Econ 2142, Fall 2013

Answers to practice final questions

1a.) $\hat{\mathbf{y}}_{t|t-1} = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\hat{\boldsymbol{\xi}}_{t|t-1}$
$\mathbf{C}_{t|t-1} = \mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R}$

b.)

$$f(y_t|\Omega_{t-1}, \boldsymbol{\theta}) = \frac{1}{(2\pi)^{n/2}|\mathbf{C}_{t|t-1}|^{1/2}} \exp\left[-\frac{(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1})'\mathbf{C}_{t|t-1}^{-1}(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1})}{2}\right]$$

c.) $\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{A} = \mu$
$\mathbf{H}' = \begin{bmatrix} 1 & \theta_1 & \theta_2 \end{bmatrix} \quad R = 0$

d.) Since the unconditional distribution is

$$\boldsymbol{\xi}_t \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}\right),$$

we should use $\hat{\xi}_{1|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{P}_{1|0} = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$

2a.) Estimate reduced-form VAR with variables ordered as given:

$$\mathbf{y}_t = \mathbf{c} + \boldsymbol{\Phi}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$$

$$\mathbf{x}_{t-1} = (1, \mathbf{y}_{t-1}',)'$$

$$\begin{bmatrix} \hat{\mathbf{c}} & \hat{\boldsymbol{\Phi}} \end{bmatrix} = \left(\sum_{t=1}^T \mathbf{y}_t\mathbf{x}_{t-1}'\right)\left(\sum_{t=1}^T \mathbf{x}_{t-1}\mathbf{x}_{t-1}'\right)^{-1}$$

$$\widehat{\frac{\partial \mathbf{y}_{t+s}}{\partial u_{3t}}} = \hat{\boldsymbol{\Phi}}^s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

b.) Suppose we can identify particular days within the sample on which there was a major change in monetary policy, and let z_t denote the change in the fed funds rate on that day in month t (with $z_t = 0$ if there was no important day identified for month t). Let $\hat{\boldsymbol{\varepsilon}}_t = \mathbf{y}_t - \hat{\mathbf{c}} - \hat{\boldsymbol{\Phi}}\mathbf{y}_{t-1}$ and $\hat{\boldsymbol{\alpha}} = T^{-1}\sum_{t=1}^{T}\hat{\boldsymbol{\varepsilon}}_t z_t$. Use

$$\widehat{\frac{\partial \mathbf{y}_{t+s}}{\partial u_{3t}}} = \hat{\boldsymbol{\Phi}}^s \begin{bmatrix} \hat{\alpha}_1/\hat{\alpha}_3 \\ \hat{\alpha}_2/\hat{\alpha}_3 \\ 1 \end{bmatrix}.$$

3a.) $\hat{c} \xrightarrow{p} 0$, $\hat{\eta} \xrightarrow{p} 0$, $\hat{\zeta} \xrightarrow{p} \zeta_0$
b.) $\hat{\eta}/\hat{\sigma}_{\hat{\eta}} \xrightarrow{L}$ Dickey-Fuller Case 2, $(\hat{\zeta} - \zeta_0)/\hat{\sigma}_{\hat{\zeta}} \xrightarrow{L} N(0,1)$
c.) $\hat{k} = \hat{c}$, $\hat{\phi}_1 = 1 + \hat{\eta} + \hat{\zeta}$, $\hat{\phi}_2 = -\hat{\zeta}$
d.) $(\hat{\phi}_1 - \phi_1^{(0)})/\hat{\sigma}_{\hat{\phi}_1} \xrightarrow{L} N(0,1)$

4a.) $\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \mathbf{v}_{t+1}$ where row j column i element of $\mathbf{P}$ is p_{ij}
b.) element j is Prob$(s_t = j|\Omega_t)$
c.) $\hat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{P}\hat{\boldsymbol{\xi}}_{t|t}$
d.) element j of $\boldsymbol{\eta}_{t+1}$ is

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y_{t+1} - \mu_j)^2}{2\sigma^2}\right]$$

e.) $\boldsymbol{\eta}_{t+1} \odot \hat{\boldsymbol{\xi}}_{t+1|t}$
f.) $\mathbf{1}'\left(\boldsymbol{\eta}_{t+1} \odot \hat{\boldsymbol{\xi}}_{t+1|t}\right)$ (sum of elements of vector in (e))

5a.) $\exp(\alpha\xi_t) \simeq \exp(\alpha\hat{\xi}_{t|t-1}) + \alpha\exp(\alpha\hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})$
$\hat{y}_{t|t-1} = \exp(\alpha\hat{\xi}_{t|t-1})$
$c_{t|t-1} = \alpha^2\exp(2\alpha\hat{\xi}_{t|t-1})P_{t|t-1} + R$
b.) $\hat{\xi}_{t+1|t} = \phi\hat{\xi}_{t|t-1} + \phi P_{t|t-1}h_t(h_t P_{t|t-1}h_t + R)^{-1}\left[y_t - \exp(\alpha\hat{\xi}_{t|t-1})\right]$
$P_{t+1|t} = \phi\left[P_{t|t-1} - P_{t|t-1}h_t(h_t P_{t|t-1}h_t + R)^{-1}h_t P_{t|t-1}\right]\phi + Q$
$h_t = \alpha\exp(\alpha\hat{\xi}_{t|t-1})$

6a.) $\xi_{t+1}^{(i)} \sim N(\phi\xi_t^{(i)}, Q)$

$$g_{t+1}(\Lambda_{t+1}^{(i)}|\Omega_t) = \frac{1}{\sqrt{2\pi Q}}\exp\left[-\frac{(\xi_{t+1}^{(i)} - \phi\xi_t^{(i)})^2}{2Q}\right] g_t(\Lambda_t^{(i)}|\Omega_{t-1})$$

b.)

$$\tilde{\omega}_{t+1}^{(i)} = \frac{\omega_t^{(i)}}{\sqrt{2\pi R}} \exp\left[-\frac{\left[y_{t+1} - \exp(\alpha\xi_{t+1}^{(i)})\right]^2}{2R}\right]$$

$$\omega_{t+1}^{(i)} = \frac{\tilde{\omega}_{t+1}^{(i)}}{\sum_{i=1}^{D} \tilde{\omega}_{t+1}^{(i)}}$$

c.) $\hat{E}(\xi_{t+1}|\Omega_{t+1}) = \sum_{i=1}^{D} \xi_{t+1}^{(i)}\omega_{t+1}^{(i)}$