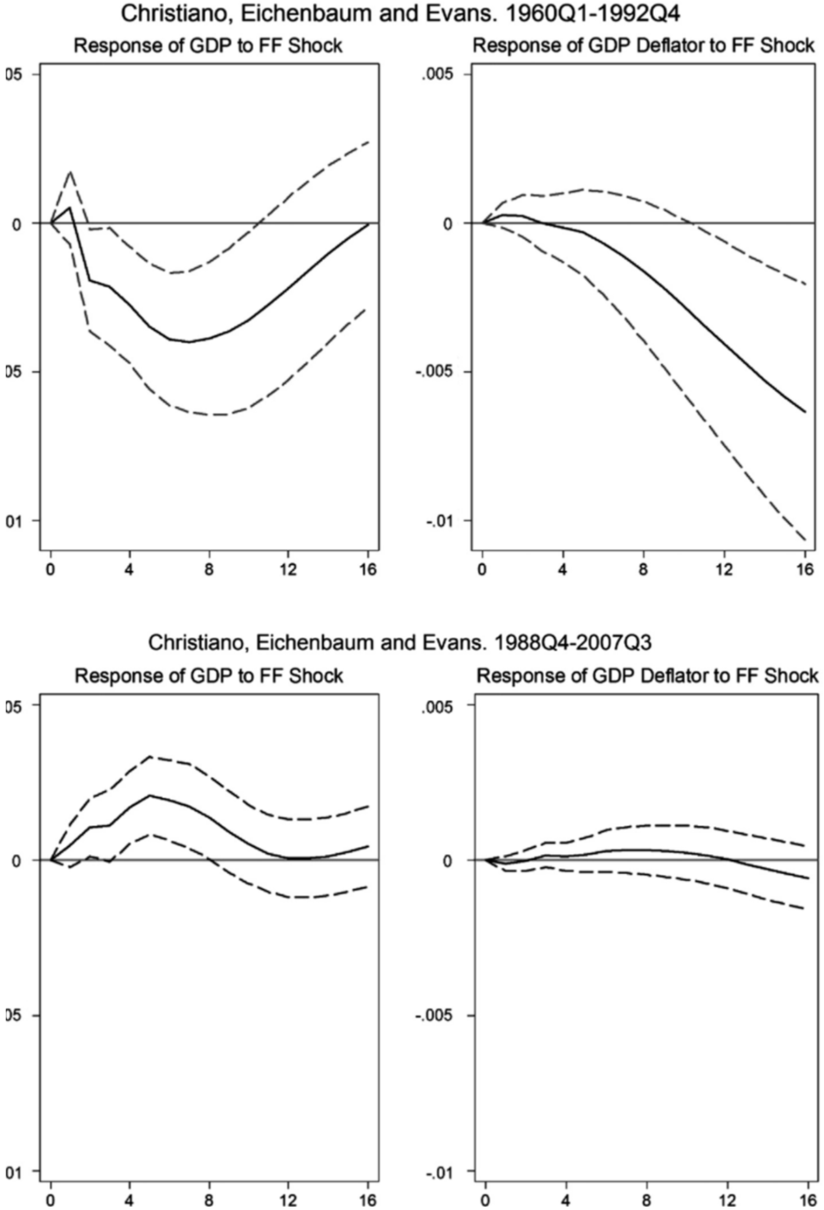
Nonorthogonalized IRF for 1991:Q1-2007:Q4 sample



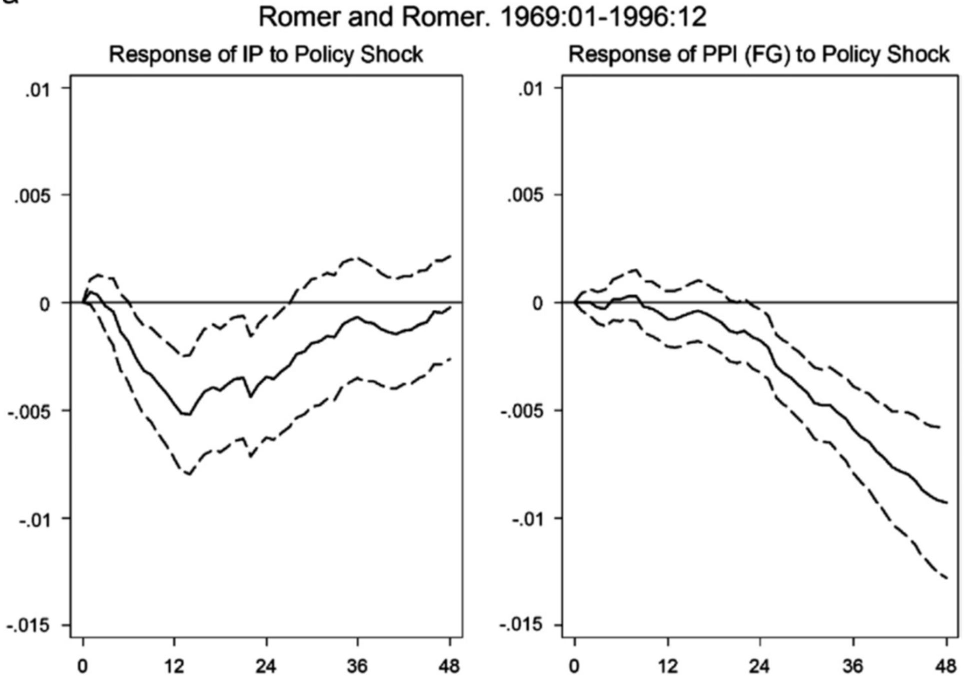
Nonorthogonalized IRF for 1954:Q3-2007:Q4 sample



Barakchian and Crowe (JME, 2013)

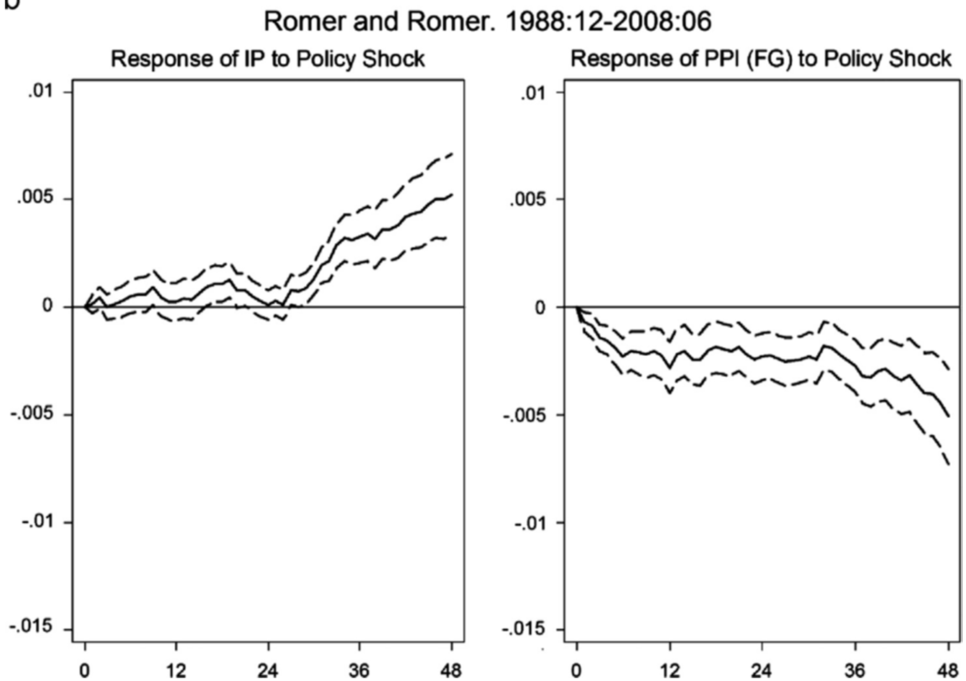


a



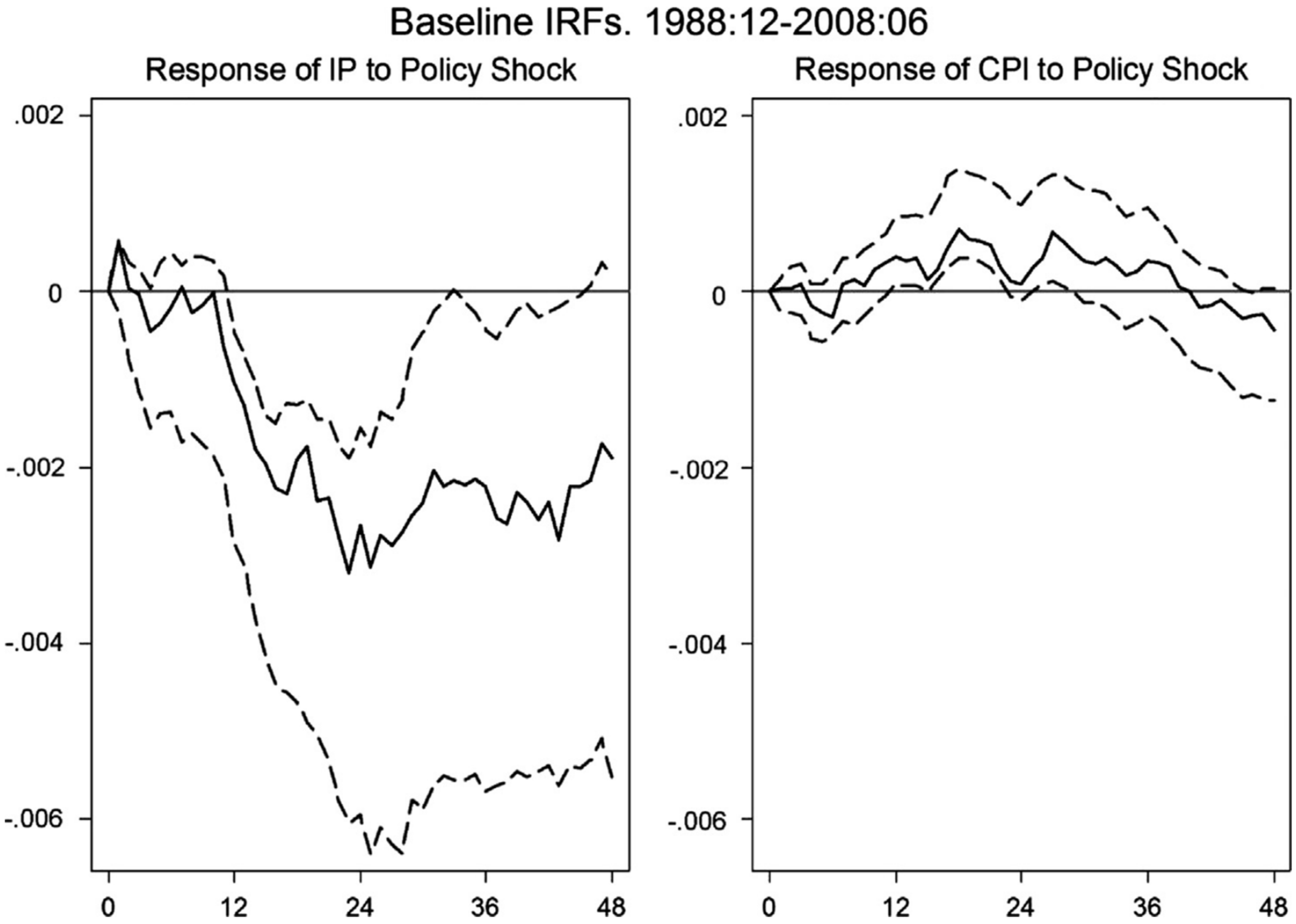
Romer and Romer. 1969:01-1996:12
Response of IP to Policy Shock
Response of PPI (FG) to Policy Shock
.01
.005
0
-.005
-.01
-.015
0
12
24
36
48

b



Romer and Romer. 1988:12-2008:06
Response of IP to Policy Shock
Response of PPI (FG) to Policy Shock
.01
.005
0
-.005
-.01
-.015
0
12
24
36
48

Barakchian and Crowe augmented VAR with accumulated shocks



E. Natural experiments in macro

- Application: permanent-income hypothesis
- Change in income at t that was perfectly anticipated at $t - 1$ should have no effect on consumption at date t
- If find effect it is evidence of borrowing constraints or some departure from neoclassical assumptions

2001 Bush tax rebates (Johnson, Parker, Souleles, AER 2006; Agarwal and Souleles, JPE 2007)

- Households notified by letter months in advance of $300-$600 rebates; substantial press coverage
- Checks delivered over 10-week period based on social security numbers
- Each dollar of rebate added 37¢ in nondurable spending within 3 months of receiving rebate

2008 Economic Stimulus Act rebates (Parker, Souleles, Johnson, McClelland, AER 2013)

- Rebates of $300-$1200 rebates
- Checks delivered or funds wired based on social security numbers
- Each dollar of rebate added 12¢ in nondurable spending within 3 months of receiving rebate (statistically significant)

Aaronson, Agarwal French (2012)	minimum wage	reject
Agarwal and Qian	2011 Singapore growth dividends	cannot reject
Browning and Collado (2011)	Spanish bonus payments scheme	cannot reject
Coulibably and Li (2006)	last mortgage payment	cannot reject
Hsieh (2003)	Alaska permanent fund	cannot reject
Scholnick (2013)	last mortgage payment	reject
Shea (1995)	union wage agreement	reject
Stephens (2008)	last auto payment	reject
Stephens (2003)	Social Security day of month	reject
Wilcox (1989)	Social Security changes	reject

Table 1: Studies of the Permanent Income Hypothesis Sorted by Size and Regularity of the Income Change

	Small	Large
Regular	Aaronson, Agarwal, and French (2012) 0.03% Parker (1999)[a] 0.00038 % Parker (1999)[b] 0.82% Shea (1995) 0.0009%	**Browning and Collado (2001) 2.61%** **Hsieh (2003) 4.79%** **Paxson (1993) -** Souleles (1999) 1.24%
Irregular	Agarwal, Liu, and Souleles (2007) 0.22% **Agarwal and Qian (2014) 0.04%** Broda and Parker (2014) 0.31% **Coulibaly and Li (2006) 0.56%** Johnson, Parker, and Souleles (2006) 0.10% Parker, Souleles, Johnson, and McClelland (2013) 0.46% Scholnick (2013) 0.45% Souleles (2002) 0.01% Stephens (2008) 0.35%	**Souleles (2000)[c] 5.24%**

Note: Papers written in bold fail to reject the Permanent Income Hypothesis. The number after each study indicates the equivalent variation associated with the respective experiment. The equivalent variation is calculated as described in the text. The online appendix provides details on the calculation of the equivalent variation for each paper.