Econ 2142, Fall 2013

Practice Midterm Exam Questions

DIRECTIONS: No books, notes, or calculators of any kind are allowed.

1.) Suppose that x_t follows an MA(1) process

$$x_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}, \quad 0 < \theta < 1$$

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t^2) = \sigma^2$$

$$E(\varepsilon_t \varepsilon_{t-j}) = 0 \qquad \text{for } j \neq 0,$$

and we then calculate a new series $y_t = (1 - \rho L)x_t$ for L the lag operator and ρ some real number.

a.) Calculate the population linear projection of y_t on $(1, \varepsilon_{t-1}, \varepsilon_{t-2})'$.

b.) For what values of ρ is y_t covariance stationary? Calculate the mean and population spectrum of y_t when those objects exist.

c.) Let $\overline{y} = T^{-1} \sum_{t=1}^{T} y_t$ denote the sample mean from a sample of size T and consider the variance of the sample mean $E[\overline{y} - E(y_t)]^2$ provided $E(y_t)$ exists. Calculate

$$\lim_{T\to\infty, \rho\to 1} TE[\overline{y} - E(y_t)]^2$$

d.) Suppose that $\rho = 2$. Can you find a moving-average representation for the process for y_t of the form

$$y_t = c + v_t + \psi_1 v_{t-1} + \psi_2 v_{t-2} + \cdots$$

where v_t are the fundamental innovations for y_t, that is,

$$v_t = y_t - \hat{E}(y_t | y_{t-1}, y_{t-2}, ...)?$$

If so, find the values for ψ_j and for $E(v_t^2)$.

2.) Suppose an $(n \times 1)$ vector $\mathbf{y}_t$ can be characterized by

$$\mathbf{y}_t = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\varepsilon}_t$ is i.i.d. $N(\mathbf{0}, \mathbf{\Omega})$.

a.) What additional assumptions (if any) would you need to make in order to conclude that $\mathbf{y}_t$ is covariance stationary?

b.) Give the formulas for the maximum likelihood estimates of $\mathbf{c}, \mathbf{\Phi}_1, \mathbf{\Phi}_2, \mathbf{\Omega}$. Note: you do not need to derive these, just state the formulas for the estimates (or accurately describing them in words is also ok).

c.) What additional assumptions (if any) would you need in order to be able to interpret the usual OLS t or F tests using the standard t and F tables?

d.) Calculate the value of the nonorthogonalized impulse-response function for horizon $s = 2$:

$$\frac{\partial \mathbf{y}_{t+2}}{\partial \boldsymbol{\varepsilon}_t'}$$

e.) A researcher estimates a just-identified structural VAR of the form

$$\mathbf{B}_0 \mathbf{y}_t = \boldsymbol{\lambda} + \mathbf{B}_1 \mathbf{y}_{t-1} + \mathbf{u}_t$$

$$\mathbf{u}_t = \mathbf{\Lambda} \mathbf{u}_{t-1} + \mathbf{v}_t$$

for $\mathbf{v}_t \sim$ i.i.d. $N(\mathbf{0}, \mathbf{I}_n)$. What is the relationship between the maximum likelihood estimates $\{\hat{\boldsymbol{\lambda}}, \hat{\mathbf{B}}_0, \hat{\mathbf{B}}_1, \hat{\mathbf{\Lambda}}\}$ and the values $\{\hat{\mathbf{c}}, \hat{\mathbf{\Phi}}_1, \hat{\mathbf{\Phi}}_2, \hat{\mathbf{\Omega}}\}$ that you specified above?

f.) Give an expression for an estimate of the effect of a structural disturbance $\mathbf{v}_t$ on the value of $\mathbf{y}_{t+s}$.