Answer key for the midterm in 2014

1a.) OLS regression coefficient, square of standard error of regression, and uncentered coefficient of determination.

b.) $\mathbf{b} = (T^{-1}\sum \mathbf{x}_t\mathbf{x}_t')^{-1}(T^{-1}\sum \mathbf{x}_t y_t) \xrightarrow{p} \mathbf{Q}^{-1}\boldsymbol{\lambda}$

c.) $s^2 = (T-k)^{-1}\sum(y_t^2 - 2y_t\mathbf{x}_t'\mathbf{b} + \mathbf{b}'\mathbf{x}_t\mathbf{x}_t'\mathbf{b})$

$\xrightarrow{p} E(y_t^2) - 2E(y_t\mathbf{x}_t')\mathbf{Q}^{-1}\boldsymbol{\lambda} + \boldsymbol{\lambda}'\mathbf{Q}^{-1}E(\mathbf{x}_t\mathbf{x}_t')\mathbf{Q}^{-1}\boldsymbol{\lambda} = \gamma - 2\boldsymbol{\lambda}'\mathbf{Q}^{-1}\boldsymbol{\lambda} + \boldsymbol{\lambda}'\mathbf{Q}^{-1}\boldsymbol{\lambda}$

d.)

$$R^2 = 1 - \frac{T^{-1}\sum e_t^2}{T^{-1}\sum y_t^2} \xrightarrow{p} 1 - \frac{\gamma - \lambda Q^{-1}\lambda}{\gamma} = \frac{\lambda^2}{Q\gamma} = \frac{[E(x_t y_t)]^2}{E(x_t^2)E(y_t^2)} = \rho^2$$

2a.) $b = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$

b.) $\hat{\beta}_{GLS} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$

$\mathbf{P}\mathbf{P}' = \mathbf{V}$

$\tilde{\mathbf{e}} = \mathbf{P}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{GLS})$

$s^2 = (T-1)^{-1}\tilde{\mathbf{e}}'\tilde{\mathbf{e}} = (T-1)^{-1}(\mathbf{y} - \mathbf{X}\hat{\beta}_{GLS})'\mathbf{V}^{-1}(\mathbf{y} - \mathbf{X}\hat{\beta}_{GLS})$

c.)

$$\frac{\hat{\beta}_{GLS}}{\sqrt{s^2(\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1}}} = \frac{\hat{\beta}_{GLS}}{\sqrt{s^2/(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})}} \sim \text{ Student } t \text{ with } (T-1) \text{ d.f.}$$

d.) $\xrightarrow{L} N(0,1)$

3.) If $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, $(\mathbf{X}'\mathbf{X})$ has full rank k, $E(\boldsymbol{\varepsilon}|\mathbf{X}) = \mathbf{0}$, $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|\mathbf{X}) = \boldsymbol{\sigma}^2\mathbf{I}_T$, $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$, $\hat{\boldsymbol{\beta}} = \mathbf{A}_{\mathbf{X}}\mathbf{y}$, $E(\hat{\boldsymbol{\beta}}|\mathbf{X}) = \boldsymbol{\beta}$, then

$$E[(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})'|\mathbf{X}] - E[(\mathbf{b} - \boldsymbol{\beta})(\mathbf{b} - \boldsymbol{\beta})'|\mathbf{X}]$$

is positive semidefinite.