Answer key for the midterm in 2017

1a.) $\mathbf{b}|\mathbf{X} \sim N(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1})$ so $\mathbf{c}'\mathbf{b}|\mathbf{X} \sim N(\mathbf{c}'\boldsymbol{\beta}, \sigma^2\mathbf{c}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c})$.

b.) For $s^2 = (T-k)^{-1}(\mathbf{y}-\mathbf{X}\mathbf{b})'(\mathbf{y}-\mathbf{X}\mathbf{b})$, use $(\mathbf{c}'\mathbf{b}-1)/\sqrt{s^2\mathbf{c}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c}}$ which is Student t with $T-k$ degrees of freedom.

c.) Now $\mathbf{b}|\mathbf{X} \sim N(\boldsymbol{\beta}, \mathbf{H})$ for $\mathbf{H} = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}\mathbf{X}(\mathbf{X}'\mathbf{X})^{1}$ so $\mathbf{c}'\mathbf{b}|\mathbf{X} \sim N(\mathbf{c}'\boldsymbol{\beta}, \mathbf{c}'\mathbf{H}\mathbf{c})$.

d.) Use $\mathbf{c}'\hat{\boldsymbol{\beta}}$ for $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{y})$.

2a.) $\mathbf{b} = \left(\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\sum \mathbf{x}_t y_t$

$\sqrt{T}(\mathbf{b}-\boldsymbol{\beta}) = \left(T^{-1}\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1}T^{-1/2}\sum \mathbf{x}_t\varepsilon_t \xrightarrow{L} \mathbf{Q}^{-1}\mathbf{v}$ for $\mathbf{v} \sim N(\mathbf{0}, \mathbf{S})$ so

$\sqrt{T}(\mathbf{b}-\boldsymbol{\beta}) \xrightarrow{L} N(\mathbf{0}, \mathbf{Q}^{-1}\mathbf{S}\mathbf{Q}^{-1})$

b.) Let $\hat{\mathbf{V}} = \hat{\mathbf{Q}}^{-1}\hat{\mathbf{S}}\hat{\mathbf{Q}}^{-1}$ for $\hat{\mathbf{Q}} = T^{-1}\sum \mathbf{x}_t\mathbf{x}_t'$ and $\hat{\mathbf{S}} = T^{-1}\sum e_t^2\mathbf{x}_t\mathbf{x}_t'$ and $e_t = y_t - \mathbf{x}_t'\mathbf{b}$. Then test statistic is

$T(\mathbf{R}\mathbf{b}-\mathbf{r})'(\mathbf{R}\hat{\mathbf{V}}\mathbf{R}')^{-1}(\mathbf{R}\mathbf{b}-\mathbf{r})$ which should be asymptotically $\chi^2(m)$.

c.) $T(\mathbf{H}\mathbf{R}\mathbf{b}-\mathbf{H}\mathbf{r})'(\mathbf{H}\mathbf{R}\hat{\mathbf{V}}\mathbf{R}'\mathbf{H}')^{-1}(\mathbf{H}\mathbf{R}\mathbf{b}-\mathbf{H}\mathbf{r}) = T(\mathbf{R}\mathbf{b}-\mathbf{r})'\mathbf{H}'(\mathbf{H}')^{-1}(\mathbf{R}\hat{\mathbf{V}}\mathbf{R}')^{-1}\mathbf{H}^{-1}\mathbf{H}(\mathbf{R}\mathbf{b}-\mathbf{r})$

$= T(\mathbf{R}\mathbf{b}-\mathbf{r})'(\mathbf{R}\hat{\mathbf{V}}\mathbf{R}')^{-1}(\mathbf{R}\mathbf{b}-\mathbf{r})$