Econ 220B, Winter 2014

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (60 points total) Consider a regression of a scalar y_t on a $(k \times 1)$ vector $\mathbf{x}_t$. Consider the following statistics calculated from a sample of size T:

$$\begin{aligned} \mathbf{b} &= \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum_{t=1}^{T} \mathbf{x}_t y_t\right) \\ s^2 &= (T-k)^{-1} \sum_{t=1}^{T} (y_t - \mathbf{x}_t'\mathbf{b})^2 \\ R^2 &= 1 - \frac{\sum_{t=1}^{T} (y_t - \mathbf{x}_t'\mathbf{b})^2}{\sum_{t=1}^{T} (y_t)^2}. \end{aligned}$$

a.) (20 points) By what names are the statistics $\mathbf{b}$, s^2 and R^2 commonly known?

b.) (20 points) Suppose you assume that the vector $(y_t, \mathbf{x}_t')'$ is stationary and ergodic with mean zero and variance-covariance matrix given by

$$\begin{bmatrix} E(y_t^2) & E(y_t \mathbf{x}_t') \\ E(\mathbf{x}_t y_t) & E(\mathbf{x}_t \mathbf{x}_t') \end{bmatrix} = \begin{bmatrix} \gamma & \boldsymbol{\lambda}' \\ \boldsymbol{\lambda} & \mathbf{Q} \end{bmatrix}$$

where γ is a positive scalar, $\boldsymbol{\lambda}$ is a $(k \times 1)$ vector, and $\mathbf{Q}$ is a positive definite $(k \times k)$ matrix. Find the plim of $\mathbf{b}$ under the above assumptions.

c.) (10 points) Show that under the above assumptions,

$$s^2 \xrightarrow{p} \gamma - \boldsymbol{\lambda}' \mathbf{Q}^{-1} \boldsymbol{\lambda}.$$

d.) (10 points) The population correlation ρ between two variables x_t and y_t is defined as

$$\rho = \frac{E[x_t - E(x_t)][y_t - E(y_t)]}{\sqrt{E[x_t - E(x_t)]^2 E[y_t - E(y_t)]^2}}.$$

Show that when $k = 1$,

$$R^2 \xrightarrow{p} \rho^2.$$

2.) (60 points total) This question refers to a regression of y_t on a scalar explanatory variable x_t. We can write this regression for a sample of size T in vector form as

$$\mathbf{y} = \mathbf{X}\beta + \boldsymbol{\varepsilon}$$

where $\mathbf{y}, \mathbf{X}$, and $\boldsymbol{\varepsilon}$ are each $(T \times 1)$ vectors and β is a scalar. For this question you should assume that $\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0},\sigma^2\mathbf{V})$ where $\mathbf{V}$ is a known $(T \times T)$ matrix and σ^2 is an unknown scalar.

a.) (10 points) Write down the expression for the value of β for which $(\mathbf{y} - \mathbf{X}\beta)'(\mathbf{y} - \mathbf{X}\beta)$ is minimized.

b.) (20 points) Write down the expression for the statistic you would use to obtain an unbiased estimate the scalar σ^2 for this example.

c.) (20 points) Write down the expression you would use to test the null hypothesis H_0: $\beta = 0$ for this example and describe its distribution.

d.) (10 points) Suppose that in addition to the above assumptions, you assumed that $\{x_t\}$ is stationary and ergodic with $E(x_t^2) = Q > 0$. State (but you do not need to derive) the asymptotic distribution of the statistic you suggested in part (c).

3.) (30 points total) State (but you do not need to prove) the Gauss-Markov Theorem.