Econ 220B, Winter 2013

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (150 points total) This question refers to a regression of y_t on a constant and two other explanatory variables,

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \varepsilon_t, \qquad \text{for } t = 1, 2, ..., T,$$

with $E(\varepsilon_t^2|\mathbf{x}_t) = \sigma^2$.

a.) (10 points) Write the model in vector form $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ and give the dimensions and definitions of the vectors $\mathbf{y}, \boldsymbol{\beta}$, and $\boldsymbol{\varepsilon}$ and the matrix $\mathbf{X}$.

b.) (10 points) Write down the formulas for the OLS estimates of $\boldsymbol{\beta}$ and σ^2.

c.) (30 points) Suppose you are willing to assume that $\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{I}_T)$ and wanted to test the hypothesis that the two explanatory variables have the identical effect on y_t:

$$H_0\text{: } \beta_1 = \beta_2.$$

Write down the formula for the OLS F test of H_0 and state (but do not derive) its distribution under the stated assumptions. Hint: if you're having trouble remembering the formula, go back to the fact that if a $(k \times 1)$ vector $\mathbf{v} \sim N(\mathbf{0}, \boldsymbol{\Omega})$, then $\mathbf{v}'\boldsymbol{\Omega}^{-1}\mathbf{v} \sim \chi^2(k)$.

d.) (20 points) Suppose instead you wanted to test the above hypothesis using a t statistic that could be either positive or negative. Write down the formula for the t statistic you could use to test the hypothesis, and state (but do not derive) its distribution.

e.) (15 points) Let's say you want to estimate the regression equation subject to the restriction that the above null hypothesis H_0 is true. Give formulas for the estimates of β_0, β_1, and β_2 that you would use. Hint: you might want to define new vectors and matrices of data $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{X}}$ as functions of the original $\mathbf{y}$ and $\mathbf{X}$ and write your expressions for $\hat{\beta}_0, \hat{\beta}_1$, and $\hat{\beta}_2$ as functions of $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{X}}$.

f.) (20 points) Returning to the original regression model, suppose that you believe that $\{\mathbf{x}_t, y_t\}$ is stationary and ergodic with $T^{-1}\sum_{t=1}^{T} \mathbf{x}_t\mathbf{x}_t' \xrightarrow{p} \mathbf{Q}$ a nonsingular matrix and that $\varepsilon_t|\mathbf{x}_t, \varepsilon_{t-1}, \mathbf{x}_{t-1}, \varepsilon_{t-2}.\mathbf{x}_{t-2}, .., \varepsilon_1, \mathbf{x}_1 \sim N(0, \sigma^2)$. Show that the vector $\varepsilon_t\mathbf{x}_t$ is a martingale difference sequence and calculate its variance-covariance matrix.

g.) (30 points) Under the assumptions in part (f), derive the asymptotic distribution of the F test that you proposed in part (c) above. Hint: if you were unable to propose an F test in part (d), derive the asymptotic distribution of the OLS estimate $\mathbf{b}$ for partial credit.

h.) (15 points) Suppose instead you believed that $\varepsilon_t|\mathbf{x}_t, \varepsilon_{t-1}, \mathbf{x}_{t-1}, \varepsilon_{t-2}.\mathbf{x}_{t-2}, .., \varepsilon_1, \mathbf{x}_1 \sim N(0, g(\mathbf{x}_t))$ where $g(.)$ is an unknown function. Suggest a test statistic you would use in place of the F test in part (c) to test the null hypothesis H_0 in this case. Hint: again just derive the asymptotic distribution of $\mathbf{b}$ under this new assumption for partial credit.