Econ 220B, Winter 2018

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (70 points total) Consider a regression with two explanatory variables,

$$y_t = x_{t1}\beta_1 + x_{t2}\beta_2 + \varepsilon_t, \qquad t = 1, 2, ..., T \tag{1}$$

for which the two explanatory variables happen to be orthogonal,

$$\textstyle\sum_{t=1}^{T} x_{t1}x_{t2} = 0. \tag{2}$$

You can assume that $(\varepsilon_1, ..., \varepsilon_T)'|(x_{11}, ..., x_{T1}, x_{12}, ..., x_{T2}) \sim N(\mathbf{0}, \sigma^2\mathbf{I}_T)$.

a.) (30 points) Write down the expression for the OLS estimate of β_1 and for the t-statistic for testing the null hypothesis $\beta_1 = 0$. Be sure you have used expression (2) to write these in the simplest possible form. What is the distribution of the t-statistic under the assumptions given? Note that you do not have to derive those properties, just state them.

b.) (5 points) Write down the expression for the OLS t-statistic for testing the null hypothesis $\beta_2 = 0$.

c.) (20 points) Write down the expression for the OLS F-statistic for testing the null hypothesis $\beta_1 = \beta_2 = 0$ and state its distribution under the given assumptions. What is the relation between your answers to (a), (b), and (c)?

d.) (15 points) Now suppose you dropped the explanatory variable x_{t2} from the regression, and just estimated

$$y_t = x_{t1}\tilde{\beta}_1 + \tilde{\varepsilon}_t \qquad \text{for } t = 1, 2, ..., T. \tag{3}$$

What is the estimate of $\tilde{\beta}_1$ for this second OLS regression (3)? What is the relation between your answers to (a) and (d)? How do you account for this?

2.) (80 points total) Suppose that for $\mathbf{x}_t$ a $(k \times 1)$ vector of explanatory variables,

$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$

$(y_t, \mathbf{x}_t')'$ is stationary and ergodic

$E(\mathbf{x}_t\mathbf{x}_t') = \mathbf{Q}$ for $\mathbf{Q}$ a nonsingular $(k \times k)$ matrix

$\varepsilon_t\mathbf{x}_t$ is a martingale difference sequence with $E(\varepsilon_t^2) = \sigma^2$ and $E(\varepsilon_t^2\mathbf{x}_t\mathbf{x}_t') = \mathbf{S}$.

a.) (40 points) Suppose you are interested in the sum of the first two elements of $\boldsymbol{\beta}$: $h = \beta_1 + \beta_2$. Propose an estimate of h and calculate its asymptotic distribution under the assumptions given. Hint: if you can't do this, propose an estimate of $\boldsymbol{\beta}$ and calculate its asymptotic distribution for partial credit.

b.) (20 points) Use your results from (a) to propose a t-statistic you might use to test the null hypothesis $h = 1$. Note you do not need to derive its asymptotic distribution, just state how you would do the test.

c.) (20 points) Would the test you proposed in (b) still be valid if $\mathbf{S} = \sigma^2\mathbf{Q}$? Would another test be better for the case when $\mathbf{S} = \sigma^2\mathbf{Q}$, and if so, what makes the other test better?