Econ 220B, Winter 2012

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (60 points total) Let y_t and z_t be stationary and ergodic scalars which are related according to

$$y_t = \phi y_{t-1} + \alpha z_t + \gamma z_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim i.i.d.\ N(0, \sigma^2)$.

a.) (20 points) Write the above equation in the form of a regression model $y_t = \mathbf{x}_t' \boldsymbol{\beta} + \varepsilon_t$. What is the value of $\mathbf{x}_t$ and $\boldsymbol{\beta}$? Use this notation to characterize the OLS estimate of the vector $(\phi, \alpha, \gamma)'$ and of σ^2.

b.) (25 points) Suppose you wanted to test the null hypothesis H_0: $\alpha = -\gamma$. Suggest a statistic that you could compare with a Student t distribution to test this null hypothesis. What are the degrees of freedom? Would the test have an exact Student t distribution or only approximately Student t under the assumptions stated?

c.) (15 points) Now propose an alternative test statistic that you could use to test the null hypothesis H_0: $\alpha = -\gamma$ based on a comparison of SSR_R, which is the sum of squared residuals of a regression in which the null hypothesis is imposed, with SSR_U, which is the sum of squared residuals from a regression in which the null hypothesis is not imposed. Explain how you would estimate each of these regressions. What is the relation between the test you proposed in (c) and the test you proposed in (b)?

2.) (60 points total) Suppose that

$y_t = \mathbf{x}_t' \boldsymbol{\beta} + \varepsilon_t$

$(y_t, \mathbf{x}_t')'$ is stationary and ergodic

$E(\mathbf{x}_t \mathbf{x}_t') = \mathbf{Q}$ for $\mathbf{Q}$ a nonsingular $(k \times k)$ matrix

$E(x_{it} x_{jt} x_{kt}) = \lambda_{ijk}$ for i, j, k any elements of $\mathbf{x}_t$

$E(\varepsilon_t^2 | \mathbf{x}_t) = \sigma^2 x_{1t}$ where x_{1t} denotes the first element of $\mathbf{x}_t$, and where x_{1t} is a time-varying random variable

$\varepsilon_t \mathbf{x}_t$ is a martingale difference sequence with $E(\varepsilon_t^2 \mathbf{x}_t \mathbf{x}_t') = \mathbf{S}$.

a.) (10 points) Calculate the value of $\mathbf{S}$ under the above assumptions.

b.) (40 points) Calculate the OLS estimator of $\boldsymbol{\beta}$ and derive its asymptotic distribution.

c.) (10 points) Calculate the GLS estimator of $\boldsymbol{\beta}$.

3.) (30 points total) Let $\mathbf{y}_t = (y_{1t}, y_{2t})'$ be a stationary and ergodic (2×1) vector with

$$E(\mathbf{y}_t) = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

$$E(\mathbf{y}_t - \boldsymbol{\mu})(\mathbf{y}_t - \boldsymbol{\mu})' = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}.$$

a.) (20 points) Consider an OLS regression of y_{1t} on y_{2t} and a constant term:

$$y_{1t} = \alpha + \gamma y_{2t} + \varepsilon_t.$$

Calculate the values α_0 and γ_0 such that $\hat{\alpha} \xrightarrow{p} \alpha_0$ and $\hat{\gamma} \xrightarrow{p} \gamma_0$.

b.) (10 points) Besides being the plim of a regression, give another reason why knowing the values of α_0 and γ_0 might be of interest to a researcher.