Econ 220B, Winter 2009
James Hamilton

Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1.) (50 points) Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon}|\mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{V}(\mathbf{X})).$$

Here $\mathbf{y}$ is a $(T \times 1)$ vector of observations on the dependent variable, $\mathbf{X}$ is a $(T \times k)$ matrix of observations on the explanatory variables with rank $k$, $\boldsymbol{\beta}$ is an unknown $(k \times 1)$ vector of parameters that must be estimated from the data, $\sigma^2$ is an unknown scalar that must be estimated from the data, and $\mathbf{V}(\mathbf{X})$ is a known nonsingular $(T \times T)$ matrix. The regression includes a constant term, so that each element of the first column of $\mathbf{X}$ is equal to unity.

a.) Give the formula for the OLS estimate of $\boldsymbol{\beta}$, denoted $\hat{\boldsymbol{\beta}}_{OLS}$.

b.) Give the formula for the GLS estimate of $\boldsymbol{\beta}$, denoted $\hat{\boldsymbol{\beta}}_{GLS}$.

c.) Consider the centered $R^2$ defined as

$$R^2 = 1 - \frac{\sum_{t=1}^{T}(y_t - \mathbf{x}_t'\hat{\boldsymbol{\beta}})^2}{\sum_{t=1}^{T}(y_t - \overline{y})^2}$$

where $y_t$ denotes the $t$th element of the original vector $\mathbf{y}$ defined above and $\mathbf{x}_t'$ denotes the $t$th row of the matrix $\mathbf{X}$. Which estimate, $\hat{\boldsymbol{\beta}}_{OLS}$ or $\hat{\boldsymbol{\beta}}_{GLS}$ , would produce the larger value for $R^2$ when plugged into the above formula? Why?

d.) Suggest a statistic you could use to test the null hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ for $\mathbf{R}$ a known $(m \times k)$ matrix and $\mathbf{r}$ a known $(m \times 1)$ vector. What distribution would you compare this statistic with to test the null hypothesis? Is this distribution an exact small-sample result or only an asymptotic approximation?

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2. (100 points) Consider the following regression model:

$$y_t = \mathbf{x}_t^{'}\boldsymbol{\beta} + \varepsilon_t$$

$$T^{-1}\sum_{t=1}^{T} \mathbf{x}_t\mathbf{x}_t^{'} \xrightarrow{p} \mathbf{Q}$$

$$E(\varepsilon_t^2) = \sigma^2$$

$$E(\varepsilon_t^2\mathbf{x}_t\mathbf{x}_t^{'}) = \mathbf{S}.$$

Here $\mathbf{x}_t$ is a $(k \times 1)$ vector of explanatory variables for the $t$th observation, $\mathbf{Q}$ has rank $k$, $\{y_t, \mathbf{x}_t\}$ are stationary and ergodic, and $\varepsilon_t\mathbf{x}_t$ is a stationary and ergodic martingale difference sequence.

a.) Prove that the OLS estimate $\mathbf{b}_T = \left(\sum_{t=1}^{T} \mathbf{x}_t\mathbf{x}_t^{'}\right)^{-1}\left(\sum_{t=1}^{T} \mathbf{x}_t y_t\right)$ is consistent.

b.) Derive the limiting distribution of $\sqrt{T}(\mathbf{b}_T - \boldsymbol{\beta})$.

c.) Suggest estimates that you might use for $\mathbf{Q}$ and $\mathbf{S}$.

d.) Assuming that the estimates you proposed in part (c) are consistent, suggest a statistic that you could use to test the null hypothesis that the first element of $\boldsymbol{\beta}$ is zero ($H_0$: $\beta_1 = 0$).

e.) Derive the asymptotic distribution of the statistic you proposed in part (d).

f.) Suggest a statistic that would have an approximate $\chi^2(m)$ distribution that you could use to test the hypothesis ($H_0$: $\mathbf{g}(\boldsymbol{\beta}) = \mathbf{0}$) for $\mathbf{g}$: $\mathbb{R}^k \rightarrow \mathbb{R}^m$ a function whose derivative is continuous.