Econ 220B, Winter 2019

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (80 points total) Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{V}).$$

Here $\mathbf{y}$ is a $(T \times 1)$ vector of observed variables, $\mathbf{X}$ is a $(T \times k)$ matrix of observed variables, $\boldsymbol{\beta}$ is a $(k \times 1)$ vector you want to estimate, σ^2 is a scalar you want to estimate, and $\mathbf{V}$ is a known nonsingular $(T \times T)$ matrix, meaning you don't have to estimate $\mathbf{V}$ from the data. Hint: if you can't answer some of the questions below, answer them for the special case when $\mathbf{V} = \mathbf{I}_T$ for partial credit.

a.) (40 points) Write down an expression for the estimates of $\boldsymbol{\beta}$ and σ^2 that you would construct from the data on $\mathbf{y}$ and $\mathbf{X}$.

b.) (30 points) Suppose you want to test the following composite hypothesis,

$$H_0 : \beta_1 = \beta_2 \textit{ and } \beta_3 = \beta_4,$$

where β_i is the ith element of $\boldsymbol{\beta}$. Write down a statistic you could use to test this hypothesis. What is the distribution of the statistic under the null hypothesis? Note you do not need to derive the distribution, simply state it.

c.) (10 points) If you wanted to estimate $\boldsymbol{\beta}$ and σ^2 subject to the constraint that H_0 is true, how would you do it?

2.) (20 points total) Suppose that $z_1, ..., z_n$ is a set of n variables that are independent and identically distributed $N(0, \sigma^2)$ variables. Consider the random variable

$$y_n = \frac{z_n}{\sqrt{(z_1^2 + z_2^2 + \cdots + z_{n-1}^2)/(n-1)}}.$$

a.) (10 points) What is the exact distribution of y_n?

b.) (10 points) Calculate the asymptotic distribution of y_n as $n \to \infty$.

3.) (50 points total) For $(x_{t1}, x_{t2}, y_t)'$ a stationary and ergodic (3×1) vector, consider the following model,

$$y_t = \beta(x_{t1} - hx_{t2}) + \varepsilon_t$$

$$E\begin{bmatrix} x_{t1}^2 & x_{t1}x_{t2} \\ x_{t2}x_{t1} & x_{t2}^2 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} = \mathbf{Q}$$

$$\text{rank}(\mathbf{Q}) = 2$$

$$E\begin{bmatrix} \varepsilon_t^2 x_{t1}^2 & \varepsilon_t^2 x_{t1}x_{t2} \\ \varepsilon_t^2 x_{t2}x_{t1} & \varepsilon_t^2 x_{t2}^2 \end{bmatrix} = \sigma^2\mathbf{Q},$$

where the (2×1) vector $(\varepsilon_t x_{t1}, \varepsilon_t x_{t2})'$ is a martingale-difference sequence.

a.) (25 points) Suppose first that the researcher knows the value of h and does not have to estimate it, and proposes to estimate β from an OLS regression of y_t on $(x_{t1} - hx_{t2})$:

$$\tilde{\beta} = \frac{\sum_{t=1}^{T}(x_{t1} - hx_{t2})y_t}{\sum_{t=1}^{T}(x_{t1} - hx_{t2})^2}.$$

Find the asymptotic distribution of $\tilde{\beta}$ under the above assumptions.

b.) (25 points) Suppose next that the researcher does not know the value of h but has a consistent estimate $\hat{h}$ such that $\hat{h} \xrightarrow{p} h$, and proposes to estimate β by a regression of y_t on $(x_{t1} - \hat{h}x_{t2})$:

$$\hat{\beta} = \frac{\sum_{t=1}^{T}(x_{t1} - \hat{h}x_{t2})y_t}{\sum_{t=1}^{T}(x_{t1} - \hat{h}x_{t2})^2}.$$

Show that under the above assumptions, $\hat{\beta} \xrightarrow{p} \beta$.