

Forecasts and VARs

- A. Intro to VARs
- B. Nonorthogonalized IRF
- C. Standard errors
- D. Jordà local projections
- E. Unit roots
- F. Instability

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A. Intro to VARs

Suppose we want to forecast y_{1t}
based on:

$$y_{1,t-1}, y_{1,t-2}, \dots, y_{1,t-p}$$

$$y_{2,t-1}, y_{2,t-2}, \dots, y_{2,t-p}$$

⋮

$$y_{n,t-1}, y_{n,t-2}, \dots, y_{n,t-p}$$

deterministic functions of t

$(1, t, \cos(\pi t/6), \text{seasonal dummies})$

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Let $\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$
($n \times 1$)

$$\mathbf{x}_t = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \dots, \mathbf{y}'_{t-p})'$$

($k \times 1$)

$$k = np + 1$$

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Suppose we consider linear forecast

$$\hat{y}_{1t|t-1} = \boldsymbol{\gamma}'_1 \mathbf{x}_t$$

Best forecast within linear class:

value of $\boldsymbol{\gamma}_1$ that minimizes

$$E(y_{1t} - \boldsymbol{\gamma}'_1 \mathbf{x}_t)^2$$

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Proposition: If \mathbf{y}_t is covariance-stationary
and $E(\mathbf{x}_t \mathbf{x}'_t)$ is nonsingular, then optimal
forecast uses

$$\boldsymbol{\gamma}_1^* = E(\mathbf{x}_t \mathbf{x}'_t)^{-1} E(\mathbf{x}_t y_t)$$

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Definition: The optimal linear forecast,

$$\hat{y}_{1t|t-1} = \boldsymbol{\gamma}_1^{*'} \mathbf{x}_t,$$

is called the “population linear projection”
of y_{1t} on \mathbf{x}_t

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Definition: Ordinary least squares (OLS) estimate is given by

$$\hat{\gamma}_1 = \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \right)^{-1} \left(\sum_{t=1}^T \mathbf{x}_t y_{1t} \right)$$

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Proposition: If \mathbf{y}_t is stationary and ergodic, then

$$\hat{\gamma}_1 \xrightarrow{p} \gamma_1^*$$

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Proof: (Law of Large Numbers)

$$\hat{\gamma}_1 = \left(T^{-1} \sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \right)^{-1} \left(T^{-1} \sum_{t=1}^T \mathbf{x}_t y_{1t} \right)$$

$$\xrightarrow{p} E(\mathbf{x}_t \mathbf{x}_t')^{-1} E(\mathbf{x}_t y_{1t})$$

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If form separate forecasting equation for each element of \mathbf{y}_t and collect in vector,

$$y_{1t} = \gamma_1' \mathbf{x}_t + \varepsilon_{1t}$$

\vdots

$$y_{nt} = \gamma_n' \mathbf{x}_t + \varepsilon_{nt}$$

$$\mathbf{y}_t = \Gamma' \mathbf{x}_t + \boldsymbol{\varepsilon}_t$$

result is called vector autoregression:

$$\mathbf{y}_t = \mathbf{c} + \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + \dots + \Phi_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t$$

Above results imply we can consistently estimate coefficients for VAR by OLS equation by equation

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$$\hat{\gamma}'_1 = \left(\sum_{t=1}^T y_{1t} \mathbf{x}_t' \right) \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \right)^{-1}$$

(1×k)

\vdots

$$\hat{\gamma}'_n = \left(\sum_{t=1}^T y_{nt} \mathbf{x}_t' \right) \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \right)^{-1}$$

(1×k)

$$\hat{\Gamma}' = \left(\sum_{t=1}^T \mathbf{y}_t \mathbf{x}_t' \right) \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \right)^{-1}$$

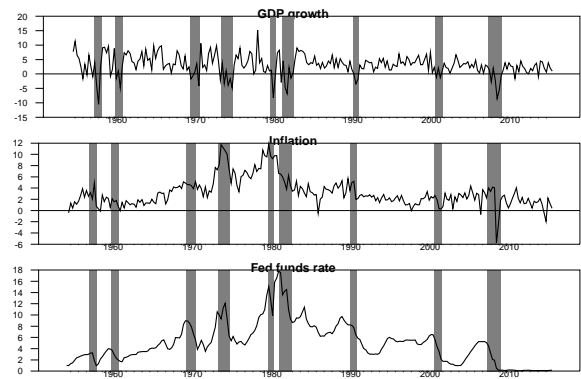
(n×k)

$$\hat{\Gamma}' = \left[\hat{\mathbf{c}} \quad \hat{\Phi}_1 \quad \hat{\Phi}_2 \quad \dots \quad \hat{\Phi}_p \right]$$

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Example

- $y_{1t} = 400 \times$ quarterly log change in real GDP
- $y_{2t} = 400 \times$ quarterly log change in PCE deflator
- $y_{3t} =$ average fed funds rate over quarter



- Estimate with 4 lags on each variable for 1960:Q1 to 1990:Q4.
- Data and code to replicate provided at course webpage.
- Sample code shows how to compare 4 versus 5 lags using hypothesis tests, AIC, or BIC.
- See Lütkepohl, Section 4.3 for description.

VAR/System - Estimation by Least Squares
 Quarterly Data From 1960:01 To 1990:04
 Usable Observations 124

Dependent Variable GDPCH
 Mean of Dependent Variable 3.4518389246
 Std Error of Dependent Variable 3.8882938217
 Standard Error of Estimate 3.3995169372
 Sum of Squared Residuals 1282.7954101
 Durbin-Watson Statistic 1.9156

Variable	Coeff	Std Error	T-Stat
1. GDPCH(1)	0.140889900	0.096939607	1.45338
2. GDPCH(2)	0.171458188	0.095324153	1.79869
3. GDPCH(3)	0.019889221	0.098968000	0.20725
4. GDPCH(4)	0.027744892	0.088859228	0.31223
5. INFLATION(1)	-0.136199595	0.255977819	-0.53208
6. INFLATION(2)	0.109623520	0.293744691	0.37319
7. INFLATION(3)	0.018771719	0.294628089	0.06711
8. INFLATION(4)	-0.002777166	0.265583119	-0.01046
9. FEDFUNDS(1)	0.073563677	0.324268492	0.22686
10. FEDFUNDS(2)	-1.515272886	0.434675848	-3.48598
11. FEDFUNDS(3)	1.138686980	0.459482934	2.47206
12. FEDFUNDS(4)	-0.002977962	0.340933515	-0.00873
13. Constant	4.458801828	1.322775244	3.37079

Dependent Variable INFLATION
 Mean of Dependent Variable 4.4308450671
 Std Error of Dependent Variable 2.7295747540
 Standard Error of Estimate 1.1933641353
 Sum of Squared Residuals 158.07709350
 Durbin-Watson Statistic 1.9869

Variable	Coeff	Std Error	T-Stat
1. GDPCH(1)	0.010693189	0.034029614	0.31247
2. GDPCH(2)	-0.017609241	0.033462527	-0.52624
3. GDPCH(3)	-0.024943566	0.038688542	-0.74042
4. GDPCH(4)	0.134451472	0.031193084	4.31030
5. INFLATION(1)	0.639779521	0.089858281	7.11987
6. INFLATION(2)	0.164641352	0.103115938	1.59666
7. INFLATION(3)	0.210029088	0.103426046	2.03072
8. INFLATION(4)	-0.043699446	0.093230119	-0.46873
9. FEDFUNDS(1)	0.162052487	0.113830993	1.42362
10. FEDFUNDS(2)	-0.195321038	0.152588317	-1.28005
11. FEDFUNDS(3)	-0.065974240	0.161296580	-0.40902
12. FEDFUNDS(4)	0.076340807	0.119681071	0.63787
13. Constant	-0.046316742	0.464346131	-0.09975

Dependent Variable FEDFUNDS
 Mean of Dependent Variable 7.1162903226
 Std Error of Dependent Variable 3.4537135588
 Standard Error of Estimate 1.0084648414
 Sum of Squared Residuals 112.88714833
 Durbin-Watson Statistic 2.0215

Variable	Coeff	Std Error	T-Stat
1. GDPCH(1)	0.092721729	0.028757081	3.22431
2. GDPCH(2)	0.055587277	0.028277858	1.96575
3. GDPCH(3)	0.016988953	0.028468855	0.59676
4. GDPCH(4)	0.006331597	0.026360042	0.24020
5. INFLATION(1)	0.227483091	0.075935680	2.99573
6. INFLATION(2)	0.066210929	0.087139202	0.75983
7. INFLATION(3)	0.007176875	0.087401262	0.08211
8. INFLATION(4)	-0.146731250	0.078785087	-1.86242
9. FEDFUNDS(1)	0.982722020	0.096194071	10.21604
10. FEDFUNDS(2)	-0.451940303	0.128946353	-3.50487
11. FEDFUNDS(3)	0.502225982	0.136305361	3.68457
12. FEDFUNDS(4)	-0.102751117	0.101137741	-1.01595
13. Constant	-0.752466690	0.392400553	-1.91760

B. Nonorthogonalized IRF

$$\begin{aligned}
 \mathbf{y}_{t+1} &= \mathbf{c} + \Phi_1 \mathbf{y}_t + \Phi_2 \mathbf{y}_{t-1} + \dots + \Phi_p \mathbf{y}_{t-p+1} + \boldsymbol{\varepsilon}_{t+1} \\
 \mathbf{y}_{t+2} &= \mathbf{c} + \Phi_1 \mathbf{y}_{t+1} + \Phi_2 \mathbf{y}_t + \dots + \Phi_p \mathbf{y}_{t-p+2} + \boldsymbol{\varepsilon}_{t+2} \\
 \mathbf{y}_{t+2} &= \mathbf{c} + \Phi_1 [\mathbf{c} + \Phi_1 \mathbf{y}_t + \Phi_2 \mathbf{y}_{t-1} + \dots + \Phi_p \mathbf{y}_{t-p+1} + \boldsymbol{\varepsilon}_{t+1}] \\
 &\quad + \Phi_2 \mathbf{y}_t + \dots + \Phi_p \mathbf{y}_{t-p+1} + \boldsymbol{\varepsilon}_{t+2} \\
 &= \mathbf{c}_2 + \Psi_0 \boldsymbol{\varepsilon}_{t+2} + \Psi_1 \boldsymbol{\varepsilon}_{t+1} + \Psi_2 \mathbf{y}_t + \mathbf{H}_{22} \mathbf{y}_{t-1} + \dots + \mathbf{H}_{2p} \mathbf{y}_{t-p+1} \\
 \Psi_0 &= \mathbf{I}_n \\
 \Psi_1 &= \Phi_1 \\
 \Psi_2 &= \Phi_1^2 + \Phi_2
 \end{aligned}$$

$$\mathbf{y}_{t+2} = \mathbf{c}_2 + \Psi_0 \boldsymbol{\varepsilon}_{t+2} + \Psi_1 \boldsymbol{\varepsilon}_{t+1} + \Psi_2 \mathbf{y}_t + \mathbf{H}_{22} \mathbf{y}_{t-1} + \dots + \mathbf{H}_{2p} \mathbf{y}_{t-p+1}$$

We know that $\boldsymbol{\varepsilon}_{t+1}$ is uncorrelated with $\mathbf{y}_t, \dots, \mathbf{y}_{t-p+1}$ by definition of the plim.

If VAR has enough lags, $\boldsymbol{\varepsilon}_{t+2}$ is also uncorrelated with $\mathbf{y}_t, \dots, \mathbf{y}_{t-p+1}$.

$$\begin{aligned}
 \Rightarrow \hat{\mathbf{y}}_{t+2|t} &= \mathbf{c}_2 + \Psi_2 \mathbf{y}_t + \mathbf{H}_{22} \mathbf{y}_{t-1} + \dots + \mathbf{H}_{2p} \mathbf{y}_{t-p+1} \\
 \frac{\partial \hat{\mathbf{y}}_{t+2|t}}{\partial \mathbf{y}_t'} &= \Psi_2
 \end{aligned}$$

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$$\begin{aligned}
 \mathbf{y}_{t+s} &= \mathbf{c}_s + \Psi_0 \boldsymbol{\varepsilon}_{t+s} + \Psi_1 \boldsymbol{\varepsilon}_{t+s-1} + \Psi_2 \boldsymbol{\varepsilon}_{t+s-2} + \dots + \Psi_{s-1} \boldsymbol{\varepsilon}_{t+1} \\
 &\quad + \Psi_s \mathbf{y}_t + \mathbf{H}_{s2} \mathbf{y}_{t-1} + \dots + \mathbf{H}_{sp} \mathbf{y}_{t-p+1} \\
 \frac{\partial \hat{\mathbf{y}}_{t+s|t}}{\partial \mathbf{y}_t'} &= \Psi_s \\
 \Psi_0 &= \mathbf{I}_n \\
 \Psi_1 &= \Phi_1 \\
 \Psi_2 &= \Phi_1^2 + \Phi_2 \\
 \Psi_s &= \Phi_1 \Psi_{s-1} + \Phi_2 \Psi_{s-2} + \dots + \Phi_p \Psi_{s-p}
 \end{aligned}$$

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$$\Psi_s = \Phi_1 \Psi_{s-1} + \Phi_2 \Psi_{s-2} + \dots + \Phi_p \Psi_{s-p}$$

Column j of Ψ_s is answer to the question:

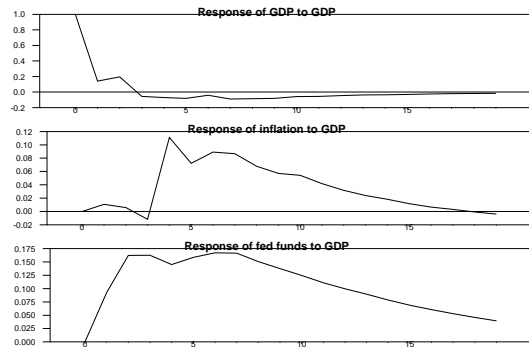
How does my forecast of \mathbf{y}_{t+s} change if I increase y_{jt} by one unit holding all other elements of \mathbf{y}_t and all elements of $\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p}$ constant.

The sequence of $n \times n$ matrices $\{\Psi_s\}_{s=0,1,2,\dots}$ is called the nonorthogonalized impulse-response function.

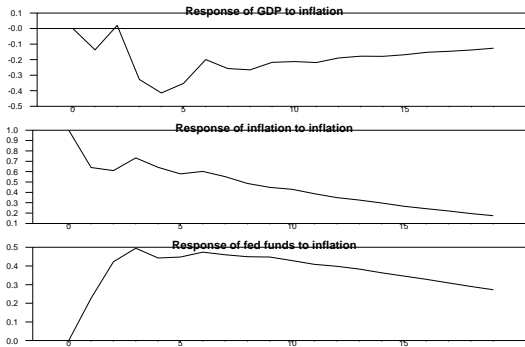
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Responses to Shock in GDPCH			
Entry	GDPCH	INFLATION	FEDFUNDS
1	1.0000000	0.0000000	0.0000000
2	0.1408899	0.0106332	0.0927217
3	0.1966809	0.0057176	0.1621894
4	-0.0564244	-0.0117519	0.1625443
5	-0.0697265	0.1110743	0.1452111
6	-0.0796774	0.0723151	0.1587723
7	-0.0409759	0.0890917	0.1672757
8	-0.0881861	0.0867922	0.1666632
9	-0.0851984	0.0678733	0.1506179
10	-0.0802485	0.0571617	0.1376026
11	-0.0569765	0.0540731	0.1247829
12	-0.0546655	0.0416753	0.1110460
13	-0.0452988	0.0316166	0.0997375
14	-0.0375426	0.0238060	0.0895958
15	-0.0336034	0.0181658	0.0785062
16	-0.0299729	0.0115640	0.0688406
17	-0.0236107	0.0066515	0.0607137
18	-0.0204608	0.0030549	0.0530267
19	-0.0181050	-0.0007017	0.0457998
20	-0.0150533	-0.0040129	0.0394487

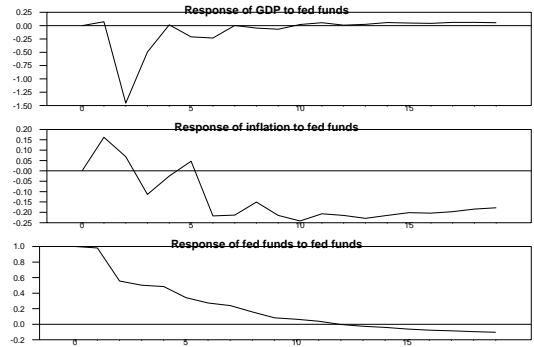
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C. Standard errors

Generate standard errors using Bayesian posterior distribution based on diffuse priors.

$$\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$$

$(n \times 1)$

$$\mathbf{x}_t = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \dots, \mathbf{y}'_{t-p})'$$

$(k \times 1)$

$$k = np + 1$$

$$\mathbf{y}_t = \Gamma' \mathbf{x}_t + \boldsymbol{\varepsilon}_t$$

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t) = \boldsymbol{\Omega}$$

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$$\hat{\Gamma}' = \left(\sum_{t=1}^T \mathbf{y}_t \mathbf{x}'_t \right) \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}'_t \right)^{-1}$$

$$\hat{\boldsymbol{\varepsilon}}_t = \mathbf{y}_t - \hat{\Gamma}' \mathbf{x}_t$$

$(n \times 1)$

$$\hat{\boldsymbol{\Omega}} = T^{-1} \sum_{t=1}^T \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}'_t$$

$(n \times n)$

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$\boldsymbol{\Omega}^{-1} | \mathbf{y}_1, \dots, \mathbf{y}_T \sim$ Wishart with $T - p$ degrees of freedom and scale matrix $T \hat{\boldsymbol{\Omega}}$

$$\text{Wishart}(k, \mathbf{H}) = \mathbf{z}_1 \mathbf{z}'_1 + \dots + \mathbf{z}_k \mathbf{z}'_k$$

$$\mathbf{z}_i \sim N(\mathbf{0}, \mathbf{H}^{-1})$$

$(n \times 1)$

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$$\text{vec}(\Gamma) | \boldsymbol{\Omega}, \mathbf{y}_1, \dots, \mathbf{y}_T \sim N\left(\text{vec}[\hat{\Gamma}], \boldsymbol{\Omega} \otimes \left[\sum_{t=1}^T \mathbf{x}_t \mathbf{x}'_t \right]^{-1}\right)$$

(1) Draw $\boldsymbol{\Omega}^{(m)}$ and $\Gamma^{(m)}$ for this distribution

(2) For each $m = 1, \dots, 10^4$ calculate $\Psi_s^{(m)}$

(3) For each i, j, s find 95% interval for row i col j element of this matrix

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D. Jordà local projections

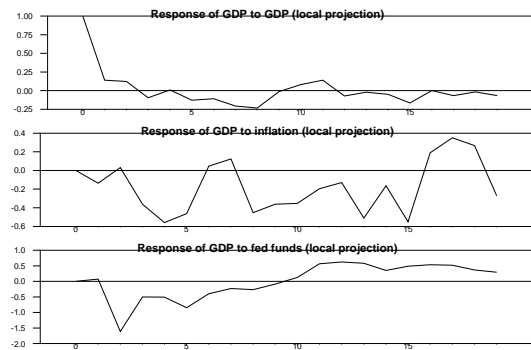
As noted by Jordà (2005), we can also estimate forecast without imposing VAR structure.

$$\mathbf{y}_{t+s} = \mathbf{c}_s + \Psi_s \mathbf{y}_t + \mathbf{H}_{s2} \mathbf{y}_{t-1} + \dots + \mathbf{H}_{sp} \mathbf{y}_{t-p+1} + \mathbf{u}_{t+s}$$

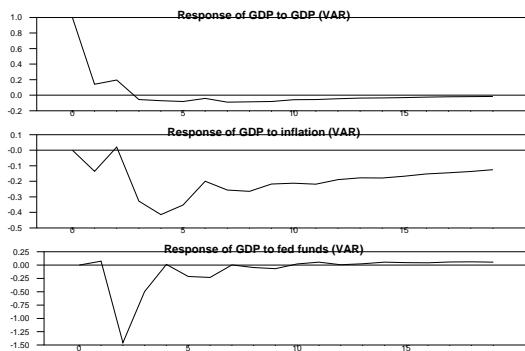
Estimate by n different regressions separately for each s .

Resulting $\{\hat{\Psi}_s\}_{s=1,2,\dots}$ is direct estimate of nonorthogonalized IRF.

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- Local projections and VAR recursion should give similar broad picture.
- Local projections likely more volatile and may give worse forecasts (Marcellino, Stock and Watson, 2006)

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E. Unit roots

In exercises so far we took y_{1t} to be growth rate of real GDP.

What if we had instead used the level of GDP without differencing?

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Coefficients for GDP equation when estimated in growth rates versus levels

Growth rate regression		Levels regression	
GDPCH{1}	0.14089	GDPLOG{1}	1.125783
GDPCH{2}	0.171458	GDPLOG{2}	0.021619
GDPCH{3}	0.019889	GDPLOG{3}	-0.14394
GDPCH{4}	0.027745	GDPLOG{4}	0.000463
INFLATION{1}	-0.1362	INFLATION{1}	-0.1873
INFLATION{2}	0.109624	INFLATION{2}	0.101033
INFLATION{3}	0.019772	INFLATION{3}	0.023217
INFLATION{4}	-0.00278	INFLATION{4}	0.033663
FEDFUNDS{1}	0.073564	FEDFUNDS{1}	0.091929
FEDFUNDS{2}	-1.51527	FEDFUNDS{2}	-1.53255
FEDFUNDS{3}	1.135869	FEDFUNDS{3}	1.127251
FEDFUNDS{4}	-0.00298	FEDFUNDS{4}	-0.11631
Constant	4.458802	Constant	-7.76291

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Suppose the correct model would use growth rates

$$\Delta y_{1t} = \zeta_1 \Delta y_{1,t-1} + \dots + \zeta_p \Delta y_{1,t-p} + \beta' \mathbf{x}_{t-1} + \varepsilon_{1t}$$

$$y_{1t} - y_{1,t-1} = \zeta_1 (y_{1,t-1} - y_{1,t-2}) + \dots + \zeta_p (y_{1,t-p} - y_{1,t-p-1}) + \beta' \mathbf{x}_{t-1} + \varepsilon_{1t}$$

This is a special case of regression in levels

$$y_{1t} = \phi_1 y_{1,t-1} + \dots + \phi_{p+1} y_{1,t-p-1} + \beta' \mathbf{x}_{t-1} + \varepsilon_{1t}$$

$$\phi_1 = 1 + \zeta_1 \quad \phi_2 = \zeta_2 - \zeta_1 \quad \phi_3 = \zeta_3 - \zeta_2$$

$$\dots \quad \phi_p = \zeta_p - \zeta_{p-1} \quad \phi_{p+1} = -\zeta_p$$

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Growth rate regression		Levels regression		predicted
GDPCH{1}	0.14089	GDPLOG{1}	1.125783	1.14089
GDPCH{2}	0.171458	GDPLOG{2}	0.021619	0.030568
GDPCH{3}	0.019889	GDPLOG{3}	-0.14394	-0.15157
GDPCH{4}	0.027745	GDPLOG{4}	0.000463	0.007856
INFLATION{1}	-0.1362	INFLATION{1}	-0.1873	-0.1362
INFLATION{2}	0.109624	INFLATION{2}	0.101033	0.109624
INFLATION{3}	0.019772	INFLATION{3}	0.023217	0.019772
INFLATION{4}	-0.00278	INFLATION{4}	0.033663	-0.00278
FEDFUNDS{1}	0.073564	FEDFUNDS{1}	0.091929	0.073564
FEDFUNDS{2}	-1.51527	FEDFUNDS{2}	-1.53255	-1.51527
FEDFUNDS{3}	1.135869	FEDFUNDS{3}	1.127251	1.135869
FEDFUNDS{4}	-0.00298	FEDFUNDS{4}	-0.11631	-0.00298
Constant	4.458802	Constant	-7.76291	

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$$y_{1t} = \phi_1 y_{1,t-1} + \dots + \phi_{p+1} y_{1,t-p-1} + \beta' \mathbf{x}_{t-1} + \varepsilon_{1t}$$

OLS minimizes $T^{-1} \sum_{t=1}^T \varepsilon_{1t}^2$

If y_{1t} has a unit root, this will be infinite unless we pick ϕ_j consistent with the growth-rate specification.

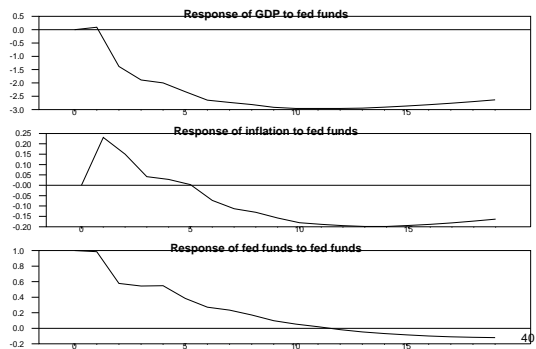
In other words, OLS should force

$$\hat{\phi}_1 + \hat{\phi}_2 + \hat{\phi}_3 + \hat{\phi}_4 \text{ close to one.}$$

Actual OLS estimate of the sum is 1.004.

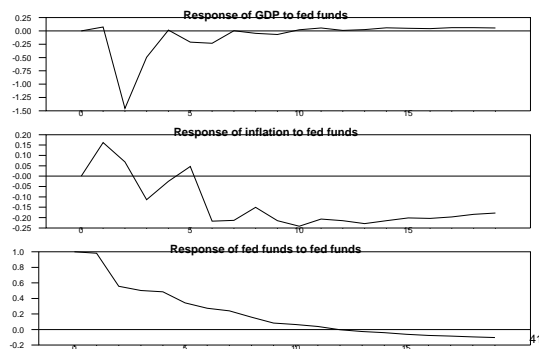
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IRF for levels specification



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IRF for growth-rate specification



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- For this example, levels regression and growth-rate regression are basically estimating the identical system
- If truth is growth-rate, when we estimate in levels we will force OLS to estimate the unit root for us
- But will have more efficient estimates if impose the unit root
- Also can avoid nonstandard distributions for hypothesis tests by using growth rates ⁴²

- Potential drawbacks to using growth rates
 - GDP may not really have a unit root
 - GDP and price level may be cointegrated
- For this example, baseline specification seems sensible (growth rate of GDP, inflation rate, level of fed funds)

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Other implications

- Does not usually make sense to throw in time trend (or quadratic time trend!) in levels regression because growth rates have no trend.
- A simple linear regression of level of a scalar on its own lagged levels is a robust, assumption-free way to remove unknown trend (much better than Hodrick-Prescott filter!)

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Proposition: if $\Delta^d y_t$ is stationary for some d , then can write y_{t+h} as a linear function of $y_t, y_{t-1}, \dots, y_{t-d+1}$ plus a stationary residual.

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Example: $d = 1$

$$u_t = \Delta y_t \sim I(0)$$

$$y_{t+h} = y_t + u_{t+1} + u_{t+2} + \dots + u_{t+h} = y_t + w_t^{(h)}$$

$$w_t^{(h)} = u_{t+1} + u_{t+2} + \dots + u_{t+h} \sim I(0)$$

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Example: $d = 2$

$$u_t = \Delta^2 y_t \sim I(0)$$

$$y_{t+h} = (h+1)y_t - hy_{t-1} + u_{t+h} + 2u_{t+h-1} + \dots + hu_{t+1}$$

$$= (h+1)y_t - hy_{t-1} + w_t^{(h)}$$

$$w_t^{(h)} = u_{t+h} + 2u_{t+h-1} + \dots + hu_{t+1} \sim I(0)$$

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If $y_t \sim I(2)$, what happens if we regress y_{t+h} on $(1, y_t, y_{t-1})'$?

- If coefficient on $y_t = h+1$ and coefficient on $y_{t-1} = -h$, then average squared residual will tend to a finite number.

- For any other coefficients, average squared residual will tend to an infinite number.

- OLS will give a consistent estimate of parameters that characterize the trend.

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If $y_t \sim I(2)$, what happens if we regress y_{t+h} on $(1, y_t, y_{t-1}, y_{t-2}, y_{t-3})'$?

- Two of the coefficients will make the residuals stationary.
- Other two coefficients will then try to forecast stationary component.

Conclusion: we don't need to know d .

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If $y_t \sim I(d)$ for some unknown $d \leq 4$, the population linear projection of y_{t+h} on $(1, y_t, y_{t-1}, y_{t-2}, y_{t-3})'$ exists and can be consistently estimated by OLS regression.

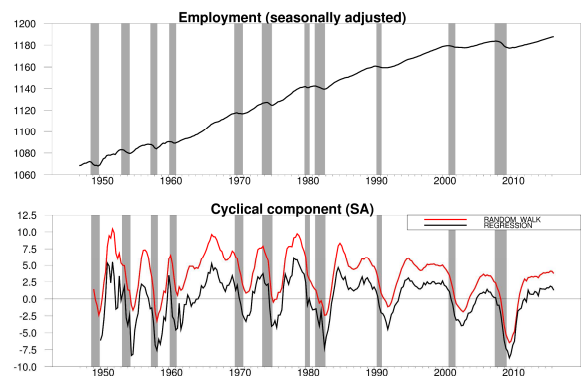
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Proposed definition: the cyclical component of y_t is part we can't predict 2 years ahead using linear regression.

For quarterly data estimate by OLS

$y_{t+8} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+8}$
interpret the residuals \hat{v}_{t+8} as the cyclical component

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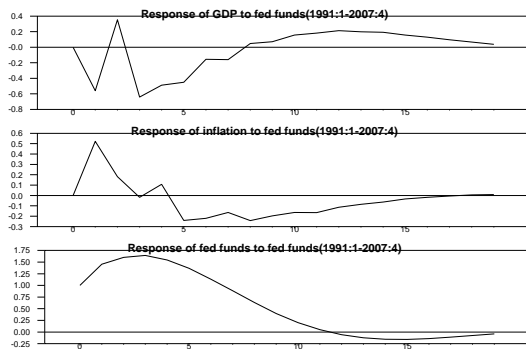
F. Instability

- What happens if we estimate VAR over 1991:Q1 to 2007:Q4?

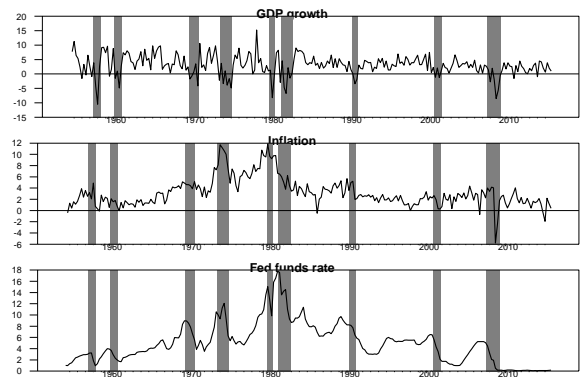
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GDP coeffs for 1991:Q1 to 2007:Q4 sample				GDP coeffs for 1960:Q1 to 1990:Q4 sample			
	coeff	std error	t stat		coeff	std error	t stat
GDPCH{1}	0.155077	0.143744	1.07884	GDPCH{1}	0.14089	0.09694	1.45338
GDPCH{2}	0.185423	0.149189	1.24288	GDPCH{2}	0.171458	0.095324	1.79869
GDPCH{3}	-0.10024	0.145688	-0.68807	GDPCH{3}	0.019889	0.095968	0.20725
GDPCH{4}	0.126178	0.145687	0.86609	GDPCH{4}	0.027745	0.088859	0.31223
INFLATION{1}	-0.05773	0.295557	-0.19531	INFLATION{1}	-0.1362	0.255978	-0.53208
INFLATION{2}	-0.16592	0.274807	-0.60378	INFLATION{2}	0.109624	0.293745	0.37319
INFLATION{3}	-0.30131	0.27268	-1.10499	INFLATION{3}	0.019772	0.294628	0.06711
INFLATION{4}	0.057866	0.278822	0.20754	INFLATION{4}	-0.00278	0.265583	-0.01046
FEDFUNDS{1}	-0.55891	0.948064	-0.58952	FEDFUNDS{1}	0.073564	0.324268	0.22686
FEDFUNDS{2}	1.282767	1.700743	0.75424	FEDFUNDS{2}	-1.51527	0.434676	-3.48598
FEDFUNDS{3}	-1.46413	1.667997	-0.87778	FEDFUNDS{3}	1.135869	0.459483	2.47206
FEDFUNDS{4}	0.670807	0.868767	0.77214	FEDFUNDS{4}	-0.00298	0.340934	-0.00873
Constant	3.253064	1.496104	2.17436	Constant	4.458802	1.322775	3.37079

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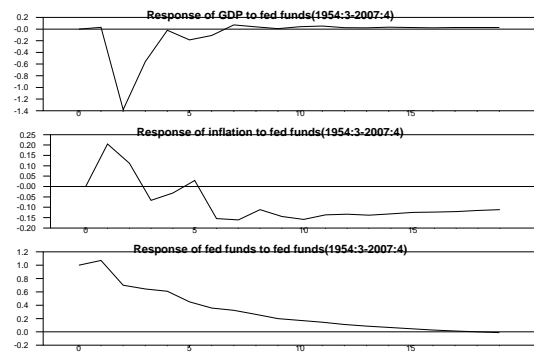


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Options for dealing with instability

- Estimate allowing for GARCH to reduce impact of outliers (Hamilton, 2010)
- Find generalization of model that is stable
- Use Bayesian methods to bring in additional information
- Estimate system with time-varying parameters or changes in regime
- Use full sample as average summary (plim of regression)

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