Econ 220B, Winter 2015

Midterm Exam

DIRECTIONS: Calculators are permitted but no books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (110 points total) This question refers to a regression of y_t on a scalar explanatory variable x_t,

$$y_t = x_t\beta + \varepsilon_t.$$

We can write this regression for a sample of size T in vector form as

$$\mathbf{y} = \mathbf{X}\beta + \boldsymbol{\varepsilon}$$

where $\mathbf{y}, \mathbf{X}$, and $\boldsymbol{\varepsilon}$ are each $(T \times 1)$ vectors and β is a scalar. For this question you can assume that all of the following conditions hold:

(i) the (2×1) vector (x_t, ε_t) is stationary and ergodic with finite fourth moments

(ii) the scalar $x_t\varepsilon_t$ is a martingale difference sequence

(iii) $E(\varepsilon_t^2|x_t) = \sigma^2 x_t^2$

(iv) $E(\boldsymbol{\varepsilon}|\mathbf{X}) = \mathbf{0}$

(v) $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|\mathbf{X}) = \sigma^2 \begin{bmatrix} x_1^2 & 0 & \cdots & 0 \\ 0 & x_2^2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & x_T^2 \end{bmatrix}$

(vi) $\text{rank}(\mathbf{X}) = 1$.

a.) (20 points) Find the asymptotic distribution of the OLS estimator $b = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$.

b.) (15 points) Use your result in (a) to propose a t-statistic of the null hypothesis $\beta = 0$ of the form $b/\hat{V}_1$ for some estimate $\hat{V}_1$, giving the formula for $\hat{V}_1$. What is the relation between your proposed estimate $\hat{V}_1$ and White heteroskedasticity-consistent standard errors?

c.) (20 points) Write down the expression for the GLS estimate $\hat{\beta}_{GLS}$ and show that it can be interpreted as the sample mean of a certain observed scalar.

d.) (20 points) Calculate the asymptotic distribution of the GLS estimator in (c).

e.) (15 points) Use your result in (d) to propose a t-statistic of the null hypothesis $\beta = 0$ of the form $\hat{\beta}_{GLS}/\hat{V}_2$ for some estimate $\hat{V}_2$, giving the formula for $\hat{V}_2$. What is the relation between your proposed estimate $\hat{V}_2$ and White heteroskedasticity-consistent standard errors?

f.) (10 points) Prove directly using your algebraic expressions for $\hat{V}_1$ and $\hat{V}_2$ that

$$\frac{\text{plim } \hat{V}_1}{\text{plim } \hat{V}_2} \geq 1.$$

Hint: Jensen's Inequality implies that for any random variable z_t, $E(z_t^2)/[E(z_t)]^2 \geq 1$.

g.) (10 points) Can you think of another theorem from regression analysis that would also lead you to know that plim $\hat{V}_1 \geq$ plim $\hat{V}_2$? State the name of the theorem and why it implies this inequality.

2.) (40 points total) Last year as an RA for a famous professor you estimated the following regressions on a sample of size $T = 22$ observations, with OLS standard errors in parentheses and $s^2 = \sum_{t=1}^{T} e_t^2/(T-k)$ for each regression:

$$y_t = \underset{(1.000)}{-0.480} + \underset{(0.500)}{3.536}x_{t1} + e_t \qquad s^2 = 0.125 \qquad (1)$$

$$y_t = \underset{(1.000)}{-0.500} + \underset{(0.500)}{3.500}x_{t1} + \underset{(0.869)}{0.780}x_{t2} - \underset{(2.000)}{1.600}x_{t3} + e_t \qquad s^2 = 0.100 \qquad (2).$$

The good news is the paper has been accepted for publication and the professor is offering to make you a co-author! The bad news is that the journal requests some additional information but you've completely lost the data and can't possibly redo the regression! The good news is that your knowledge of econometrics is so deep that you know the answers to the journal's questions without even having to do the regressions over!

a.) (15 points) One thing the journal asks you to do is calculate the F statistic for the joint hypothesis that the coefficients on x_{t2} and x_{t3} are both zero. Fortunately you remember something about how to calculate that statistic if you were stuck on a desert island using the formula $m^{-1}(SSR_0 - SSR_1)/[SSR_1/(T-k)]$. Calculate the F statistic and find the p-value associated with the hypothesis using the attached table of the F distribution. Indicate the number you look at in the table that leads you to conclude whether H_0 should be rejected.

b.) (10 points) The professor says to you, "I don't know why the journal is bugging us about this F test anyway. It's obvious from the t-statistics that ~~x_{t1}~~ x_{t2} and ~~x_{t2}~~ x_{t3} don't belong in the regression." What do you say to the professor about this statement? Hint: try to answer even if you could not come up with an answer to part (a).

c.) (10 points) The journal also asks to know the centered R^2 for regression (1). Fortunately you know how to use the above desert-island formula along with the t-statistic from regression (1) to calculate what the R^2 for regression (1) must have been. What is it?

d.) (5 points) What was the R^2 for regression (2)?