Econ 220B, Winter 2010

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (20 points) Consider the following equations:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}.$$

Here $\mathbf{y}$ is a $(T \times 1)$ vector of observations on the dependent variable, $\mathbf{X}$ is a $(T \times k)$ matrix of observations on the explanatory variables with rank k, and $\boldsymbol{\beta}$ is the true value of a $(k \times 1)$ vector of parameters. Let b_2 denote the second element of the vector $\mathbf{b}$ and let β_2 denote the second element of the vector $\boldsymbol{\beta}$. Consider the following theorem:

> Let $\hat{\beta}_2$ be any unbiased estimator of β_2 that is constructed from some function of $\mathbf{X}$ and $\mathbf{y}$. If condition A holds, then $\mathrm{Var}(\hat{\beta}_2|\mathbf{X}) \geq \mathrm{Var}(b_2|\mathbf{X})$.

a.) Write down an equation for what it means to say that $\hat{\beta}_2$ is an unbiased estimator of β_2.

b.) State a sufficient condition or conditions A for which the above theorem is true.

2.) (30 points) Suppose that $y_t \sim$ i.i.d. $N(\mu, \sigma^2)$ with $\mu \neq 0$ and let $q_T = T/\sum_{t=1}^{T} y_t$. Prove that $\sqrt{T}(q_T - \mu^{-1}) \xrightarrow{L} N(0, v)$ and calculate v. Does $v = \lim_{T\to\infty} E(q_T - \mu^{-1})^2$?

3.) (100 points) Suppose $y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$ where $\mathbf{x}_t$ is a $(k \times 1)$ vector of explanatory variables and

$$E[\varepsilon_t \mathbf{x}_t | \varepsilon_{t-1}\mathbf{x}_{t-1}, \varepsilon_{t-2}\mathbf{x}_{t-2}, ..., \varepsilon_1 \mathbf{x}_1] = \mathbf{0}$$

with $\{\varepsilon_t, \mathbf{x}_t\}$ stationary and ergodic and $E(\varepsilon_t^2 \mathbf{x}_t \mathbf{x}_t') = \sigma^2 \mathbf{Q}$. You can further assume that $T^{-1}\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \xrightarrow{p} \mathbf{Q}$ which has rank k. The OLS estimates are given by $\mathbf{b} = \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum_{t=1}^T \mathbf{x}_t y_t\right)$ and $s^2 = (T-k)^{-1}\sum_{t=1}^T (y_t - \mathbf{x}_t'\mathbf{b})$.

a.) Derive the asymptotic distribution of $\mathbf{b}$ under the stated assumptions.

b.) Write down the formula for the standard OLS F test for testing the null hypothesis that $\beta_1 = \beta_2$, where β_1 denotes the first element of the vector $\boldsymbol{\beta}$ and β_2 denotes the second element.

c.) Derive the asymptotic distribution of the test you proposed in part (b) under the stated assumptions. Be sure your derivation accurately describes this particular example rather than repeating an argument or notation you may have seen in some more general case. [HINT: You can assume that $s_T^2 \xrightarrow{p} \sigma^2$ and need not prove this separately].

d.) Propose a t test you could use to test the null hypothesis that $\beta_1 = \beta_2$.

e.) A colleague followed the advice you gave in parts (b) and (d) but got different answers, rejecting the null hypothesis when she did test (b) but failing to reject the null hypothesis when she did test (d). What would you tell her is the most likely explanation for why she got different answers?

f.) How would you modify your answer to (b) if you instead assume that $E(\varepsilon_t^2 \mathbf{x}_t \mathbf{x}_t') = \mathbf{S} \neq \sigma^2 \mathbf{Q}$ where $\mathbf{S}$ can be consistently estimated fom $\hat{\mathbf{S}} = T^{-1}\sum_{t=1}^T e_t^2 \mathbf{x}_t \mathbf{x}_t'$ for $e_t = y_t - \mathbf{x}_t'\mathbf{b}$? [Note you do not have to derive the distribution, just state the form of the test statistic you would use under the new assumption]