Answer key for the midterm in 2015

1a.) Notice $E(\varepsilon_t x_t)^2 = E[x_t^2 \bullet E(\varepsilon_t^2|x_t)] = E(x_t^2 \bullet \sigma^2 x_t^2) = \sigma^2 E(x_t^4)$.

$\sqrt{T}(b-\beta) = (T^{-1}\sum x_t^2)^{-1}(T^{-1/2}\sum x_t\varepsilon_t)$

$T^{-1}\sum x_t^2 \xrightarrow{p} E(x_t^2)$

$T^{-1/2}\sum x_t\varepsilon_t \xrightarrow{L} N(0,\sigma^2 E(x_t^4))$

$\sqrt{T}(b-\beta) \xrightarrow{L} N(0,V_1)$

$V_1 = \sigma^2 E(x_t^4)/[E(x_t^2)]^2$

1b.) For $\hat{V}_1 = [T^{-1}\sum x_t^2 e_t^2/(T^{-1}\sum x_t^2)^2]$ with $e_t = y_t - x_t b$ t-stat would be $b/(T^{1/2}\hat{V}_1^{1/2})$. This is identical to the White standard error. An alternative consistent estimator is $\tilde{V}_1 = [T^{-1}\sum(e_t^2/x_t^2)][T^{-1}\sum x_t^4]/[T^{-1}\sum x_t^2]^2$, which would not be identical to White but $\hat{V}_1$ and $\tilde{V}_1$ have the same plim.

1c.) $\hat{\beta}_{GLS} = (\sum \tilde{x}_t^2)^{-1}(\sum \tilde{x}_t\tilde{y}_t)$

$\tilde{x}_t = x_t/x_t = 1$

$\tilde{y}_t = y_t/x_t$

$\hat{\beta}_{GLS} = T^{-1}\sum(y_t/x_t)$

1d.) $\sqrt{T}(\hat{\beta}_{GLS}-\beta) = T^{-1/2}\sum\varepsilon_t/x_t \xrightarrow{L} N(0,\sigma^2)$

1e.) $\hat{V}_2 = \left[T^{-1}\sum(\tilde{y}_t - \hat{\beta}_{GLS})^2\right]$ with t-stat $b/[T^{1/2}\hat{V}_2^{1/2}]$. $\hat{V}_2$ is not the same as $\hat{V}_1$, the White standard errors

1f.) $\frac{\text{plim } T^{1/2}\hat{V}_1}{\text{plim } T^{1/2}\hat{V}_2} = \left[\frac{\sigma^2 E(x_t^4)/[E(x_t^2)]^2}{\sigma^2}\right]^{1/2} \geq 1$ by Jensen's Inequality defining $z_t = x_t^2$

1g.) Note b and $\hat{\beta}_{GLS}$ are both linear in **y** and are both unbiased. Gauss-Markov Theorem says that $\hat{\beta}_{GLS}$ has the smallest variance within the class of such estimators.

2a.) $SSR_0/20 = 0.125 \Rightarrow SSR_0 = 2.5$

$SSR_1/18 = 0.1 \Rightarrow SSR_1 = 1.8$

$m^{-1}(SSR_0 - SSR_1)/[SSR_1/(T-k)] = 2^{-1}(2.5-1.8)/0.1 = 3.5$

The 5% critical value for an $F(2,18)$ variable is 3.55, so we just fail to reject H_0 with a p-value slightly above 0.05.

2b.) It is perfectly possible (as almost occurs in this case) to have insignificant t statistics but a significant F statistic. This arises when the estimated coefficients are correlated, so that although we can not be sure that either one might be zero, we can be confident that both are not zero. The famous professor evidently does not understand some basic facts about econometrics.

2c.) Let $Q = \sum(y_t - \bar{y})^2$. The square of the t-statistic in (1) (namely $(3.536/0.5)^2 = 50$) could also have been calculated from the SSR expression as

$$50 = \frac{Q - SSR_0}{SSR_0/(T-k)} = \frac{Q-2.5}{0.125}$$

so $Q = 8.75$. Hence $R^2 = 1 - SSR_0/Q = 1-(2.5/8.25) = 0.70$

2d.) $R^2 = 1-(1.8/8.25) = 0.78$