Answer key for the midterm in 2011

1a.)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} n_M & 0 \\ 0 & n_F \end{bmatrix}^{-1} \begin{bmatrix} \sum y_t x_{t1} \\ \sum y_t x_{t2} \end{bmatrix}$$

b_1 is exactly equal to the average wage earned by male workers in the sample.

b.) $s^2 = (T-2)^{-1} \sum (y_t - x_{t1} b_1 - x_{t2} b_2)^2$

c.)

$$\frac{(b_1 - b_2)^2}{\begin{bmatrix} 1 & -1 \end{bmatrix} s^2 \begin{bmatrix} n_M^{-1} & 0 \\ 0 & n_F^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}} = \frac{(b_1 - b_2)^2}{s^2 (n_M^{-1} + n_F^{-1})}$$

d.) One could not add a constant term because $x_{3t} = x_{1t} + x_{2t}$ and $(\mathbf{X}'\mathbf{X})^{-1}$ would not exist.

2a.) $F(m, n)$

b.) $(1/m)$ times a $\chi^2(m)$

3a.) $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$

b.) $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ (also correct is $\boldsymbol{\beta} = [E(\mathbf{x}_t \mathbf{x}_t')]^{-1} E(\mathbf{x}_t y_t)$)

c.) $\hat{\boldsymbol{\beta}} = \{\mathbf{X}'[\mathbf{V}(\mathbf{X})]^{-1}\mathbf{X}\}^{-1}\mathbf{X}'[\mathbf{V}(\mathbf{X})]^{-1}\mathbf{y}$ (also correct is $\hat{\boldsymbol{\beta}} = (\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}'\tilde{\mathbf{y}}$ where row t of $\tilde{\mathbf{X}}$ is $\mathbf{x}_t/\sqrt{x_{t1}}$ and element t of $\tilde{\mathbf{y}}$ is $y_t/\sqrt{x_{t1}}$

4a.)

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$$s^2 = (T-k)^{-1} \sum (y_t - \mathbf{x}_t'\mathbf{b})^2$$

$$\mathbf{G} = \left. \frac{\partial \mathbf{g}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}'} \right|_{\boldsymbol{\beta}=\mathbf{b}}$$

$$[\mathbf{g}(\mathbf{b})]' \left\{ s^2 \mathbf{G}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{G}' \right\}^{-1} [\mathbf{g}(\mathbf{b})]$$

m degrees of freedom

b.)

$$\mathbf{S} = \sum e_t^2 \mathbf{x}_t \mathbf{x}_t'$$

$$[\mathbf{g}(\mathbf{b})]' \left\{ \mathbf{G}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{S}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{G}' \right\}^{-1} [\mathbf{g}(\mathbf{b})]$$

m degrees of freedom