Orthogonalized VARs

A. Recursively orthogonalized VAR
B. Variance decomposition
C. Historical decomposition
D. Structural interpretation
E. Generalized IRFs

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A. Recursively orthogonalized VARs

Nonorthogonal IRF:
$$\mathbf{\Psi}_s = \frac{\partial E(\mathbf{y}_{t+s}|\mathbf{y}_t, \mathbf{y}_{t-1}, ..., \mathbf{y}_{t-p})}{\partial \mathbf{y}_t'}$$
$$(n \times n)$$

Column 1 $= \frac{\partial E(\mathbf{y}_{t+s}|y_{1t}, y_{2t}, ..., y_{nt}, \mathbf{y}_{t-1}, ..., \mathbf{y}_{t-p})}{\partial y_{1t}}$

e.g., already have data on $y_{2t}, ..., y_{nt}$ and ask how a 1-unit change in y_{1t} affects forecast.

2

Could instead ask $\frac{\partial E(\mathbf{y}_{t+s}|y_{1t}, \mathbf{y}_{t-1}, ..., \mathbf{y}_{t-p})}{\partial y_{1t}}$

e.g., don't have any data from period t except for y_{1t} and ask how 1-unit change in y_{1t} affects forecast.

Knowing y_{1t} gives us information about $y_{2t}, ..., y_{nt}$ if VAR forecast errors are correlated.

3

How calculate $\mathbf{h}_{s1} = \frac{\partial E(\mathbf{y}_{t+s}|y_{1t}, \mathbf{y}_{t-1}, ..., \mathbf{y}_{t-p})}{\partial y_{1t}}$?
$$(n \times 1)$$

Method 1: local projection
Estimate by n OLS equations
$$\mathbf{y}_{t+s} = \mathbf{c}_s + \mathbf{h}_{s1} y_{1t} + \mathbf{H}_{s2} \mathbf{y}_{t-1} + \cdots + \mathbf{H}_{sp} \mathbf{y}_{t-p+1} + \mathbf{u}_{t+s}$$

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Method 2: calculate answer implied by VAR
$$\mathbf{y}_t = (y_{1t}, y_{2t}, ..., y_{nt})'$$
$$(n \times 1)$$
$$\mathbf{x}_t = (1, \mathbf{y}_{t-1}', \mathbf{y}_{t-2}', ..., \mathbf{y}_{t-p}')'$$
$$(k \times 1)$$
$$k = np + 1$$
$$\mathbf{y}_t = \mathbf{\Gamma}' \mathbf{x}_t + \mathbf{\varepsilon}_t$$
$$E(\mathbf{\varepsilon}_t \mathbf{\varepsilon}_t') = \mathbf{\Omega}$$

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Given parameters, observation of $y_{1t}, \mathbf{y}_{t-1}, ..., \mathbf{y}_{t-p}$
allows us to observe
$$\varepsilon_{1t} = y_{1t} - \mathbf{\gamma}_1' \mathbf{x}_t$$

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1

Can calculate optimal forecast of ε_{it} given ε_{1t} as

$$E(\varepsilon_{it}|\varepsilon_{1t}) = \frac{\sigma_{i1}}{\sigma_{11}}\varepsilon_{1t}$$

$$E(\boldsymbol{\varepsilon}_t|\varepsilon_{1t}) = \begin{bmatrix} 1 \\ \sigma_{21}/\sigma_{11} \\ \vdots \\ \sigma_{n1}/\sigma_{11} \end{bmatrix} \varepsilon_{1t} = \mathbf{a}_1\varepsilon_{1t}$$

$$\frac{\partial E(\mathbf{y}_t|y_{1t},\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial y_{1t}} = \mathbf{a}_1$$

$$\frac{\partial E(\mathbf{y}_{t+s}|y_{1t},\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial y_{1t}} = \frac{\partial E(\mathbf{y}_{t+s}|\mathbf{y}_t,\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial \mathbf{y}_t'} \frac{\partial E(\mathbf{y}_t|y_{1t},\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial y_{1t}}$$

$$= \boldsymbol{\Psi}_s\mathbf{a}_1$$

$$\underset{(n\times k)}{\hat{\boldsymbol{\Gamma}}'} = \left(\sum_{t=1}^{T}\mathbf{y}_t\mathbf{x}_t'\right)\left(\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\right)^{-1} \Rightarrow \hat{\boldsymbol{\Psi}}_s$$

$$\underset{(n\times 1)}{\hat{\boldsymbol{\varepsilon}}_t} = \mathbf{y}_t - \hat{\boldsymbol{\Gamma}}'\mathbf{x}_t$$

$$\underset{(n\times n)}{\hat{\boldsymbol{\Omega}}} = T^{-1}\sum_{t=1}^{T}\hat{\boldsymbol{\varepsilon}}_t\hat{\boldsymbol{\varepsilon}}_t' \Rightarrow \hat{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ \hat{\sigma}_{21}/\hat{\sigma}_{11} \\ \vdots \\ \hat{\sigma}_{n1}/\hat{\sigma}_{11} \end{bmatrix}$$

Could also do this using Cholesky factor:

$$\hat{\boldsymbol{\Omega}} = \hat{\mathbf{P}}\hat{\mathbf{P}}' \quad (\hat{\mathbf{P}} \text{ lower triangular})$$

$$\hat{\mathbf{a}}_1 = \hat{\mathbf{p}}_1/\hat{p}_{11}$$

$\hat{\mathbf{p}}_1$ = column 1 of $\hat{\mathbf{P}}$

$\hat{p}_{11}$ = row 1 col 1 element of $\hat{\mathbf{P}}$

$$\boldsymbol{\Psi}_s\mathbf{a}_1 = \frac{\partial E(\mathbf{y}_{t+s}|y_{1t},\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial y_{1t}}$$

is effect of one-unit increase in y_{1t} or ε_{1t} on forecast

$\boldsymbol{\Psi}_s\mathbf{p}_1$ is effect of one-standard-deviation increase in ε_{1t} on forecast.

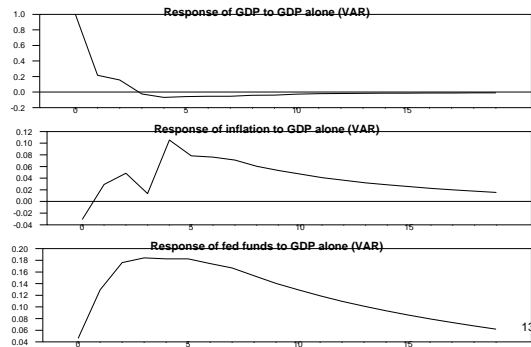$\boldsymbol{\Psi}_s\mathbf{a}_1$ shows IRF to shock in observed units

$\boldsymbol{\Psi}_s\mathbf{p}_1$ shows IRF to shock of typical size
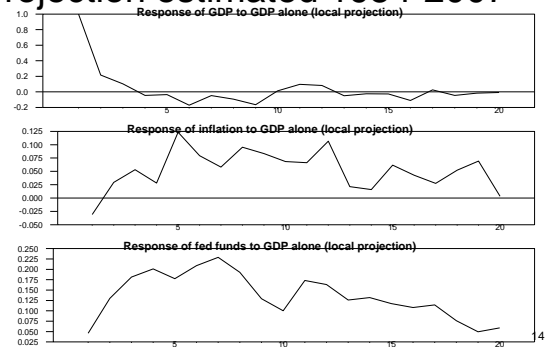
Plots will look identical just with different units on vertical axis $\boldsymbol{\Psi}_s\mathbf{p}_1 = \boldsymbol{\Psi}_s\mathbf{a}_1 p_{11}$

Recursively orthgonalized VAR estimated 1954-2007

Response of GDP to GDP alone (VAR)
Response of inflation to GDP alone (VAR)
Response of fed funds to GDP alone (VAR)

Recursively orthgonalized local projection estimated 1954-2007

Response of GDP to GDP alone (local projection)
Response of inflation to GDP alone (local projection)
Response of fed funds to GDP alone (local projection)

Suppose next that we've observed y_{1t} and y_{2t} but not $y_{3t}, y_{4t}, \ldots, y_{nt}$.

What is effect on forecast of changing y_{2t}?

$$\frac{\partial E(\mathbf{y}_t | y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{2t}} = \mathbf{a}_2 = \mathbf{p}_2 / p_{22}$$

$\mathbf{p}_2$ = column 2 of $\mathbf{P}$
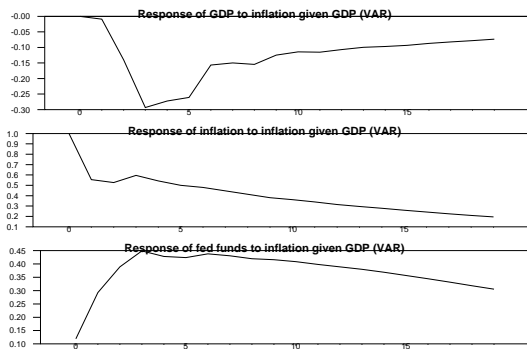
p_{22} = row 2 col 2 element of $\mathbf{P}$

first element of $\mathbf{a}_2$ is zero

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$$\frac{\partial E(\mathbf{y}_{t+s} | y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{2t}} = \frac{\partial E(\mathbf{y}_{t+s} | \mathbf{y}_t, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial \mathbf{y}_t'} \cdot \frac{\partial E(\mathbf{y}_t | y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{2t}}$$

$$= \mathbf{\Psi}_s \mathbf{a}_2$$

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Response of GDP to inflation given GDP (VAR)
Response of inflation to inflation given GDP (VAR)
Response of fed funds to inflation given GDP (VAR)

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$(n \times n)$ matrix of recursively orthogonalized shocks:

$$\mathbf{\Psi}_s \mathbf{P} \text{ or } \mathbf{\Psi}_s \mathbf{A}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} = \mathbf{P}[\text{diag}(\mathbf{P})]^{-1}$$

Note last col of $\mathbf{\Psi}_s \mathbf{A}$ is identical to last col of $\mathbf{\Psi}_s$

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We have broken down the news arriving in period t into n separate uncorrelated components

$\varepsilon_{1t} = y_{1t} - \boldsymbol{\gamma}_1' \mathbf{x}_t$ = news about y_{1t}

$u_{2t} = \varepsilon_{2t} - a_{21}\varepsilon_{1t}$ = news about y_{2t} not already revealed by y_{1t}

$\vdots$

$u_{nt} = \varepsilon_{nt} - E(\varepsilon_{nt}|\varepsilon_{1t},\ldots,\varepsilon_{n-1,t})$

$\quad$ = news about y_{nt} not already revealed by

$y_{1t},\ldots,y_{n-1,t}$

Simple way to summarize these components:

$$\mathbf{v}_t \underset{(n\times1)}{=} \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t = \begin{bmatrix} p^{11} & 0 & \cdots & 0 \\ p^{21} & p^{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ p^{n1} & p^{n2} & \cdots & p^{nn} \end{bmatrix}\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix}$$

$E(\mathbf{v}_t\mathbf{v}_t') = \mathbf{P}^{-1}E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\mathbf{P}'^{-1} = \mathbf{P}^{-1}\boldsymbol{\Omega}\mathbf{P}'^{-1}$

$\quad = \mathbf{P}^{-1}\mathbf{P}\mathbf{P}'\mathbf{P}'^{-1} = \mathbf{I}_n$

$\mathbf{v}_t$ is a linear combination of $\boldsymbol{\varepsilon}_t$ whose elements are uncorrelated with each other

$v_{1t} = p^{11}\varepsilon_{1t}$

$\quad$ = rescaled error forecasting y_{1t}

$v_{2t} = p^{21}\varepsilon_{1t} + p^{22}\varepsilon_{2t}$

$\quad$ = rescaled error forecasting ε_{2t} from ε_{1t}

$v_{nt} = p^{n1}\varepsilon_{1t} + p^{n2}\varepsilon_{2t} + \cdots + p^{nn}\varepsilon_{nt}$

$\quad$ = rescaled error forecasting ε_{nt} from $\varepsilon_{1t},\ldots,\varepsilon_{n-1,t}$

B. Variance decomposition

$\mathbf{y}_{t+s} = \hat{\mathbf{y}}_{t+s|t} + \boldsymbol{\Psi}_0\boldsymbol{\varepsilon}_{t+s} + \boldsymbol{\Psi}_1\boldsymbol{\varepsilon}_{t+s-1} + \boldsymbol{\Psi}_2\boldsymbol{\varepsilon}_{t+s-2} + \cdots + \boldsymbol{\Psi}_{s-1}\boldsymbol{\varepsilon}_{t+1}$

$E(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t}) = \sum_{m=0}^{s-1}\boldsymbol{\Psi}_m\boldsymbol{\Omega}\boldsymbol{\Psi}_m'$

$\boldsymbol{\varepsilon}_t = \mathbf{P}\mathbf{v}_t = \mathbf{p}_1 v_{1t} + \mathbf{p}_2 v_{2t} + \cdots + \mathbf{p}_n v_{nt}$

Contribution of $v_{i,t+1}, v_{i,t+2}, \ldots, v_{i,t+s}$ to forecast error:

$\boldsymbol{\Psi}_0\mathbf{p}_i v_{i,t+s} + \boldsymbol{\Psi}_1\mathbf{p}_i v_{i,t+s-1} + \cdots + \boldsymbol{\Psi}_{s-1}\mathbf{p}_i v_{i,t+1}$

$E(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})$

$\quad = \sum_{m=0}^{s-1}\boldsymbol{\Psi}_m\mathbf{p}_1\mathbf{p}_1'\boldsymbol{\Psi}_m' + \cdots + \sum_{m=0}^{s-1}\boldsymbol{\Psi}_m\mathbf{p}_n\mathbf{p}_n'\boldsymbol{\Psi}_m'$

First term: amount by which could reduce MSE if we knew the values of $\varepsilon_{1,t+1},\ldots,\varepsilon_{1,t+s}$

Second term: amount by which we could reduce MSE if we knew the values of $u_{2,t+1},\ldots,u_{2,t+s}$

```
Decomposition of Variance for Series GDPCH
Step   Std Error      GDPCH    INFLATION  FEDFUNDS
   1  3.08741913    100.000     0.000      0.000
   2  3.15963628     99.994     0.001      0.005
   3  3.36710884     90.098     0.218      9.684
   4  3.41017407     87.881     1.147     10.972
   5  3.43030191     87.226     1.928     10.846
   6  3.45041262     86.487     2.627     10.886
   7  3.45981410     86.245     2.871     10.883
   8  3.46834423     86.053     3.093     10.854
   9  3.47514728     85.850     3.331     10.819
  10  3.47997093     85.727     3.483     10.789
  11  3.48341375     85.612     3.612     10.776
  12  3.48663478     85.487     3.744     10.769
  13  3.48920625     85.388     3.856     10.756
  14  3.49137054     85.302     3.954     10.745
  15  3.49341827     85.216     4.047     10.737
  16  3.49526789     85.139     4.132     10.729
  17  3.49686734     85.072     4.206     10.721
  18  3.49830813     85.011     4.273     10.716
  19  3.49960777     84.956     4.333     10.711
  20  3.50074296     84.907     4.385     10.708
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Decomposition of Variance for Series INFLATION
Step    Std Error     GDPCH   INFLATION FEDFUNDS
    1 1.12790671      0.711     99.289    0.000
    2 1.30103516      1.009     97.570    1.421
    3 1.43945588      1.909     96.580    1.511
    4 1.58901603      1.634     97.023    1.343
    5 1.73329210      4.906     93.947    1.147
    6 1.83842144      6.087     92.878    1.034
    7 1.93358191      6.981     91.717    1.302
    8 2.01323981      7.629     90.802    1.569
    9 2.07617743      7.983     90.376    1.641
   10 2.12893320      8.188     89.988    1.824
   11 2.17571089      8.286     89.663    2.051
   12 2.21501613      8.322     89.481    2.197
   13 2.24826561      8.329     89.336    2.335
   14 2.27737512      8.303     89.209    2.488
   15 2.30262070      8.268     89.109    2.623
   16 2.32420833      8.229     89.031    2.740
   17 2.34293779      8.186     88.958    2.856
   18 2.35921378      8.141     88.891    2.968
   19 2.37321184      8.097     88.834    3.069
   20 2.38526891      8.056     88.782    3.163
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Decomposition of Variance for Series FEDFUNDS
Step    Std Error     GDPCH   INFLATION FEDFUNDS
    1 0.78294284      3.309      2.939   93.752
    2 1.24088336     11.710      8.195   80.095
    3 1.51928430     20.625     13.752   65.623
    4 1.76776027     25.568     18.317   56.116
    5 1.97194315     28.715     20.684   50.601
    6 2.13303190     31.520     22.672   45.807
    7 2.27057752     33.433     24.707   41.860
    8 2.39054533     34.802     26.385   38.813
    9 2.49021257     35.694     27.902   36.404
   10 2.57500835     36.215     29.393   34.392
   11 2.64925525     36.494     30.775   32.732
   12 2.71414804     36.608     32.042   31.350
   13 2.77123546     36.605     33.230   30.165
   14 2.82208295     36.523     34.335   29.142
   15 2.86734779     36.391     35.348   28.261
   16 2.90764045     36.226     36.275   27.499
   17 2.94364156     36.040     37.125   26.836
   18 2.97580221     35.843     37.897   26.260
   19 3.00448248     35.642     38.597   25.761
   20 3.03006847     35.441     39.230   25.329
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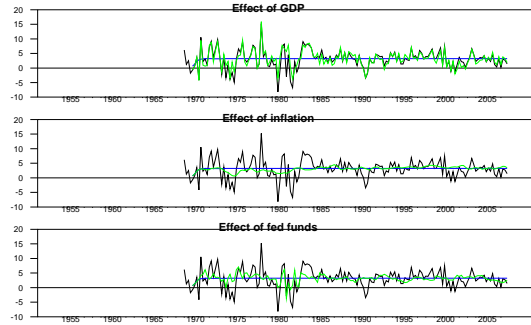
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C. Historical decomposition

$$\mathbf{y}_{t+s} = \hat{\mathbf{y}}_{t+s|t} + \sum_{m=0}^{s-1} \boldsymbol{\Psi}_m \boldsymbol{\varepsilon}_{t+s-m}$$

$$= \hat{\mathbf{y}}_{t+s|t} + \sum_{m=0}^{s-1} \boldsymbol{\Psi}_m [\mathbf{p}_1 v_{1,t+s-m} + \cdots + \mathbf{p}_n v_{n,t+s-m}]$$

Can decompose the observed value for any variable at any date into component that could have been predicted as of some earlier date plus innovations in individual $v_{i,t+m}$ since then.

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Historical Decomposition of GDP

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D. Structural interpretation

Suppose we hypothesized the following structural model for the behavior of the Fed:

$$i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + \mathbf{b}'_{31}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{3p}\mathbf{y}_{t-p} + u_{3t}$$

i_t = fed funds rate

y_t = GDP growth rate

π_t = inflation rate

ψ_y, ψ_π = coefficients in Taylor Rule

$\mathbf{b}_{3m}$ allow for inertia in monetary policy

u_{3t} = serially uncorrelated shock to monetary policy

    = deviation from Fed's usual rule, uncorrelated

    with $\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ by definition

Would like to know $\partial \mathbf{y}_{t+s}/\partial u_{3t}$

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Suppose I also thought there was a Phillips Curve of the form

$$\pi_t = \lambda_2 + \alpha y_t + \mathbf{b}'_{21}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{2p}\mathbf{y}_{t-p} + u_{2t}$$

α = slope of Phillips Curve

$\mathbf{b}_{2m}$ allow for inertia in PC

u_{2t} = unpredictable shock to PC

u_{2t} uncorrelated with $\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ by definition

u_{2t} also assumed to be uncorrelated with u_{3t}

    (assumption that monetary policy shocks

    take more than one period to affect inflation)

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Model equilibrium output as

$$y_t = \lambda_1 + \mathbf{b}'_{11}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{1p}\mathbf{y}_{t-p} + u_{1t}$$

u_{1t} = error forecasting GDP one period ahead

u_{1t} uncorrelated with $\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p}$ by definition

u_{1t} also assumed to be uncorrelated with u_{2t}, u_{3t}
(assumption that PC and monetary shocks
take more than one period to affect output)

$$i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + \mathbf{b}'_{31}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{3p}\mathbf{y}_{t-p} + u_{3t}$$

Above assumptions mean u_{3t} uncorrelated
with y_t and π_t.

$\Rightarrow$ could estimate by OLS

$\hat{\psi}_y$ and $\hat{\psi}_\pi$ are same as step 0 Jordá projection

$\hat{\psi}_y$ and $\hat{\psi}_\pi$ are same as $\hat{a}_{31}$ and $\hat{a}_{32}$

$$\pi_t = \lambda_2 + \alpha y_t + \mathbf{b}'_{21}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{2p}\mathbf{y}_{t-p} + u_{2t}$$

Above assumptions mean u_{2t} uncorrelated with y_t.

$\Rightarrow$ could estimate by OLS

$\hat{\alpha}$ is same as step 0 Jordá projection

$\hat{\alpha}$ is same as $\hat{a}_{21}$

Conclusion: under above assumptions with

$$\mathbf{A} = \mathbf{P}[\text{diag}(\mathbf{P})]^{-1}$$

$$\mathbf{u}_t = \mathbf{A}^{-1}\boldsymbol{\varepsilon}_t$$

$$u_{1t} = \varepsilon_{1t}$$

The error I make forecasting y_{1t} given
$\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to equilibrium output.

The error I make forecasting y_{2t} given
$y_{1t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to PC.

The error I make forecasting y_{3t} given
$y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to monetary policy.
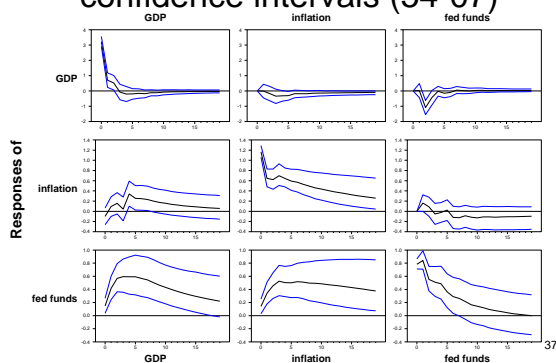

$$\frac{\partial E(\mathbf{y}_{t+s}|y_{1t}, y_{2t}, y_{3t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{3t}} = \frac{\partial \mathbf{y}_{t+s}}{\partial u_{3t}}$$

Recursively orthogonalized VAR gives the dynamic
effects of monetary policy.

Orthogonal Cholesky IRF with 95% confidence intervals (54-07)

- A monetary contraction (higher fed funds rate) is followed by slower GDP growth 2-3 quarters later
- But unanticipated monetary policy shocks account for only 10% of variance of output
- Most of variation in fed funds rate comes from predictable response of monetary policy to output and inflation
- A monetary contraction is followed by higher inflation (known as "price puzzle")

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- Assumption-free statement of price puzzle:
  - if you tell me that fed funds rate is higher than I would have predicted given current output, inflation, and lags, then I will revise my expectation of future inflation up.
- Natural interpretation:
  - Fed raised funds rate because it anticipated future inflation.
  - Our 3-variable equation is too simplistic a description of Fed

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- Popular "fix" for price puzzle:
  - Add other variables that better capture information about future inflation (such as commodity prices) to Fed policy equation

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Christiano, Eichenbaum, Evans (1996)

y_{1t} = log of real GDP

y_{2t} = log of GDP deflator

y_{3t} = index of sensitive commodity prices

y_{4t} = fed funds rate

y_{5t} = nonborrowed reserves

y_{6t} = total reserves

y_{7t} = one of a set of macro variables

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Structural model:

$\mathbf{B}_0 \mathbf{y}_t = \mathbf{B} \mathbf{x}_t + \mathbf{u}_t$

$\mathbf{x}_t = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \ldots, \mathbf{y}'_{t-p})'$

$\mathbf{u}_t$ = vector of structural shocks

$E(\mathbf{u}_t \mathbf{u}'_t) = \mathbf{D}$ (diagonal)

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$$\mathbf{B}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ x & 1 & 0 & 0 & 0 & 0 & 0 \\ x & x & 1 & 0 & 0 & 0 & 0 \\ x & x & x & 1 & 0 & 0 & 0 \\ x & x & x & x & 1 & 0 & 0 \\ x & x & x & x & x & 1 & 0 \\ x & x & x & x & x & x & 1 \end{bmatrix}$$

Variable 4 is fed funds rate, equation 4 is monetary policy equation.

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Note that
$$\frac{\partial E(\mathbf{y}_{t+s}|y_{1t},y_{2t},y_{3t},y_{4t},\mathbf{y}_{t-1},\mathbf{y}_{t-2},...,\mathbf{y}_{t-p})}{\partial y_{4t}} = \frac{\partial E(\mathbf{y}_{t+s}|y_{2t},y_{1t},y_{3t},y_{4t},\mathbf{y}_{t-1},\mathbf{y}_{t-2},...,\mathbf{y}_{t-p})}{\partial y_{4t}}$$

Will have the identical answer for effect of variable 4 any way we order variables 1-3 and any way we order variables 5-7.
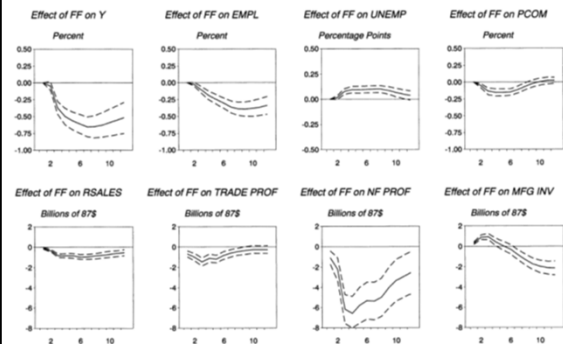Jordá estimate identical if reorder (keeping 4 in place).

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If all we care about is effect of monetary policy, we only need to assume block-recursive

$$\mathbf{B}_0 = \begin{bmatrix} x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 1 & 0 & 0 & 0 \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \end{bmatrix}$$

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67% confidence bands

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E. Generalized IRFs

- If we put fed funds fourth, estimated effect of monetary policy does not depend on how we order variables 1-3.
- But if we switch fed funds from 4 to 3, results could change

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- Pesaran and Shin (1998): "generalized impulse-response function"
  - Put variable #1 first to find effect of variable 1
  - Put variable #2 first to find effect of variable 2
  - Put variable #n first to find effect of variable n

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GIRF: for every i, calculate

$$\frac{\partial E(\mathbf{y}_{t+s}|y_{it},\mathbf{y}_{t-1},...,\mathbf{y}_{t-p})}{\partial y_{it}}$$

- Conclusion: any IRF or GIRF is giving answer to a forecasting question.
- Best practice: describe forecasting question explicitly and explain the reason that question is interesting.