Econ 2142, Fall 2013

Answers to practice midterm questions

1a.)

$$x_t = \mu + (1+\theta L)\varepsilon_t$$
$$(1-\rho L)x_t = (1-\rho L)\mu + (1-\rho L)(1+\theta L)\varepsilon_t$$
$$y_t = (1-\rho)\mu + [1+(\theta-\rho)L - \theta\rho L^2]\varepsilon_t$$
$$\hat{P}(y_t|1,\varepsilon_{t-1},\varepsilon_{t-2}) = (1-\rho)\mu + (\theta-\rho)\varepsilon_{t-1} - \theta\rho\varepsilon_{t-2}$$

1b.) $y_t \sim$ MA(2) which is stationary for all ρ

$$E(y_t) = (1-\rho)\mu$$

$$s_Y(\omega) = \frac{\sigma^2(1+\theta e^{-i\omega})(1-\rho e^{-i\omega})(1+\theta e^{i\omega})(1-\rho e^{i\omega})}{2\pi}$$

1c.) This is proportional to the spectrum at frequency 0, which from part (b) is zero.

1d.) Can equivalently think of starting with $(1-\rho L)\varepsilon_t$ and then filtering by $(1+\theta L)$. The representation $(1-\rho L)\varepsilon_t$ is noninvertible, but its autocovariances $\gamma_0 = (1+\rho^2)\sigma^2$ and $\gamma_1 = \rho\sigma^2$ are identical to those of $(1-\rho^{-1}L)v_t$ with $E(v_t^2) = \tau^2$. Solving the equation for γ_1 we have $2\sigma^2 = 0.5\tau^2$ or $\tau^2 = 4\sigma^2$. The invertible representation is thus

$$\begin{aligned} y_t &= (1-\rho)\mu + (1-\rho^{-1}L)(1+\theta L)v_t \\ &= c + v_t + (\theta - 0.5)v_{t-1} - 0.5\theta v_{t-2}. \end{aligned}$$

2a.) Need eigenvalues of $\mathbf{F}$ inside unit circle for

$$\mathbf{F} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{I}_n & \mathbf{0} \end{bmatrix}$$

or equivalently that solutions z to $|\mathbf{I}_n - \mathbf{\Phi}_1 z - \mathbf{\Phi}_2 z^2| = 0$ are outside unit circle.

2b.) Let

$$\mathbf{x}_t = \begin{bmatrix} 1 \\ \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \end{bmatrix}$$

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{c} & \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \end{bmatrix}$$

$$\hat{\mathbf{\Pi}} = \left(\sum_{t=1}^T \mathbf{y}_t\mathbf{x}_t'\right)\left(\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}$$

or equivalently row i of $\hat{\mathbf{\Pi}}$ is obtained by OLS regression of y_{it} on $\mathbf{x}_t$. Also

$$\hat{\mathbf{\Omega}} = T^{-1} \sum_{t=1}^{T} \left(\mathbf{y}_t - \hat{\mathbf{\Pi}}\mathbf{x}_t\right) \left(\mathbf{y}_t - \hat{\mathbf{\Pi}}\mathbf{x}_t\right)'.$$

2c.) Stated assumptions along with stationarity condition are sufficient.

2d.)

$$\frac{\partial \mathbf{y}_{t+2}}{\partial \boldsymbol{\varepsilon}_t'} = \mathbf{\Phi}_1^2 + \mathbf{\Phi}_2$$

2e.)

$$(\mathbf{I}_n - \mathbf{\Lambda} L)\mathbf{B}_0\mathbf{y}_t = (\mathbf{I}_n - \mathbf{\Lambda} L)\boldsymbol{\lambda} + (\mathbf{I}_n - \mathbf{\Lambda} L)\mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{v}_t$$
$$\mathbf{B}_0\mathbf{y}_t = \mathbf{k} + \mathbf{C}_1\mathbf{y}_{t-1} + \mathbf{C}_2\mathbf{y}_{t-1} + \mathbf{v}_t$$

for $\mathbf{k} = (\mathbf{I}_n - \mathbf{\Lambda})\boldsymbol{\lambda}$, $\mathbf{C}_1 = \mathbf{\Lambda}\mathbf{B}_0 + \mathbf{B}_1$, $\mathbf{C}_2 = -\mathbf{\Lambda}\mathbf{B}_1$. Thus

$$\hat{\mathbf{B}}_0^{-1}(\hat{\mathbf{B}}_0^{-1})' = \hat{\mathbf{\Omega}}$$

$$\hat{\mathbf{B}}_0^{-1}(\hat{\mathbf{\Lambda}}\hat{\mathbf{B}}_0 + \hat{\mathbf{B}}_1) = \hat{\mathbf{\Phi}}_1$$
$$-\hat{\mathbf{B}}_0^{-1}\hat{\mathbf{\Lambda}}\hat{\mathbf{B}}_1 = \hat{\mathbf{\Phi}}_2$$
$$\hat{\mathbf{B}}_0^{-1}(\mathbf{I}_n - \hat{\mathbf{\Lambda}})\hat{\boldsymbol{\lambda}} = \hat{\mathbf{c}}$$

2e.)

$$\frac{\partial \mathbf{y}_{t+s}}{\partial \mathbf{v}_t'} = \hat{\mathbf{\Psi}}_s\hat{\mathbf{B}}_0^{-1}$$

where $\hat{\mathbf{\Psi}}_s$ denotes the first n rows and n columns of $\hat{\mathbf{F}}^s$ and

$$\hat{\mathbf{F}} = \begin{bmatrix} \hat{\mathbf{\Phi}}_1 & \hat{\mathbf{\Phi}}_2 \\ \mathbf{I}_n & \mathbf{0} \end{bmatrix}$$