Econ 220B, Winter 2016

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (75 points total) Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$E(\boldsymbol{\varepsilon}|\mathbf{X}) = \mathbf{0}$$

$$E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|\mathbf{X}) = \lambda^2\mathbf{X}\mathbf{X}' + \sigma^2\mathbf{I}_T$$

where $\mathbf{y}$ and $\boldsymbol{\varepsilon}$ are each $(T \times 1)$ vectors, $\mathbf{X}$ is a $(T \times k)$ matrix of rank k, $\boldsymbol{\beta}$ is a $(k \times 1)$ vector, and λ and σ are scalars.

a.) (10 points) What name do we use to describe the above set of assumptions under the special case when $\lambda = 0$?

b.) (25 points) Calculate $E(\mathbf{b})$ for $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ the OLS coefficient vector. Describe in words the meaning of this result.

c.) (20 points) Calculate $E[(\mathbf{b} - \boldsymbol{\beta})(\mathbf{b} - \boldsymbol{\beta})'|\mathbf{X}]$ being sure to simplify if possible. By what name is the (1,1) element of this matrix more commonly known? Hint: check your work by making sure that your formula is correct for the special case when $\lambda = 0$.

d.) (20 points) Calculate $E(s^2)$ for $s^2 = \mathbf{e}'\mathbf{e}/(T - k)$ and $\mathbf{e} = (\mathbf{y} - \mathbf{X}\mathbf{b})$. Hint: recall that $\mathbf{e} = \mathbf{M}\boldsymbol{\varepsilon}$ for $\mathbf{M} = \mathbf{I}_T - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$.

2.) (75 points total) The following question leads you in steps to find the asymptotic distribution of the OLS estimate s^2. Hint: if you are unable to complete any step, just take the result from that step as given and go on to the next part.

a.) (10 points) Suppose that ε_t is an i.i.d. sequence with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$ and $E(\varepsilon_t^4) = 3\sigma^4$. Find the asymptotic distribution of $\sqrt{T}\left(T^{-1}\sum_{t=1}^{T}(\varepsilon_t^2 - \sigma^2)\right)$ and state the theorem you used to obtain this.

b.) (10 points) Suppose that $\mathbf{x}_t$ is stationary and ergodic with finite fourth moments. Show that $T^{-1}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t' \xrightarrow{p} \mathbf{Q}$. State the theorem you used to obtain this and give a description of the matrix $\mathbf{Q}$ that does not involve the concept of a probability limit.

c.) (10 points) Consider the regression $y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$ where $\mathbf{x}_t'$ is a $(1 \times k)$ vector. Show that if $\mathbf{Q}$ is full rank and ε_t is independent of $\mathbf{x}_s$ for all t and s, then $\sqrt{T}(\mathbf{b}-\boldsymbol{\beta}) \xrightarrow{L} N(\mathbf{0},\sigma^2\mathbf{Q}^{-1})$ for $\mathbf{b} = \left(\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(\sum_{t=1}^{T}\mathbf{x}_t y_t'\right)$.

d.) (10 points) Show that results (b) and (c) imply that

$$T^{-1/2}\sum_{t=1}^{T}[\mathbf{x}_t'(\boldsymbol{\beta} - \mathbf{b})]^2 \xrightarrow{p} 0.$$

e.) (10 points) Show that

$$\sum_{t=1}^{T}\varepsilon_t^2 = \sum_{t=1}^{T}(y_t - \mathbf{x}_t'\boldsymbol{\beta})^2 = \sum_{t=1}^{T}e_t^2 + \sum_{t=1}^{T}[\mathbf{x}_t'(\boldsymbol{\beta} - \mathbf{b})]^2$$

f.) (15 points) Use results (a),(d) and (e) to find the asymptotic distribution of $\sqrt{T}(s^2 - \sigma^2)$ for $s^2 = (T-k)^{-1}\sum_{t=1}^{T}e_t^2$.

g.) (10 points) Explain how you could use the result in (f) to test the null hypothesis that $\sigma^2 = 2$.