Econ 220B, Winter 2017

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (75 points total) Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0},\sigma^2\mathbf{I}_T)$$

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

where $\mathbf{y}$ and $\boldsymbol{\varepsilon}$ are each $(T \times 1)$ vectors, $\mathbf{X}$ is a $(T \times k)$ matrix of rank k, $\boldsymbol{\beta}$ is a $(k \times 1)$ vector, and σ is a scalar. Our interest in this question is on a particular linear combination $\mathbf{c}'\boldsymbol{\beta}$ where $\mathbf{c}$ is a known $(k \times 1)$ vector.

a.) (25 points) Calculate the distribution of $\mathbf{c}'\mathbf{b}$ conditional on $\mathbf{X}$ under the assumptions stated.

b.) (20 points) How would you use the results from (a) to calculate a t-test of the null hypothesis $\mathbf{c}'\boldsymbol{\beta} = 1$? What are the degreees of freedom for this t-statistic? Note you only need to write down the formula for the t-statistic, and do not need to derive its distribution.

c.) (15 points) Suppose now instead that $\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0},\sigma^2\mathbf{V})$ for $\mathbf{V}$ a known $(T \times T)$ matrix. Calculate the distribution of $\mathbf{c}'\mathbf{b}$ conditional on $\mathbf{X}$ under this alternative assumption.

d.) (15 points) Can you suggest an unbiased estimate of $\mathbf{c}'\boldsymbol{\beta}$ which under the assumptions in part (c) would have a smaller variance than $\mathbf{c}'\mathbf{b}$? Note that you only need to write down the formula for this estimator, and do not need to prove that it has a smaller variance.

2.) (75 points total) Suppose that for $\mathbf{x}_t$ a $(k \times 1)$ vector of explanatory variables,

$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$

$(y_t, \mathbf{x}_t')'$ is stationary and ergodic

$E(\mathbf{x}_t\mathbf{x}_t') = \mathbf{Q}$ for $\mathbf{Q}$ a nonsingular $(k \times k)$ matrix

$\varepsilon_t\mathbf{x}_t$ is a martingale difference sequence with $E(\varepsilon_t^2\mathbf{x}_t\mathbf{x}_t') = \mathbf{S}$.

a.) (40 points) Calculate the OLS estimator of $\boldsymbol{\beta}$ and derive its asymptotic distribution.

b.) (20 points) Suggest a test that you could use under the above assumptions to test the null hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ for $\mathbf{R}$ a known $(m \times k)$ matrix and $\mathbf{r}$ a known $(m \times 1)$ vector. Note that you do not need to derive its asymptotic distribution, just describe how you would do the test.

c.) (15 points) An alert student observes that the null hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ could equivalently be written as $\mathbf{H}\mathbf{R}\boldsymbol{\beta} = \mathbf{H}\mathbf{r}$ where $\mathbf{H}$ could be any nonsingular $(m \times m)$ matrix. Demonstrate to the student that the test statistic you proposed in part (b) would produce the numerically identical answer for any nonsingular $\mathbf{H}$.