New meeting times for Econ 210D

Mondays 8:00-9:20 a.m. in Econ 300

Wednesdays 11:00-12:20 in Econ 300

Identification using nonrecursive structure, long-run restrictions and heteroskedasticity

General statement of problem of structural interpretation:

Can observe in the data:

$\varepsilon_{1t},\ldots,\varepsilon_{nt}$ = errors I make forecasting variables from lagged values.

Think of these as resulting from n structural shocks:

$$u_{1t} = \text{shock to technology}$$

$$u_{2t} = \text{shock to price markup}$$

$$u_{3t} = \text{shock to monetary policy}$$

$$\boldsymbol{\varepsilon}_t = \mathbf{H}\mathbf{u}_t$$

$$\boldsymbol{\varepsilon}_t = \mathbf{H}\mathbf{u}_t$$

$$E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{\Omega}$$ (can observe in the data)

$$E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{D}$$ (unknown variance of structural shocks)

$$\boldsymbol{\Omega} = \mathbf{H}\mathbf{D}\mathbf{H}'$$

A. Nonrecursive Orthogonalized VARs

Structural model:

$$\mathbf{B}_0\mathbf{y}_t = \mathbf{B}\mathbf{x}_t + \mathbf{u}_t$$

$$\mathbf{x}_t = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \ldots, \mathbf{y}'_{t-p})'$$

$\mathbf{u}_t =$ vector of structural shocks

$E(\mathbf{u}_t\mathbf{u}'_t) = \mathbf{D}$ (diagonal)

recursive identification assumed $\mathbf{B}_0$ was lower triangular

If $\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{D})$ then log likelihood is

$$-(Tn/2)\log(2\pi) - (T/2)\log|\mathbf{B}_0^{-1}\mathbf{D}(\mathbf{B}_0^{-1})'|$$

$$-(1/2)\sum_{t=1}^{T}(\mathbf{B}_0\mathbf{y}_t - \mathbf{B}\mathbf{x}_t)'\mathbf{D}^{-1}(\mathbf{B}_0\mathbf{y}_t - \mathbf{B}\mathbf{x}_t)$$

If $\mathbf{B}$ is unrestricted, MLE of $\mathbf{B}_0$ and $\mathbf{D}$ are values that maximize

$$(T/2)\log|\mathbf{B}_0|^2 - (T/2)\log|\mathbf{D}|$$

$$-(T/2)\text{trace}\{(\mathbf{B}_0'\mathbf{D}^{-1}\mathbf{B}_0)\hat{\boldsymbol{\Omega}}\}$$

If model is just identified, estimates will satisfy

$$\hat{\mathbf{B}}_0^{-1}\hat{\mathbf{D}}(\hat{\mathbf{B}}_0^{-1})' = \hat{\boldsymbol{\Omega}}$$

B. Identification using long-run conditions

x_t log of productivity in quarter t

(log GDP minus log of hours worked)

n_t log of hours worked in nonag establishments

1948:Q1-1994:Q4

$$\mathbf{y}_t = \begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix} \sim I(0)$$

VAR (reduced-form) ($p = 4$)

$$\mathbf{y}_t = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \cdots + \mathbf{\Phi}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t$$

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^{'}) = \mathbf{\Omega}$$

Structural model:

$$\mathbf{B}_0\mathbf{y}_t = \boldsymbol{\lambda} + \mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{B}_2\mathbf{y}_{t-2} + \cdots + \mathbf{B}_p\mathbf{y}_{t-p} + \mathbf{u}_t$$

$E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{I}_2$ (normalization)

Relation between representations:

$$\mathbf{u}_t = \mathbf{B}_0 \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\Omega} = \mathbf{B}_0^{-1} (\mathbf{B}_0^{-1})'$$

Premultiply structural model,

$$\mathbf{B}(L)\mathbf{y}_t = \boldsymbol{\lambda} + \mathbf{u}_t$$

by $\mathbf{C}(L) = \mathbf{B}(L)^{-1}$:

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}_0\mathbf{u}_t + \mathbf{C}_1\mathbf{u}_{t-1} + \mathbf{C}_2\mathbf{u}_{t-2} + \cdots$$

which gives structural MA representation

$$\mathbf{u}_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

u_{1t} technology shock

u_{2t} nontechnology shock

Assumption: only technology shocks can have a permanent effect on productivity

$$\lim_{s\to\infty} \frac{\partial x_{t+s}}{\partial u_{2t}} = 0$$

Notice

$$\frac{\partial x_{t+s}}{\partial u_{2t}} = \frac{\partial(x_{t+s}-x_{t+s-1})}{\partial u_{2t}} + \frac{\partial(x_{t+s-1}-x_{t+s-2})}{\partial u_{2t}} + \cdots + \frac{\partial(x_{t}-x_{t-1})}{\partial u_{2t}}$$

$$\mathbf{y}_t = \begin{bmatrix} x_t - x_{t-1} \\ n_t - n_{t-1} \end{bmatrix}$$

$$\frac{\partial(x_t - x_{t-1})}{\partial u_{2t}} = \frac{\partial y_{1t}}{\partial u_{2t}}$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}_0 \mathbf{u}_t + \mathbf{C}_1 \mathbf{u}_{t-1} + \mathbf{C}_2 \mathbf{u}_{t-2} + \cdots$$

$$\frac{\partial \mathbf{y}_{t+m}}{\partial \mathbf{u}_t'} = \mathbf{C}_m$$

$$\frac{\partial x_{t+s}}{\partial u_{2t}} = \frac{\partial(x_{t+s} - x_{t+s-1})}{\partial u_{2t}} + \frac{\partial(x_{t+s-1} - x_{t+s-2})}{\partial u_{2t}} + \cdots + \frac{\partial(x_t - x_{t-1})}{\partial u_{2t}}$$

is given by the row 1 column 2 element of

$$\mathbf{C}_0 + \mathbf{C}_1 + \mathbf{C}_2 + \cdots + \mathbf{C}_s$$

$$\lim_{s\to\infty} \frac{\partial x_{t+s}}{\partial u_{2t}} = 0$$

requires that the following matrix is lower triangular:

$$\mathbf{C}_0 + \mathbf{C}_1 + \mathbf{C}_2 + \cdots = \mathbf{C}(1)$$

Goal: find structural disturbances $\mathbf{u}_t$ that are a linear combination of the VAR innovations,

$$\mathbf{u}_t = \mathbf{B}_0 \boldsymbol{\varepsilon}_t,$$

such that:

(1) $E(\mathbf{u}_t \mathbf{u}_t') = \mathbf{I}_2$

$$\Rightarrow \mathbf{B}_0 \boldsymbol{\Omega} \mathbf{B}_0' = \mathbf{I}_2$$

$$\Rightarrow \boldsymbol{\Omega} = (\mathbf{B}_0^{-1})(\mathbf{B}_0^{-1})'$$

(2) $\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}(L)\mathbf{u}_t$

(3) $\mathbf{C}(1)$ is lower triangular

$$\mathbf{\Phi}(L)\mathbf{y}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\varepsilon}_t = \mathbf{B}_0^{-1}\mathbf{u}_t$$

$$\Rightarrow \mathbf{\Phi}(L)\mathbf{y}_t = \mathbf{c} + \mathbf{B}_0^{-1}\mathbf{u}_t$$

$$\Rightarrow \mathbf{y}_t = \boldsymbol{\mu} + [\mathbf{\Phi}(L)]^{-1}\mathbf{B}_0^{-1}\mathbf{u}_t$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}(L)\mathbf{u}_t$$

$$\Rightarrow \mathbf{C}(1) = [\mathbf{\Phi}(1)]^{-1}\mathbf{B}_0^{-1}$$

$$\mathbf{C}(1) = [\mathbf{\Phi}(1)]^{-1}\mathbf{B}_0^{-1}$$

$$\mathbf{C}(1)[\mathbf{C}(1)]' =$$

$$[\mathbf{\Phi}(1)]^{-1}\mathbf{B}_0^{-1}(\mathbf{B}_0^{-1})'\left\{[\mathbf{\Phi}(1)]^{-1}\right\}'$$

$$\mathbf{C}(1)[\mathbf{C}(1)]' = [\mathbf{\Phi}(1)]^{-1}\mathbf{\Omega}\left\{[\mathbf{\Phi}(1)]^{-1}\right\}'$$

Can estimate $\mathbf{\Phi}(1)$ and $\mathbf{\Omega}$ from VAR

$$\hat{\mathbf{\Omega}} = T^{-1}\sum_{t=1}^{T}\hat{\boldsymbol{\varepsilon}}_t\hat{\boldsymbol{\varepsilon}}_t'$$

$$\hat{\mathbf{\Phi}}(1) = \mathbf{I}_2 - \hat{\mathbf{\Phi}}_1 - \hat{\mathbf{\Phi}}_2 - \hat{\mathbf{\Phi}}_3 - \hat{\mathbf{\Phi}}_4$$

Want: Lower triangular matrix $\mathbf{C}(1)$ such that

$$\mathbf{C}(1)[\mathbf{C}(1)]' = [\mathbf{\Phi}(1)]^{-1}\mathbf{\Omega}\left\{[\mathbf{\Phi}(1)]^{-1}\right\}'$$

Conclusion: $\mathbf{C}(1)$ is Cholesky factor of

$$[\mathbf{\Phi}(1)]^{-1}\mathbf{\Omega}\left\{[\mathbf{\Phi}(1)]^{-1}\right\}'$$

To get $\mathbf{B}_0$ we then use fact that

$$\mathbf{C}(1) = [\boldsymbol{\Phi}(1)]^{-1}\mathbf{B}_0^{-1}$$

$$\mathbf{B}_0 = [\mathbf{C}(1)]^{-1}[\boldsymbol{\Phi}(1)]^{-1}$$

Summary:

(1) Estimate VAR by OLS

$$\mathbf{y}_t = \begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix}$$

$$\mathbf{y}_t = \hat{\mathbf{c}} + \hat{\mathbf{\Phi}}_1 \mathbf{y}_{t-1} + \hat{\mathbf{\Phi}}_2 \mathbf{y}_{t-2} + \cdots + \hat{\mathbf{\Phi}}_p \mathbf{y}_{t-p} + \hat{\boldsymbol{\varepsilon}}_t$$

$$\hat{\mathbf{\Omega}} = T^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t'$$

(2) Find Cholesky factor or lower triangular matrix $\widehat{\mathbf{C}}$ such that

$$\widehat{\mathbf{C}}\widehat{\mathbf{C}}' = \hat{\mathbf{Q}}\hat{\mathbf{\Omega}}\hat{\mathbf{Q}}'$$

$$\hat{\mathbf{Q}} = \left(\mathbf{I}_2 - \hat{\mathbf{\Phi}}_1 - \hat{\mathbf{\Phi}}_2 - \cdots - \hat{\mathbf{\Phi}}_p\right)^{-1}$$

(3) Technology shock and transitory shocks for date t are first and second elements of

$$\hat{\mathbf{u}}_t = \hat{\mathbf{B}}_0 \hat{\boldsymbol{\varepsilon}}_t$$

where

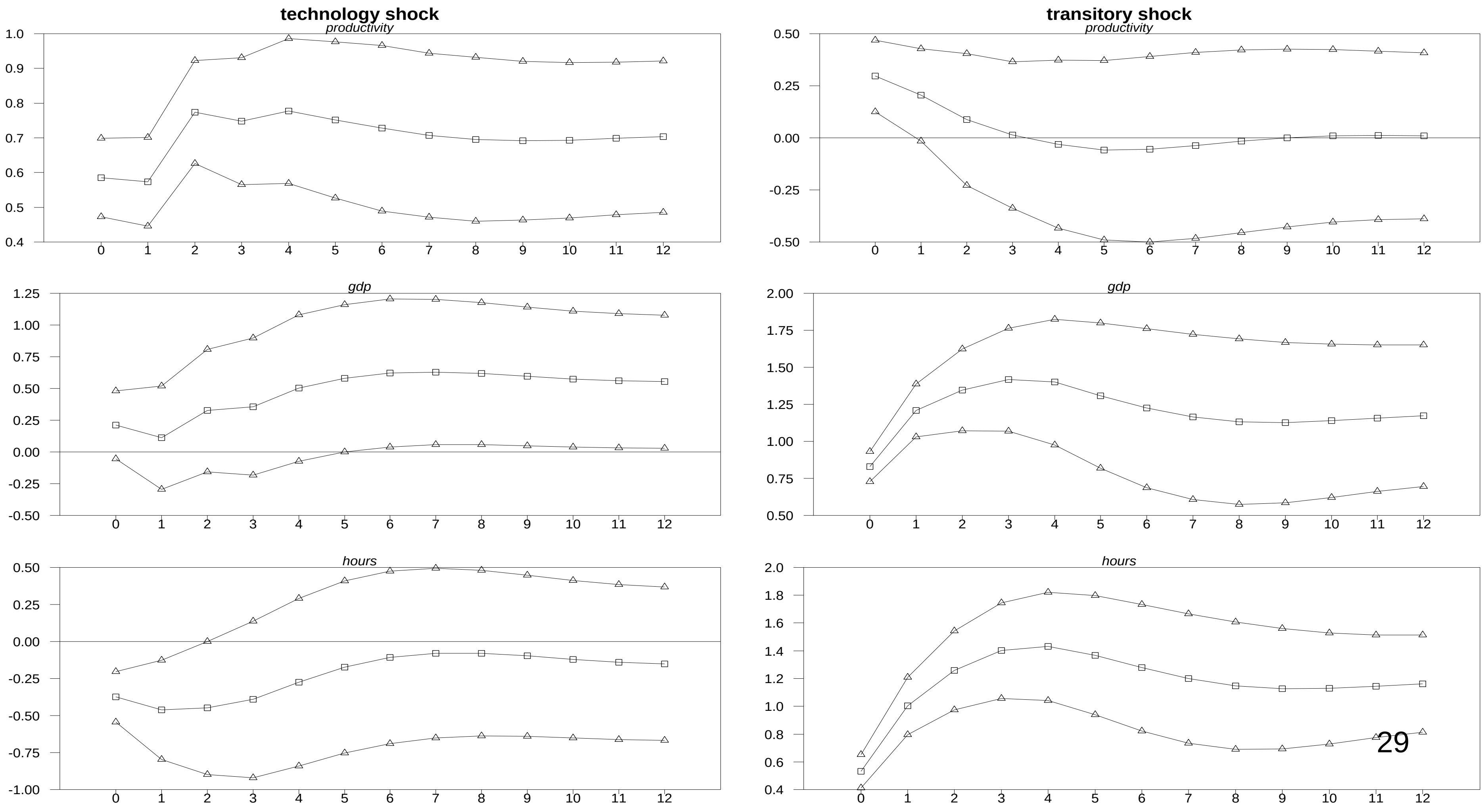
$$\hat{\mathbf{B}}_0 = \widehat{\mathbf{C}}^{-1} \hat{\mathbf{Q}}$$

(4) Effect of tech shock and transitory shock at date t on $\mathbf{y}_{t+s}$ are given by first and second columns, respectively, of

$$\frac{\partial \mathbf{y}_{t+s}}{\partial \mathbf{u}_t'} = \mathbf{\Psi}_s \mathbf{B}_0^{-1}$$

Estimated structural IRF with 95% CI (Fig 2 in Galí)

Figure 2 : US



- Conclusion: technology shock raises productivity and lowers hours worked

$$\hat{\boldsymbol{\Omega}} = \begin{bmatrix} 0.43 & -0.06 \\ -0.06 & 0.42 \end{bmatrix}$$

small negative correlation between VAR resids

$$\lim_{s\to\infty}\begin{bmatrix} \frac{\partial x_{t+s}}{\partial \varepsilon_{1t}} & \frac{\partial x_{t+s}}{\partial \varepsilon_{2t}} \\ \frac{\partial n_{t+s}}{\partial \varepsilon_{1t}} & \frac{\partial n_{t+s}}{\partial \varepsilon_{2t}} \end{bmatrix} = [\hat{\boldsymbol{\Phi}}(1)]^{-1} = \begin{bmatrix} 0.89 & -0.50 \\ 0.87 & 1.73 \end{bmatrix}$$

For u_{2t} to have zero long-run effect on x_t, it must be interpreted as something that moves ε_{1t} and ε_{2t} in the same direction.

For u_{1t} to be uncorrelated with u_{2t}, it must be interpreted as something that moves ε_{1t} and ε_{2t} in opposite directions.

So if technology shock raises productivity, it must increase ε_{1t} and decrease ε_{2t}.

$$\boldsymbol{\varepsilon}_t = \mathbf{B}_0^{-1}\mathbf{u}_t$$

$$\hat{\mathbf{B}}_0^{-1} = \begin{bmatrix} 0.59 & 0.30 \\ -0.37 & 0.53 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\partial \varepsilon_{1t}}{\partial u_{1t}} & \dfrac{\partial \varepsilon_{1t}}{\partial u_{2t}} \\ \dfrac{\partial \varepsilon_{2t}}{\partial u_{1t}} & \dfrac{\partial \varepsilon_{2t}}{\partial u_{2t}} \end{bmatrix} = \begin{bmatrix} 0.59 & 0.30 \\ -0.37 & 0.53 \end{bmatrix}$$

$$\lim_{s\to\infty}\begin{bmatrix} \dfrac{\partial x_{t+s}}{\partial u_{1t}} & \dfrac{\partial x_{t+s}}{\partial u_{2t}} \\ \dfrac{\partial n_{t+s}}{\partial u_{1t}} & \dfrac{\partial n_{t+s}}{\partial u_{2t}} \end{bmatrix} = [\hat{\mathbf{\Phi}}(1)]^{-1}\mathbf{B}_0^{-1}$$

$$= \begin{bmatrix} 0.89 & -0.50 \\ 0.87 & 1.73 \end{bmatrix}\begin{bmatrix} 0.59 & 0.30 \\ -0.37 & 0.53 \end{bmatrix}$$

$$= \begin{bmatrix} 0.71 & 0 \\ -0.14 & 1.18 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial \varepsilon_{1t}}{\partial u_{1t}} & \frac{\partial \varepsilon_{1t}}{\partial u_{2t}} \\ \frac{\partial \varepsilon_{2t}}{\partial u_{1t}} & \frac{\partial \varepsilon_{2t}}{\partial u_{2t}} \end{bmatrix} = \begin{bmatrix} 0.59 & 0.30 \\ -0.37 & 0.53 \end{bmatrix}$$

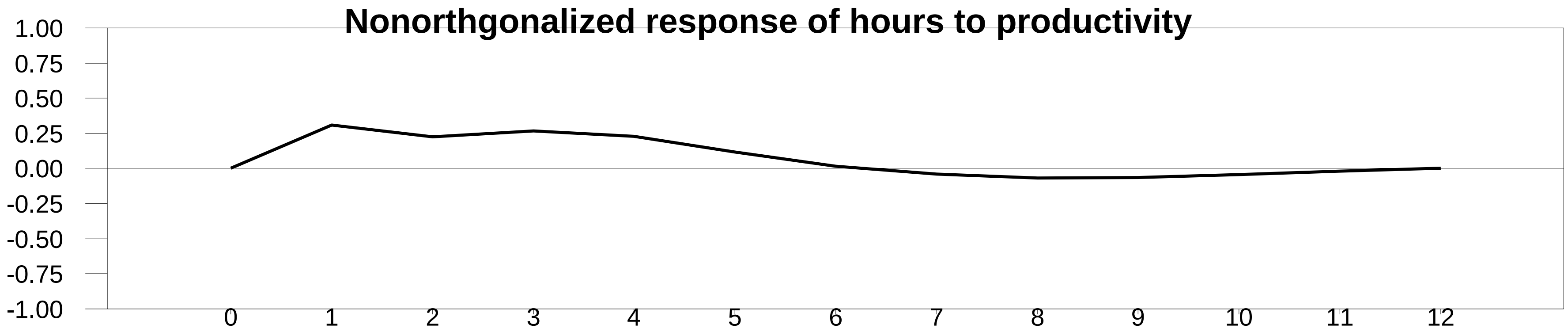
A one-unit shock to u_{2t} is interpreted as causing a 0.30 increase in ε_{1t} and 0.53 increase in ε_{2t}

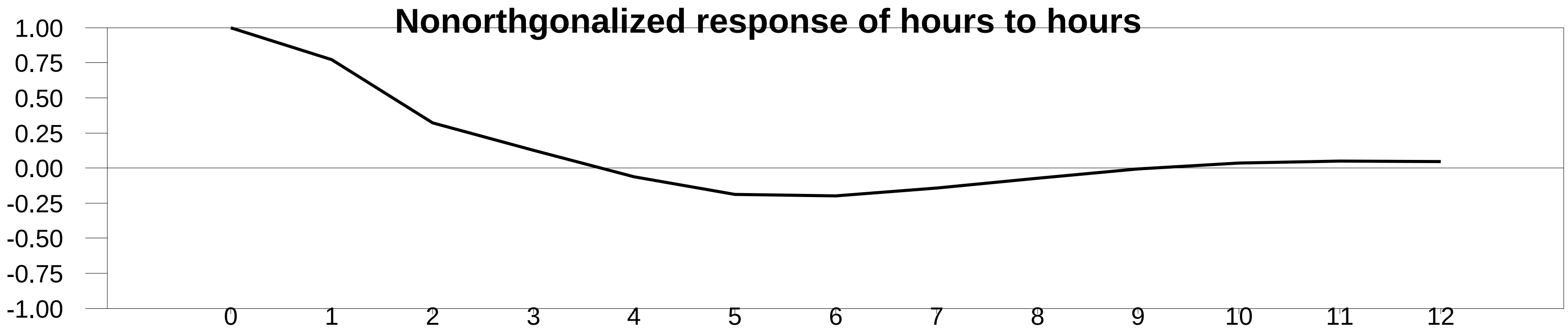
$$\lim_{s \to \infty} \frac{\partial x_{t+s}}{\partial u_{2t}} = (0.89)(0.30) - (0.50)(0.53) = 0$$

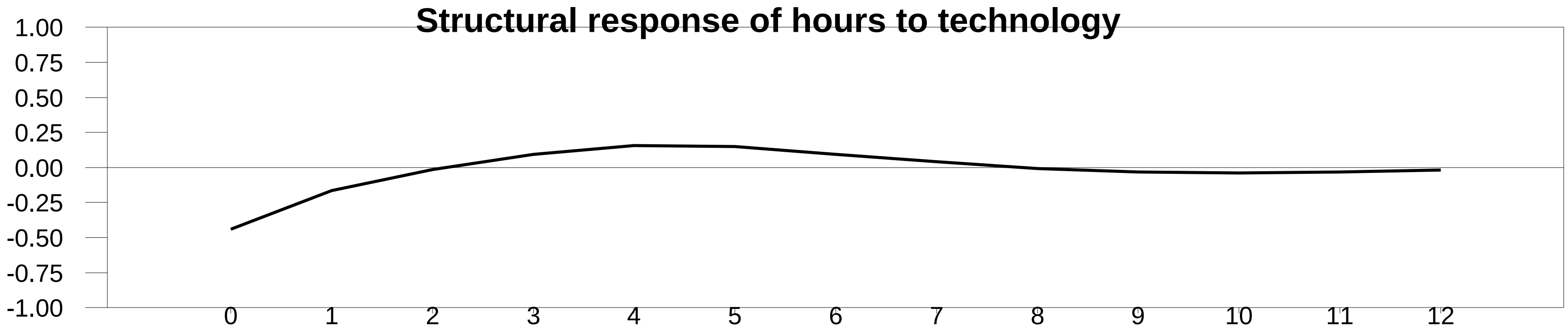
$$\begin{bmatrix} \frac{\partial \varepsilon_{1t}}{\partial u_{1t}} & \frac{\partial \varepsilon_{1t}}{\partial u_{2t}} \\ \frac{\partial \varepsilon_{2t}}{\partial u_{1t}} & \frac{\partial \varepsilon_{2t}}{\partial u_{2t}} \end{bmatrix} = \begin{bmatrix} 0.59 & 0.30 \\ -0.37 & 0.53 \end{bmatrix}$$

A one-unit shock to u_{1t} is interpreted as causing a 0.59 increase in ε_{1t} and 0.37 decrease in ε_{2t}

$$\frac{\partial n_{t+s}}{\partial u_{1t}} = 0.59 \frac{\partial n_{t+s}}{\partial \varepsilon_{1t}} - 0.37 \frac{\partial n_{t+s}}{\partial \varepsilon_{1t}}$$


Nonorthgonalized response of hours to productivity
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0 1 2 3 4 5 6 7 8 9 10 11 12


Nonorthgonalized response of hours to hours
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0 1 2 3 4 5 6 7 8 9 10 11 12


Structural response of hours to technology
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0 1 2 3 4 5 6 7 8 9 10 11 12

Additional comments:

(1) $\hat{\mathbf{Q}} = \left(\mathbf{I}_2 - \hat{\mathbf{\Phi}}_1 - \hat{\mathbf{\Phi}}_2 - \cdots - \hat{\mathbf{\Phi}}_p\right)^{-1}$

is estimated poorly, sensitive to p

Note: algorithm may be a little more robust if instead use

$$\hat{\mathbf{Q}} = \hat{\mathbf{\Psi}}_0 + \hat{\mathbf{\Psi}}_1 + \cdots + \hat{\mathbf{\Psi}}_m \text{ for some } m$$

Chari, Kehoe and McGrattan (2008) give example of DSGE for which when this kind of procedure is applied does not uncover technology shock.

(2) technology shock could be temporary

(e.g., delay in adoption of discovered technology)

(3) demand shock could be permanent

(e.g., lost human capital)

C. Identification using heteroskedasticity (Wright, 2012)

$t =$ daily Nov 3, 2008 to Sep 30, 2011

$y_{1t} =$ 2-year Treasury yield

$y_{2t} =$ 10-year Treasury yield

$y_{3t} =$ 5-year TIPS break-even

(nominal yield minus TIPS yield)

$y_{4t} =$ 5-10-year TIPS forward break-even

($2 \times$ 10-year TIPS break-even minus

5-year TIPS break-even)

$y_{5t} =$ BAA yield

$y_{6t} =$ AAA yield

$$\mathbf{y}_t = \mathbf{c} + \boldsymbol{\Phi}_1 \mathbf{y}_{t-1} + \cdots + \boldsymbol{\Phi}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\varepsilon}_t = \mathbf{B}_0^{-1} \mathbf{u}_t$$

u_{1t} = monetary policy shock

want to estimate $\mathbf{b}^{(1)}$ (first column of $\mathbf{B}_0^{-1}$)

Suppose we believed that:

(1) monetary policy shocks have higher variance on particular days

$$E(u_{1t}^2) = \begin{cases} d_{11}^{(0)} + \lambda & \text{if } t \in S \\ d_{11}^{(0)} & \text{if } t \notin S \end{cases}$$

Set S is known (Wright uses FOMC dates and dates of monetary policy announcements)

Date	Event	Time
11/25/2008	*Fed announces purchases of MBS and agency bonds*	*08:15*
12/1/2008	*Bernanke states Treasuries may be purchased*	*13:45*
12/16/2008	*FOMC Meeting*	*14:15*
1/28/2009	*FOMC Meeting*	*14:15*
3/18/2009	*FOMC Meeting*	*14:15*
4/29/2009	FOMC Meeting	14:15
6/24/2009	FOMC Meeting	14:15
8/12/2009	FOMC Meeting	14:15
9/23/2009	FOMC Meeting	14:15
11/4/2009	FOMC Meeting	14:15
12/16/2009	FOMC Meeting	14:15
1/27/2010	FOMC Meeting	14:15
3/16/2010	FOMC Meeting	14:15
4/28/2010	FOMC Meeting	14:15
6/23/2010	FOMC Meeting	14:15
8/10/2010	*FOMC Meeting*	*14:15*
8/27/2010	*Bernanke speech at Jackson Hole*	*10:00*
9/21/2010	*FOMC Meeting*	*14:15*
10/15/2010	*Bernanke speech at Boston Fed*	*08:15*
11/3/2010	*FOMC Meeting*	*14:15*
12/14/2010	FOMC Meeting	14:15
1/26/2011	FOMC Meeting	14:15
3/15/2011	FOMC Meeting	14:15
4/27/2011	FOMC Meeting	12:30
6/2/2011	FOMC Meeting	12:30
8/9/2011	*FOMC Meeting*	*14:15*
8/26/2011	*Bernanke Speech at Jackson Hole*	*10:00*
9/21/2011	*FOMC Meeting*	*14:15*

(2) A monetary policy shock of given size would have the same effects on these dates as others

(3) Variance and effects of other shocks same on these dates as others

Then

$$E(\mathbf{u}_t\mathbf{u}_t') = \begin{cases} \mathbf{D} + \lambda\mathbf{e}_1\mathbf{e}_1' & \text{if } t \in S \\ \mathbf{D} & \text{if } t \notin S \end{cases}$$

$\mathbf{e}_1$ = col 1 of $\mathbf{I}_n$

$$\boldsymbol{\varepsilon}_t = \mathbf{B}_0^{-1}\mathbf{u}_t = \sum_{i=1}^{n} \mathbf{b}^{(i)} u_{it}$$

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \begin{cases} \mathbf{B}_0^{-1}\mathbf{D}(\mathbf{B}_0^{-1})' + \lambda \mathbf{b}^{(1)}(\mathbf{b}^{(1)})' & \text{if } t \in S \\ \mathbf{B}_0^{-1}\mathbf{D}(\mathbf{B}_0^{-1})' & \text{if } t \notin S \end{cases}$$

$$\hat{\mathbf{\Omega}}_1 = T_1^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t' \delta(t \in S)$$

$$T_1 = \sum_{t=1}^{T} \delta(t \in S)$$

$$\hat{\mathbf{\Omega}}_0 = T_0^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t' \delta(t \notin S)$$

$$T_0 = \sum_{t=1}^{T} \delta(t \notin S)$$

$$\hat{\mathbf{\Omega}}_1 - \hat{\mathbf{\Omega}}_0 \xrightarrow{p} \lambda \mathbf{b}^{(1)} (\mathbf{b}^{(1)})'$$

so we can estimate $\mathbf{b}^{(1)}$ up to an unknown scale, e.g.: normalize $\lambda = 1$

$$\sqrt{T_1}\,[\text{vech}(\hat{\boldsymbol{\Omega}}_1) - \text{vech}(\boldsymbol{\Omega}_1)]$$

$$\xrightarrow{L} N(\mathbf{0}, \mathbf{V}_1)$$

element of $\mathbf{V}_1$ corresponding to covariance between $\hat{\sigma}_{ij}$ and $\hat{\sigma}_{\ell m}$ given by $(\sigma_{i\ell}\sigma_{jm} + \sigma_{im}\sigma_{j\ell})$ (Hamilton, TSA, p. 301).

(1) Test null hypothesis that $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega}_1$

$$\hat{\mathbf{q}}'[\hat{\mathbf{V}}_1/T_1 + \hat{\mathbf{V}}_0/T_0]^{-1}\hat{\mathbf{q}} \xrightarrow{L} \chi^2(n(n+1)/2)$$

$\hat{\mathbf{q}} = \text{vech}(\hat{\boldsymbol{\Omega}}_1) - \text{vech}(\hat{\boldsymbol{\Omega}}_0)$

or bootstrap critical value

This statistic is 67.9.

asymptotic: $P[\chi^2(21) > 46.8] = 0.001$

bootstrap p-value $= 0.005$

$\Longrightarrow$ reject $H_0 : \mathbf{\Omega}_0 = \mathbf{\Omega}_1$

Variance on announcement days different from others so this assumption of framework is correct.

(2) Estimate $\mathbf{b}^{(1)}$ by minimum chi square:

$$\hat{\mathbf{b}}^{(1)} = \arg\min_{\mathbf{b}^{(1)}} \tilde{\mathbf{q}}'[\hat{\mathbf{V}}_1/T_1 + \hat{\mathbf{V}}_0/T_0]^{-1}\tilde{\mathbf{q}}$$

$$\tilde{\mathbf{q}} = \hat{\mathbf{q}} - \text{vech}[\mathbf{b}^{(1)}(\mathbf{b}^{(1)})']$$

$$\widehat{\frac{\partial \mathbf{y}_{t+s}}{\partial u_{1t}}} = \hat{\mathbf{\Psi}}_s \hat{\mathbf{b}}^{(1)}$$

(3) Test null hypothesis restriction valid: value of objective function asymptotically $\chi^2(n(n-1)/2)$ or bootstrap critical value. This test does not reject

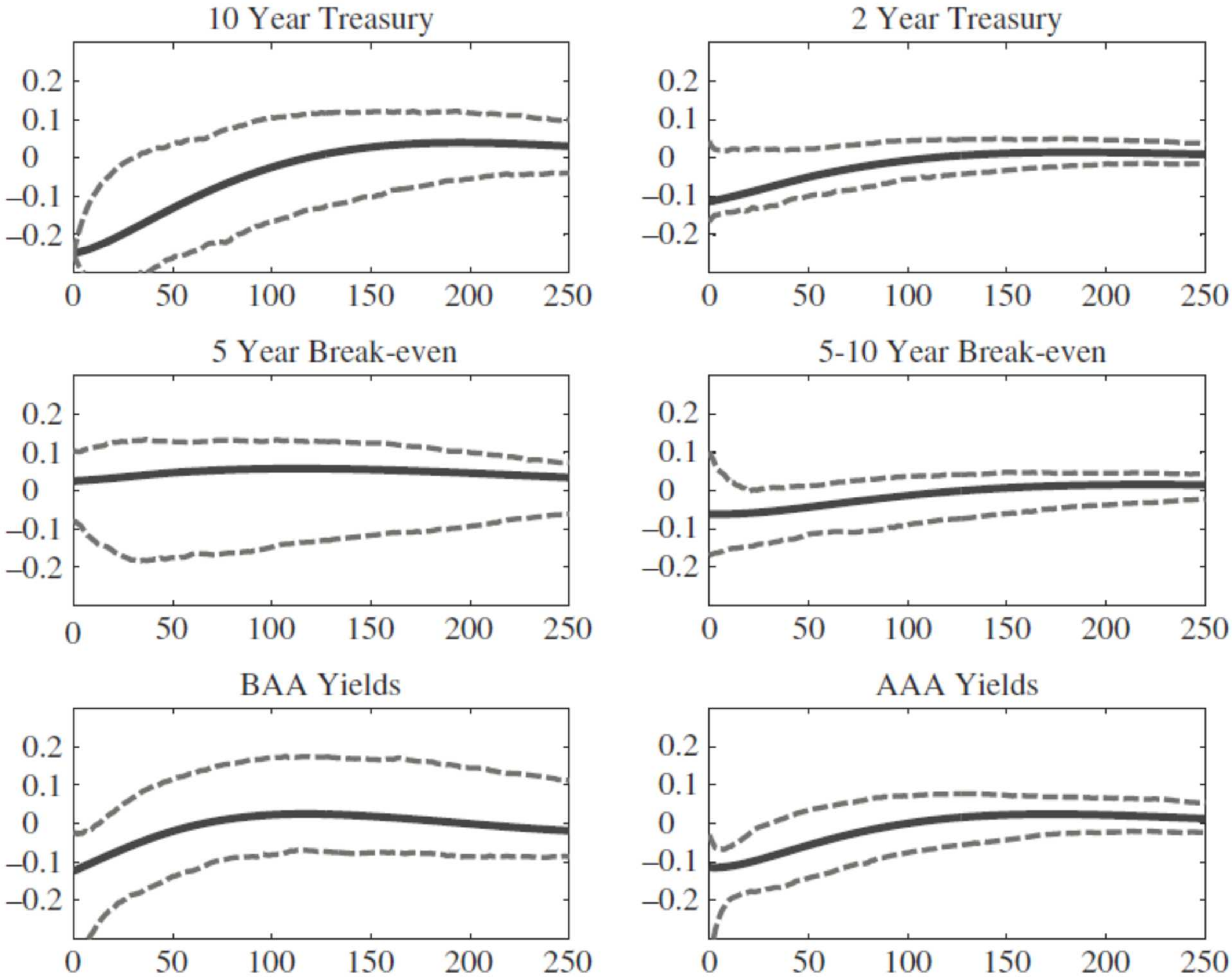
$\Rightarrow$ assumption that $\mathbf{\Omega}_1 = \mathbf{\Omega}_0 + \mathbf{b}^{(1)}\mathbf{b}^{(1)\prime}$

is consistent with observed data.

Normalization: second element of

$$\mathbf{b}^{(1)} = -0.25$$

Monetary policy shock lowers 10-year yield by 25 bp.



10 Year Treasury
2 Year Treasury
5 Year Break-even
5-10 Year Break-even
BAA Yields
AAA Yields
0.2
0.1
0
−0.1
−0.2
0
50
100
150
200
250

- No evidence that unconventional monetary policy works through changes in expected inflation or risk premia.