Econ 220B, Winter 2011
James Hamilton

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (55 points total) An economist is interested in comparing the wages earned by male and female workers. Let

$$x_{t1} = \begin{cases} 1 & \text{if worker } t \text{ is male} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{t2} = \begin{cases} 1 & \text{if worker } t \text{ is female} \\ 0 & \text{otherwise} \end{cases}$$

$$y_t = \text{ wage earned by worker } t.$$

We have an observed sample of $t = 1, ..., T$ workers of whom n_M are male and n_F are female, so that $n_M + n_F = T$. Consider the following regression model:

$$y_t = x_{t1}\beta_1 + x_{t2}\beta_2 + \varepsilon_t$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{I}_T).$$

a.) (15 points) Calculate the OLS estimates of β_1 and β_2. What is the relation between the OLS estimate b_1 and the average wage earned by male workers in the sample?

b.) (15 points) Propose an unbiased estimate of σ^2. (You do not need to prove that it is unbiased– just state the formula).

c.) (15 points) Give a formula for the OLS F test of the null hypothesis that $\beta_1 = \beta_2$.

d.) (10 points) What would be the advantages and disadvantages of adding a constant term ($x_{t3} = 1$ for all t) to the regression?

2.) (20 points total) Let $\{Z_1, ..., Z_m\}$ and $\{W_1, ..., W_n\}$ be sets of independent standard Normal variables, that is, each variable is $N(0, 1)$ and is independent of all the others.

a.) (10 points) What name would you give for the distribution of the following variable?

$$\frac{(Z_1^2 + Z_2^2 + \cdots + Z_m^2)/m}{(W_1^2 + W_2^2 + \cdots + W_n^2)/n}$$

b.) (10 points) What name would you give for the limiting distribution of the above variable as $n \to \infty$?

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3.) (45 points total) Let $\{y_t, \mathbf{x}_t\}$ be strictly stationary and ergodic and consider

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t.$$

Let $\mathbf{y}$ be a $(T \times 1)$ vector of observed values for y_t and $\mathbf{X}$ a $(T \times k)$ matrix of observations on $\mathbf{x}_t$. Suppose there is a true value $\boldsymbol{\beta}_0$ such that for $\boldsymbol{\varepsilon} = \mathbf{y} - \mathbf{X}\boldsymbol{\beta}_0$ it is the case that

$$E(\boldsymbol{\varepsilon}|\mathbf{X}) = \mathbf{0}$$

$$E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|\mathbf{X}) = \sigma^2\mathbf{V}(\mathbf{X})$$

$$\underset{(T\times T)}{\mathbf{V}(\mathbf{X})} = \begin{bmatrix} x_{11} & 0 & \cdots & 0 \\ 0 & x_{21} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & x_{T1} \end{bmatrix}$$

where x_{t1} denotes the first element of $\mathbf{x}_t$. You can also assume that $\mathbf{X}$ has rank k. In answering the questions below you do not have to derive the results: just give the formula for the correct answer.

a.) (15 points) Calculate the value of $\boldsymbol{\beta}$ for which $T^{-1}\sum_{t=1}^{T}(y_t - \mathbf{x}_t'\boldsymbol{\beta})^2$ is minimized.

b.) (15 points) Calculate the value of $\boldsymbol{\beta}$ for which $E(y_t - \mathbf{x}_t'\boldsymbol{\beta})^2$ is minimized.

c.) (15 points) Let $\mathbf{c}$ be a known $(k \times 1)$ vector and let $\hat{\boldsymbol{\beta}}$ be an estimate of $\boldsymbol{\beta}$ based on the data. Calculate the value of $\hat{\boldsymbol{\beta}}$ for which $E\left(\mathbf{c}'\hat{\boldsymbol{\beta}} - \mathbf{c}'\boldsymbol{\beta}_0\right)^2$ is minimized.

4.) (30 points total) Consider a regression model $y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$ with $\mathbf{x}_t$ a $(k \times 1)$ vector of explanatory variables and an observed sample of observations $t = 1, ..., T$. Suppose we are interested in testing the hypothesis $\mathbf{g}(\boldsymbol{\beta}) = \mathbf{0}$ for $\mathbf{g}(.)$ a known $(m \times 1)$ vector-valued continuous function. In answering the following questions, you do not need to derive anything: just state the formula if you know it.

a.) (15 points) Give the formula for the OLS χ^2 test of H_0 based on the usual OLS standard errors. What would be its degrees of freedom?

b.) (15 points) Give a formula for the χ^2 test of H_0 based on the White standard errors (also known as "robust standard errors" or "heteroskedasticity-consistent standard errors"). What would be its degrees of freedom?