



Chapter 11 Career-Based Incentives

Julian Betts, Economics 136

Note: We'll cover all of the
appendix



Main Questions

- 1) How can promotions affect effort?
- 2) What determines the raise that a worker should get upon promotion in a promotion tournament?
- 3) How many levels of jobs should there be?
- 4) How does a firm ensure that it does not overpay the Chief Executive Officer (CEO)?
- 5) What are the advantages/disadvantages of promotion tournaments versus piece rates?



Main Questions (II)

- 6) What are the relative (dis)advantages of promotion tournaments versus promotions based on absolute standards?
- 7) How to handle variations across workers in ability or personality?
- 8) Should a firm promote from within or hire senior candidates from outside?



Main Questions (III)

- **Seniority Pay and Incentives**
- 1) When people unlikely to get promoted, what are alternatives to provide motivation?
- 2) What about pay raises as experience rises? Risks to firm and worker
- 3) Can overpaying older workers cause them to stay too long past normal retirement age?

1) How can promotions affect effort?

Sports metaphor of tournaments is hard to miss!



1) How can promotions affect effort?



- Promotions are sometimes like a tournament, where promotion/winning based on output *relative* to that of other workers, rather than on *absolute* level of output
- Prospect of a wage hike increases effort
- The bigger the potential reward, the harder people will work for it
 - Much stronger incentive than changing job title without raise!



Related insight...

- May be one reason why the size of raises tends to increase near the top of a firm's hierarchy. See "Acme" case Fig. 11.1.



What determines the raise that a worker should get upon promotion?

- For most jobs is element of luck in how much net revenue worker produces per period. Examples:
 - Recession
 - Mechanical breakdowns
 - Change in input prices
- Typical worker will work less hard to gain promotion if there is lots of “noise”, that is, randomness, in the promotion decision



Promotions as Tournaments

- Consider 2 workers in workplace, with 1 winning/getting the promotion and earning a prize/a higher wage
- Think about the parallels with tennis or golf tournaments
- It is relative, not absolute performance that matters



Promotions as Tournaments

- Prospect of wage hike increases effort
- Worker effort is not the only thing that matters in the promotion decision, there is a factor besides effort that affects who wins
 - E.g.
 - a tennis player who trains really hard, but on the day of an important match happens to be sick
 - equipment failure for someone making a product while using that equipment
 - a hard drive crash on the day a paper is due
 - a supervisor not rating performance perfectly accurately



How does the luck/noise factor relate to effort?

- Suppose this course were graded in the following way:
 - Your percentage score on the exams counted for **one tenth of your grade**
 - A roll of the dice counted for **nine tenths of your grade**
 - (A=1, B=2, C=3, D=4, F=5, Incomplete=6))
 - Would you this grading scheme cause you to put in more or less time studying?
 - A. More
 - B. Less
 - C. Same

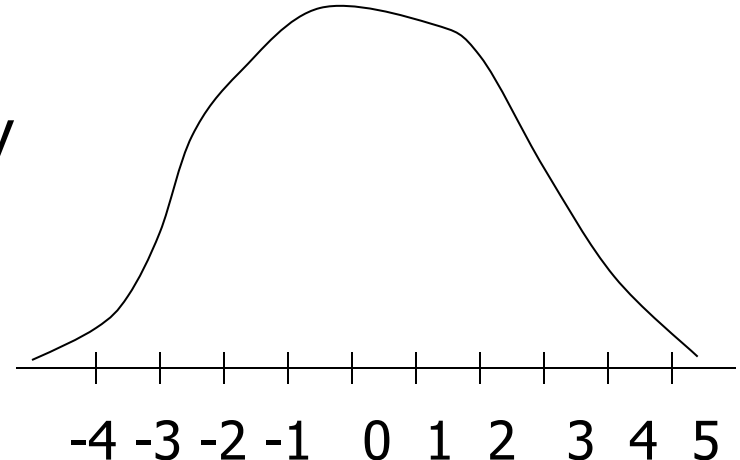


Modeling the noise factor

- Intuition
 - There is noise or luck in the promotion decision
 - Typical worker works less hard if there is more noise
- We can use random variables to model noise in a tournament, where the winner of the tournament is based on who produces more output, and where each worker's output depends on effort (m) and a random shock/noise (e)

Random Variable Review

- A Random Variable maps outcomes to real numbers
- One way to characterize distribution of a RV is through a Probability Density Function (PDF)
- PDF gives density of RV at each real number

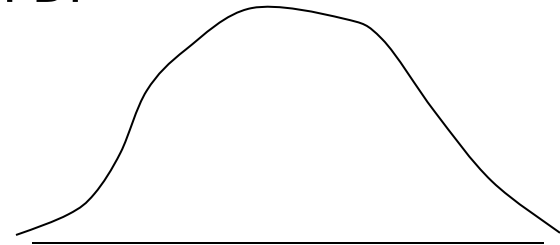


Probability Density Function (pdf)

- A function $g(x)$ is a pdf of a continuous random variable x if and only if

$$\text{Prob}(a \leq x \leq b) = \int_a^b g(x) dx$$

PDF

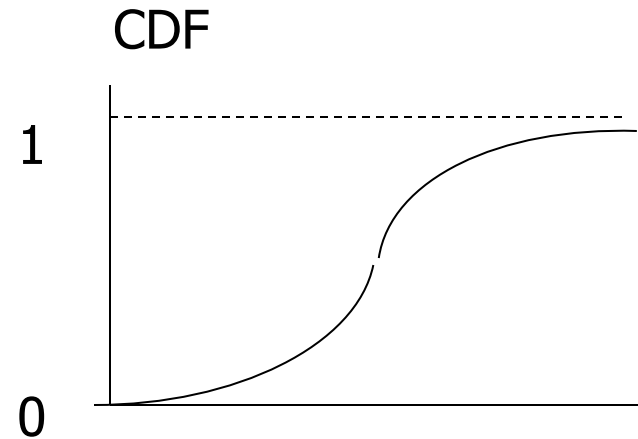
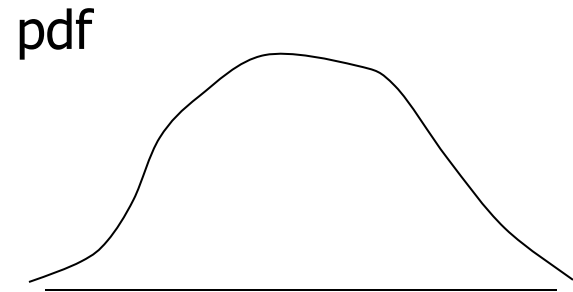


Cumulative Distribution Function (CDF)

- If X is a continuous random variable, then function given by

$$G(c) = \text{Prob}(x \leq c) = \int_{-\infty}^c g(t) dt$$

- where $g(t)$ is probability density function of x at t , is called the Cumulative Distribution Function or CDF.



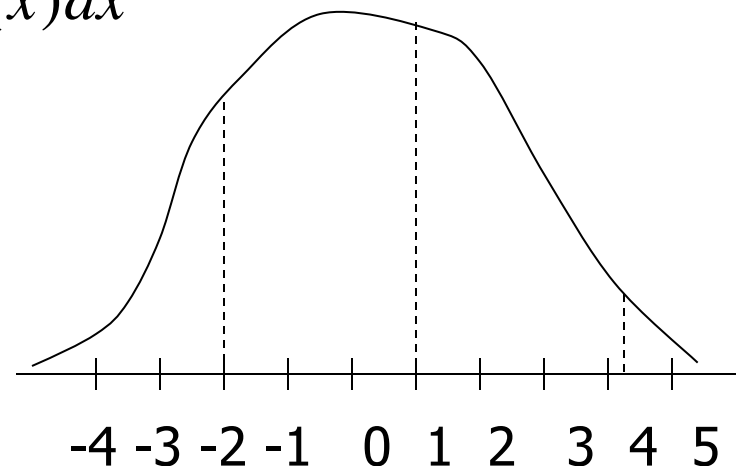
Theorem

- For constants $a \leq b$ (and given some conditions on G and g):

$$\text{Prob}(a \leq x \leq b) = G(b) - G(a) = \int_a^b g(x) dx$$

$$g(x) = \frac{dG(x)}{dx}$$

- $a = -2, b = 1$
- $G(a)$ area under the curve to the left of $x = -2$
- $G(b)$ area under the curve to the left of $x = 1$





Example: Uniform Distribution

- Probability spread evenly across an interval
- If the interval is wide, then the outcomes are more dispersed

pdf for Uniform Distribution

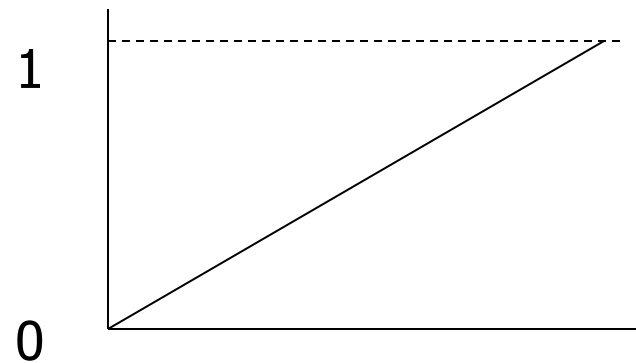
Where b is the upper bound and a is the lower bound of the interval

- Pdf for uniform distribution given by

PDF

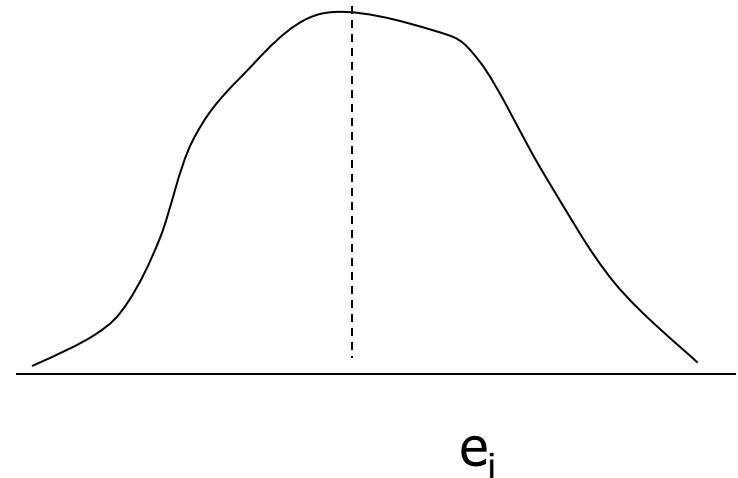


CDF



Set up for tournament model

- Two workers
- Worker with larger output in per 1 becomes "boss" and gets $w_1 > w_2$, where w_1 = wage of boss
- Output of worker 1 = q_1 .
- Output of worker 2 = q_2 .
- $q_i = m_i + e_i$
 - m_i effort
 - e_i noise
 - Noise can be + or -
- $C(m_i)$ effort cost
- $C' > 0$ and $C'' > 0$ (Increasing marginal disutility of effort)
- Utility = $E(\text{wage}) - C(m)$





Set up (continued)

- Price of output = \$1
- Utility for worker j
 - Expected (wage)-C(m_j)
- Max w₁*P+w₂*(1-P)-c(m_j)
 - P=probability win
 - (function of effort).
- F.O.C.:

$$(M^I - M^S) \frac{\partial w^j}{\partial B} = \frac{\partial w^j}{\partial C}$$

- Marginal benefit
- Marginal cost

What is P?

- $P = \text{Prob}(m_j + e_j > m_k + e_k)$
 $= \text{Prob}(e_k - e_j < m_j - m_k)$

- Let $G = \text{CDF}$ of $e_k - e_j$

- Then $P = G(m_j - m_k)$

- $dP/dm_j = g(m_j - m_k)$

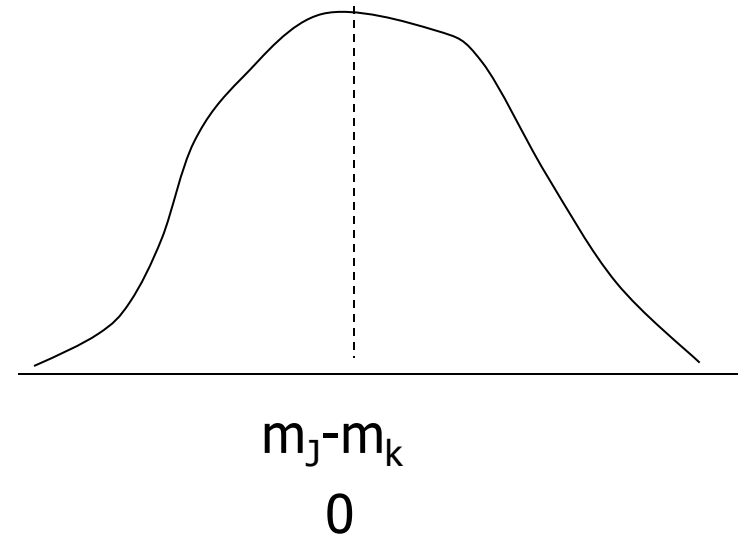
- F.O.C.

$$(w_1 - w_2) \frac{dP}{dm_j} = \frac{dC}{dm_j}$$

$$(w_1 - w_2) g(m_j^* - m_k) = \frac{dC}{dm_j}$$

- But $m_j^* = m_k^*$, because workers identical.

$$(w_1 - w_2) g(0) = \frac{dC}{dm_j}$$

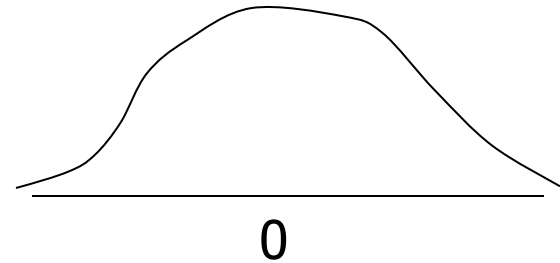


Analysis of Worker's

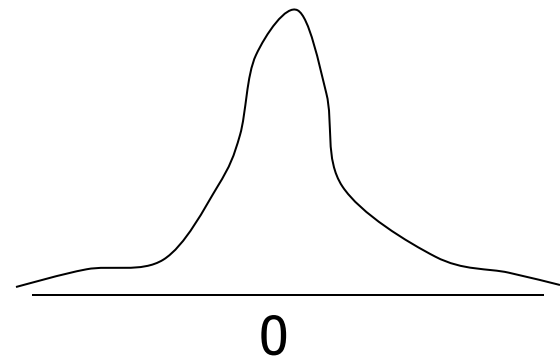
$$\text{FOC: } (w_1 - w_2)g(0) = \frac{dC}{dm_j}$$

A

- Note RHS increases with m
 - (due to increasing disutility of effort assumption, slope of effort cost curve increases as m increases, i.e., $C'' > 0$)
- If $(w_1 - w_2)$ rises, then m rises
 - Bigger pay gap implies higher effort
- if $g(0)$ rises, then m rises
 - less noise means workers have more incentive to work hard – put differently, lower $g(0)$ more noise results in less incentive to work hard



B



Firm's Problem (Assuming Expected Utility U must be at least 0)

- **2. Max $m_j + m_k - (w_1 + w_2)$**

w_1, w_2

s.t. $E(U_i) = 0$ or...

3. $(w_1 + w_2) / 2 = c(m)$

- Because $m^* = m_j^* = m_k^*$ the firm's problem becomes:
- Max $2m - (w_1 + w_2)$
 w_1, w_2
- Substitute **3** into objective function
- Max $2m - 2c(m)$
 w_1, w_2

- F.O.C.s

$$2 \frac{dm}{dw_1} - 2c'(m) \frac{dm}{dw_1} = 0$$

$$2 \frac{dm}{dw_2} - 2c'(m) \frac{dm}{dw_2} = 0$$

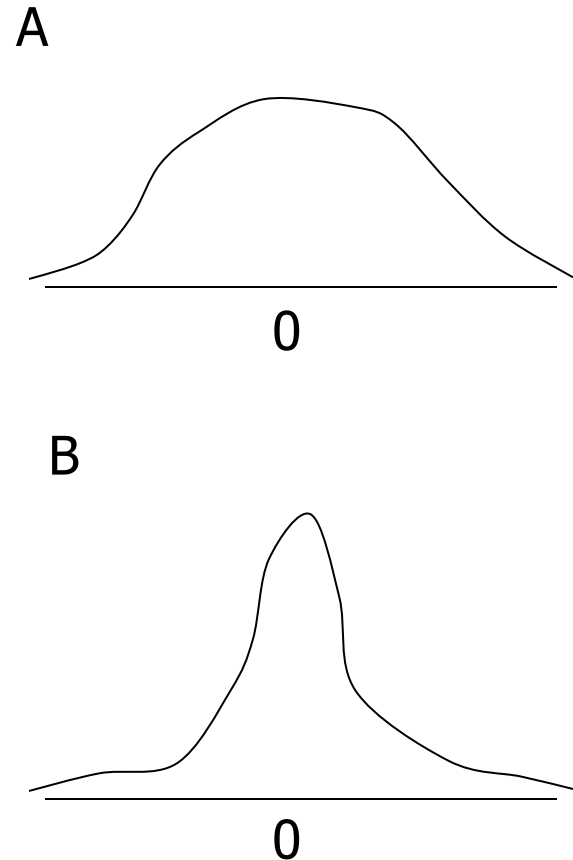
$$2(1 - c'(m)) \frac{dm}{dw_1} = 0$$

$$2(1 - c'(m)) \frac{dm}{dw_2} = 0$$

- Therefore $C'(m) = 1$
- M.C. of effort = Marginal benefit of effort to firm
 - (price of good = 1 in this problem, 1 unit of effort = 1 unit of good)

Combining Worker and Firm Problems

- $c'(m)=1$ (This gives us m)
- Sub into 1 from Worker's FOC:
- $(w_1-w_2)*g(0)=c'(m)$
- $(w_1-w_2)*g(0)=1$
- 4. $w_1-w_2=1/g(0)$
- The more noise in the process, the lower $g(0)$ and the higher must w_1-w_2 be – tells us the relationship between noise and wage spread. Note that average wage doesn't change if $g(0)$ changes, but the **spread** does.
- To solve for w_1, w_2 , combine 4 (worker's FOC) with condition that $U=0$:
- 3. $(w_1+w_2)/2=c(m)$
- Two equations, two unknowns



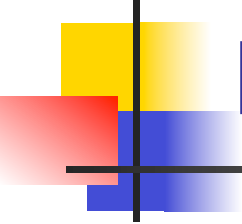
Example

- Two workers, j and k , make baskets.
- $q_i = m_i + e_i$
- q is number of baskets, m is effort, e is luck factor.
- Basket price: \$24 each.
- $x = e_k - e_j$ takes on values between $-1/4$ and $1/4$ with a uniform prob distribution
- Worker cost of effort: $C(m) = 3m^2$
- At end of period 1, worker who produces the most baskets gets w_1 and other gets w_2 .



(Remaining) Main Questions

- 3) How many levels of jobs should there be?
- 4) How does a firm ensure that it does not overpay the Chief Executive Officer (CEO)?
- 5) What are the advantages/disadvantages of promotion tournaments and piece rates?
- 6) What are the relative (dis)advantages of promotion tournaments versus promotions based on absolute standards?
- 7) How to handle variations across workers in ability or personality?
- 8) Should a firm promote from within or hire senior candidates from outside?



Questions 3) and 4) Setting right number of job levels and right pay hike for promotion to CEO

- Can't answer either exactly but some rules of thumb:
- a) If decide to pay based on relative performance, then need to ensure that pay raises between levels are consistent with risk of not being promoted



Getting the size of pay hikes “right”

- Typically easier to get promotion in lower than in higher tiers
 - Example: $\frac{1}{2}$ of factory floor workers might get promotion to better factory floor job but only $\frac{1}{4}$ of those eventually become floor managers
 - ... and near the top maybe 15 vice presidents vying to become CEO

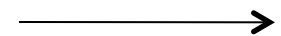
Pay Hikes Rise with Seniority

Pay (\$)



Job Tier

Level of
Responsibility





b) A Second Reason for Large Pay Hikes in Higher Tiers

- Part of incentive to work hard in lower tiers is prospect of promotion to higher tiers later in career.
 - If win the promotion tournament to next level this year, it grants worker the *option* to compete for jobs in higher tiers in later years



c) If heterogeneous work force then create many job tiers

- Also, don't load most pay hikes asymmetrically into top tiers
- Explanation for both rules:
 - Want worker productivity to be similar within each job tier. If not, then some workers give up because they don't think they can compete to get to the next level



5) What are the advantages/ disadvantages of promotion tournaments and piece rates?

- Advantages of tournaments:
 - 1) If hard to measure workers' absolute level of productivity but easy to rank workers' output, then use tournaments
 - Examples: managers and “information” workers
 - 2) If some of the variability in workers' output, or in supervisor' s *perceptions* of worker output, is correlated across workers, then tournaments may be better than piece rates



Why Correlated Productivity Shocks Encourage Tournaments: Insurance for Risk Averse Workers

- Tournaments remove this variability, insuring workers against wages changing for reasons beyond their control
- Example: Suppose all salespeople working in a given region suffer random variations in sales due to state of economy
 - A piece rate would lower all workers' earnings during recession
 - A tournament would leave *average* wage unchanged during a recession



Other Examples of How Tournaments Can Reduce Risk to Workers

- If managers vary in their probability of recommending workers of the same quality for promotion firm can give all managers the same \$ to distribute
 - Insurance against having an overly tough manager



Examples of how tournaments can reduce risk to workers (continued)

- John Abowd paper: Evidence that CEO pay in part based on firm's performance relative to other firms in the industry.
 - Partly removes from CEO's wages variability related to shocks to industry profitability

Two Potential Problems with Tournaments



- 1) Collusion among workers
 - Can decrease incentive effects
 - Less likely to happen if
 - Large number of competitors and/or
 - Not all competitors know each other
 - Hire outside contractors!!

- 2)
 - Excessive competition: cheating and sabotage!



6) What are the relative (dis)advantages of promotion tournaments versus promotions based on absolute standards?

- 1) If hierarchy rigid, then promoting 1 candidate in a tournament better than promoting all who meet an absolute standard
 - But tournaments may produce winners who are quite variable in quality from one year to next
- 2) Firm may prefer to use Relative Performance Evaluation to absolute standards if hard to measure absolute output but easier to measure relative output

Advantages of Tournaments (continued)



- 3) Tournaments may reduce risk caused by shocks that affect both participants.
 - But the difference between performance of two workers could still have higher variance than the worker's own absolute production. Read p. 300.
- 4) Tournament can reduce cooperation between workers and even promote sabotage.
 - Less likely with absolute standards. See p. 301.

7) How to handle variations across workers in ability or personality?

- Tournaments not effective if competitors vary a lot in ability.

- One solution is to have many levels in the hierarchy to create more homogeneity within a given job title
- Managers may give lower appraisals to better workers and unduly optimistic appraisals to worse workers to maintain tournament incentives



Role of Personality Variations

- Aggressive sabotegers versus peaceful cooperators
- Not a good mix as aggressive workers are likely to do even more sabotage against a cooperative worker in a tournament as less chance of retaliation
- This is another reason to sort workers into separate tournaments by not only ability, but aggressiveness!



8) Should firm promote from within or hire from outside?

- Two reasons to hire from outside:

- 1)

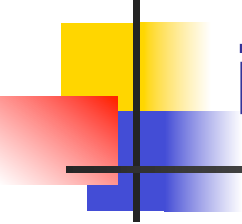
- 2) IF lots of internal collusion, hiring outside candidate, or *threatening* to do so, can decrease collusion internally

- (Collusion becomes a much bigger issue if the tournament repeated regularly, so that participants take turns winning)



8) Should firm promote from within or hire from outside? (Continued)

- Two reasons to promote from within:
 - 1) Firm-specific human capital
 - 2) Opening up competition to outsiders decreases insiders' chances of being promoted, reducing effort



Example of second point: (hiring from outside reduces incentives for insiders)

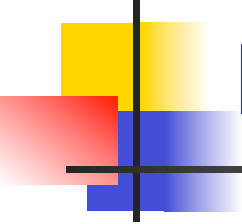
- Suppose you are competing against one internal competitor for a promotion. You each have identical productivity except for a random shock to your productivity. You both have a 50:50 chance of having a high vs. low shock.
 - So, there are four equally likely outcomes (HH, LL, HL and LH) and only with HL will you win: a $\frac{1}{4}$ chance of winning



Exercise

- Prove that if you are also competing with one external competitor, who like the two of you internal competitors has same average productivity and a 50:50 chance of a high/low random productivity shock, that your chances of winning drop to $1/8$.

Implications of External Competition for the Wage Hike Associated with Promotion



- Mathematically, external competition decreases incentive to work hard because $g(0)$ falls as tournament becomes riskier
- Solution: Increase $W_1 - W_2$
- But only a partial solution if workers highly risk averse:
 - Internal workers would prefer less competition and smaller wage hike

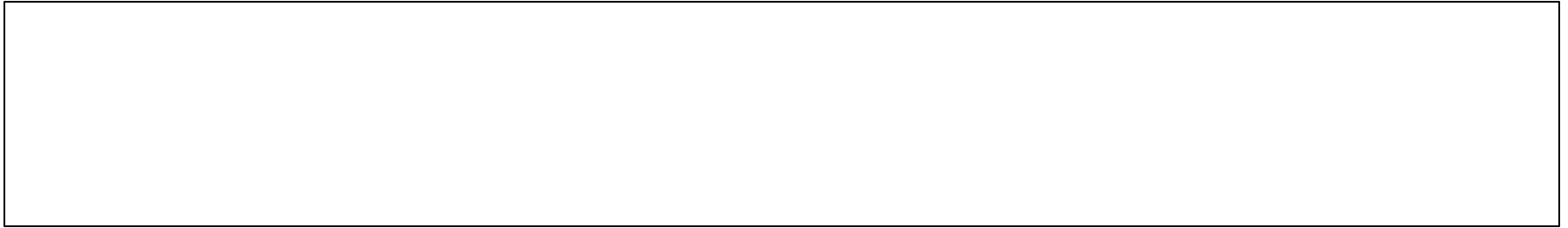
Seniority Pay and Incentives



- 1) When people unlikely to get promoted, what are alternatives to provide motivation?
- 2) What about pay raises as experience rises? Risks to firm and worker
- 3) Can overpaying older workers cause them to stay too long past normal retirement age?

1) When people unlikely to get promoted, what are alternatives to provide motivation?

- Can be huge incentive to goof off or “shirk”



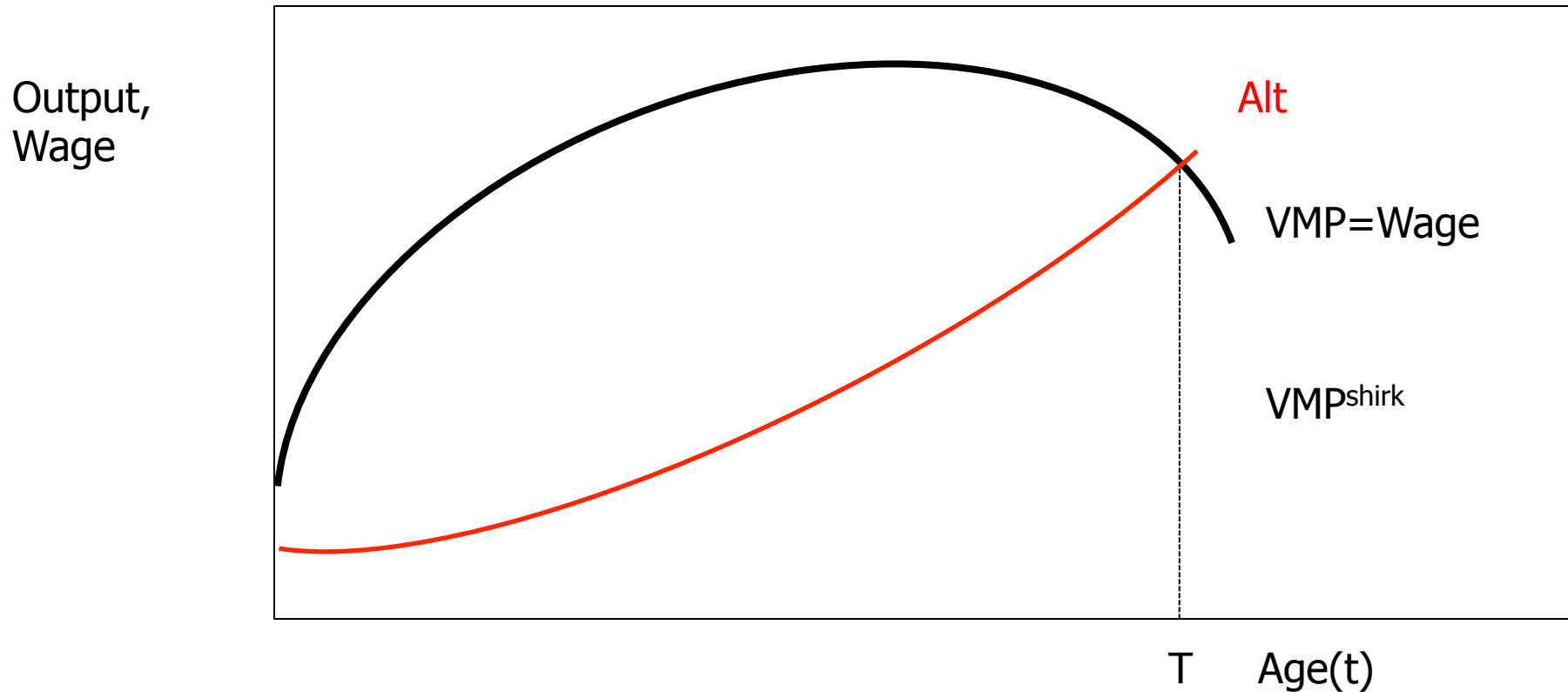
- If no chance of promotion, and hard to monitor effort all the time, worker may decide to increase “leisure” on the job at a small risk of being caught and fired
- Cost of being fired = $W(t) - Alt(t)$ where $Alt(t)$ is utility at worker’s next-best opportunity



Shirking could become worse near retirement age

- Toward retirement $Alt(t)$ rises as value of leisure goes up.
- Also $VMP(t)$ tends to fall due to depreciating human capital
 - (Workers toward retirement find it optimal to reduce training – including “refresher courses” – because payback period is short)

Incentive to shirk rises as approach retirement age T



Comparing Gains and Losses from Shirking

- Gain = extra leisure
- Loss = $\text{Prob}(\text{fire}) * (\text{VMP} - \text{Alt}) \rightarrow 0$
as $t \rightarrow T$
- Solution?

2) What about pay raises as experience rises? Risks to firm and worker

- Underpay worker when young and overpay when old

■ Note:

- So firm does not lose money, and worker still gets paid present value of VMP:

$$\frac{\sum W_t}{(1+r)^t} = \frac{\sum VMP_t}{(1+r)^t}$$

Effects of Allowing W to Vary from VMP



- Older worker now has LESS incentive to shirk because $(W - Alt) \gg 0$ even near retirement.

Why Would Worker Accept Low Wage Initially?



- Overall, paid the same as if $W=VMP$ each period
- Worker realizes that a firm offering $W_t=VMP_t$ for all ages t will have to lower wages for, or fire, many older workers because they will be tempted to shirk near retirement
- In practice PENSIONS play role of motivator as their value goes up with years of tenure



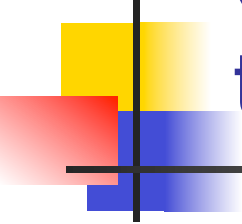
Risks to Worker

- 1) Workers is making “loan” to firm that is supposed to be paid back towards retirement. Gives firm incentive to lay off older workers and “pocket the change”
 - Firms’ concern about reputation, as well as Age Discrimination in Employment Act, limit this problem
- Risk of firm bankruptcy



Risks to Firm

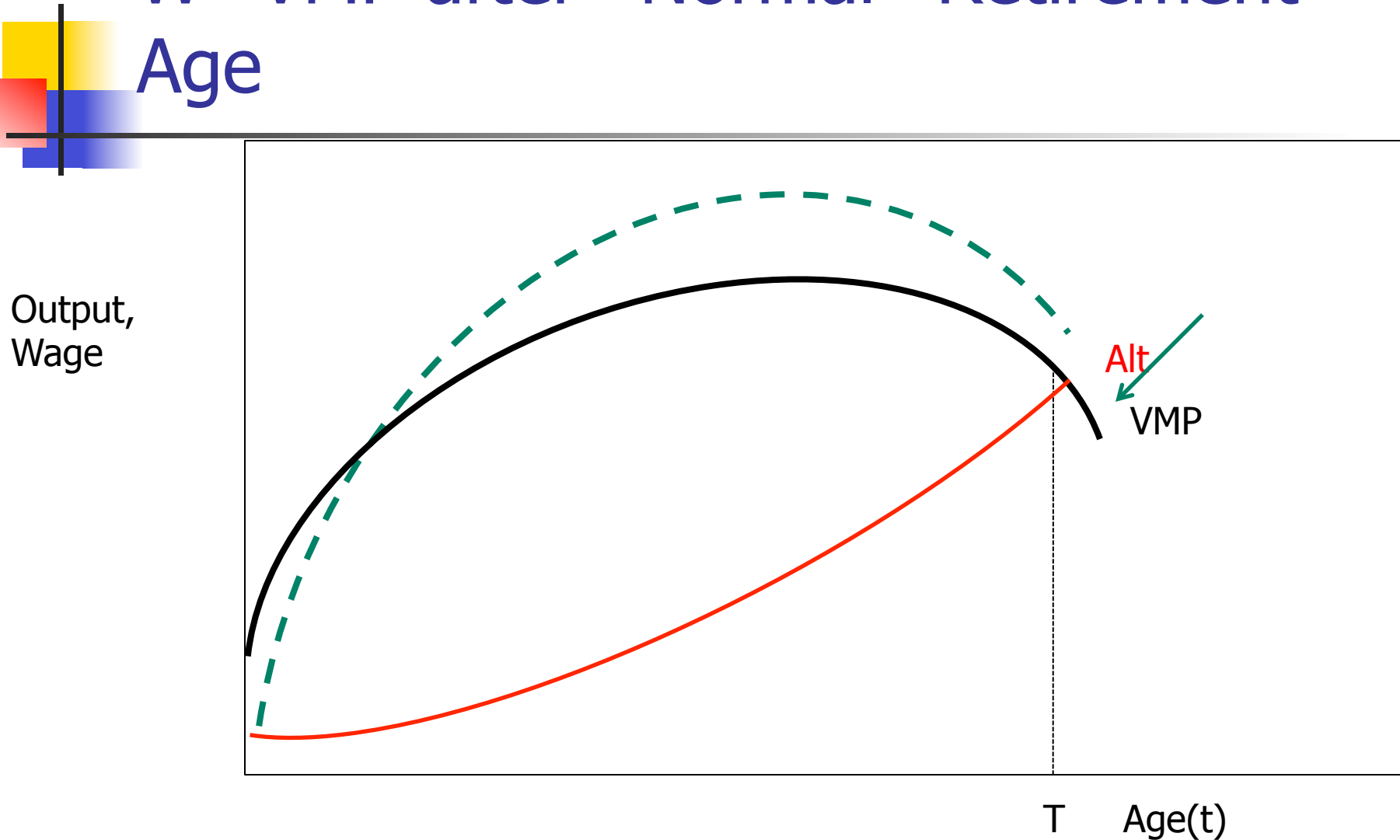
- At normal time of retirement, period T , if $W(T) > Alt(T)$ then worker won't want to retire.
- Lazear (1979) suggested this was why in the U.S. retirement at age 65 used to be mandatory.



Question 3): How to Reduce Risk that Overpaying Older Workers Causes Them to Stay Past Normal Retirement Age?

- But mandatory retirement now virtually outlawed. Solutions?
- i) Reduce W to VMP after retirement age. But this decreases effort and could cause lawsuits
- ii) A better solution:

So Modify Our Earlier Graphs: Set $W=VMP$ after “Normal” Retirement Age





Two Conditions Which Make Positively Sloped Wage Profile Especially Useful

- 1) Output hard to measure
 - By creating a huge potential loss if get caught shirking, firm decreases shirking.
 - The larger the punishment, the less often supervisor has to monitor.
- 2) Situations where cooperation is important, so that tournaments don't work well

What About Real World, Where We See Raises and Promotions?

- Goal is to do both to create incentives throughout career:

Wage



Tenure

In Practice, How Does Manager Decide How Steep the Wage:Tenure Profile Should Be?



- Human resources departments hire compensation consultants who conduct salary surveys. Can use to:
 - a) Calibrate firm's pay scale to market wages
 - Most common use
 - b) Estimate how worker effort responds to initial salary and salary hikes



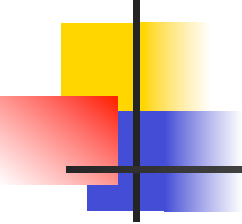
The Basic Idea

- 1) Use regression analysis to estimate hours worked (or productivity) as function of initial salary, and wage gains over career.
- 2) Estimate $d(\text{hours worked})/d(\text{wage gain})$ and the related cost
- 3) Compare cost to:
 - $d(\text{Net Revenues})/d(\text{hours worked})$

Review Question Spring 2008

Test 3 #3

- 3. (20 points) Senior management asks you to devise a promotion tournament for two workers, where the worker who produces more earns W_1 and the worker who produces less earns a smaller amount W_2 . The two workers have identical productivity (on average) and the same utility function:
- $U = E(\text{wage}_i) - m_i^3$ for workers $i=k,j$ where m_i is effort.
- Each worker's output is given by
- $q_i = m_i + e_i$ where m is effort and e is a random luck factor. Each unit of output brings in \$6 of revenues to your company. However, for each unit of output there are additional costs of \$1 for electricity and \$2 for raw materials. These are costs in addition to the cost of labor.
- $x = e_k - e_j$ takes on values between $-1/4$ and $+1/4$ with a uniform probability distribution.
- To ensure that both workers accept the job, you must pay an expected wage for a given level of effort to ensure that expected utility equals 0.
- a) Solve the workers' problems and the firm's problem. What is the firm's total profit from hiring the two workers?

- 
-
- b) (4 points) You solved part a) under the assumption that the worker's expected utility had to be 0 for him or her to accept a job. Suppose that because of the strengthening of the economy, wages rise to the point where workers will accept a job only if expected utility obeyed $E(U) = 2$ or more. Without working through the entire solution to part a) again, explain which parts of your answer to part a) would change, and from this work out the new optimal W_1 , W_2 and profits from hiring the two workers.