Econ 220B, Winter 2010

Answers to midterm exam

1.) a.) $E(\hat{\beta}_2) = \beta_2$. b.) $\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \sigma \mathbf{I}_T)$

2.) Let $a(\mu) = 1/\mu$ and

$$A(\mu) = \frac{\partial a}{\partial \mu} = -\frac{1}{\mu^2}.$$

Then for $\hat{\mu}_T = T^{-1}\sum_{t=1}^T y_t$ by CLT $\sqrt{T}(\hat{\mu}_T - \mu) \xrightarrow{L} N(0, \sigma^2)$ and from Hayashi Lemma 2.5 (the delta method)

$$\sqrt{T}[a(\hat{\mu}_T) - a(\mu)] \xrightarrow{L} N(0, [A(\mu)]^2\sigma^2).$$

Hence $v = \sigma^2/\mu^4$. But $v \neq \lim_{T\to\infty} E(q_T - \mu^{-1})^2$ because $E(q_T)$ does not exist.

3.) a.)

$$\mathbf{b}_T = \boldsymbol{\beta} + \left(\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(\sum_{t=1}^T \mathbf{x}_t\varepsilon_t\right)$$

$$\sqrt{T}(\mathbf{b}_T - \beta) = \left(T^{-1}\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(T^{-1/2}\sum_{t=1}^T \mathbf{x}_t\varepsilon_t\right)$$

which converges in distribution to $\mathbf{Q}^{-1}$ times a $N(0, \sigma^2\mathbf{Q})$ variable, meaning $\sqrt{T}(\mathbf{b}_T - \beta) \xrightarrow{L} N(\mathbf{0}, \sigma^2\mathbf{Q}^{-1})$.

b.) Let $\mathbf{R} = (1, -1, 0, 0, ..., 0)$. Then

$$F = (b_1 - b_2)\left[s^2\mathbf{R}\left(\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\mathbf{R}'\right]^{-1}(b_1 - b_2)$$

c.) Note

$$\begin{aligned} F &= \frac{(b_1 - b_2)^2}{s^2\mathbf{R}\left(\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\mathbf{R}'} \\ &= \frac{\left[\sqrt{T}\mathbf{R}(\mathbf{b} - \boldsymbol{\beta})\right]^2}{s^2\mathbf{R}\left(T^{-1}\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\mathbf{R}'} \\ &\xrightarrow{L} \frac{\left[\sqrt{T}\mathbf{R}(\mathbf{b} - \boldsymbol{\beta})\right]^2}{\sigma^2\mathbf{R}\mathbf{Q}^{-1}\mathbf{R}'}. \end{aligned}$$

This is the square of a zero-mean Normal variable divided by its variance, so$F_T \xrightarrow{L} \chi^2(1)$.

d.)

$$t = \frac{b_1 - b_2}{\sqrt{s^2 \mathbf{R} \left(\sum \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \mathbf{R}'}}$$

e.) The answers should be numerically identical, no matter what is assumed. Even if the colleague used a one-sided test for t, that could only reject more often than the two-sided test. Hence tell your colleague she simply made a mistake.

f.)

$$F = \frac{(b_1 - b_2)^2}{T\mathbf{R}\left[\left(\sum \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \hat{\mathbf{S}} \left(\sum \mathbf{x}_t \mathbf{x}_t'\right)^{-1}\right] \mathbf{R}'}$$