Economics 136
Chapter 10 Rewarding Performance

Appendix: You are responsible for all of appendix except for the final section on the ratchet effect

Key Questions

- 1) Relative advantages/disadvantages of variable pay (piece rates) vs. straight salary
- 2) For piece rates, what is the best commission rate to pay?
- 3) Should commission be based on total revenues or revenues net of costs (profits)?
- 4) 7 factors affecting how strong incentives should be
- 5) How to reduce problems with variable pay?

Piece Rates: Advantages

- 1) Screen out less productive workers
  - (Similar to probation idea in chapter 2)
- 2) Provides incentive to work harder
  - In chapters 1 and 2 we assumed only that workers vary in ability. Now their effort can vary too.
  - In general, to get workers to work harder, must increase pay more than proportionately
- (We'll discuss disadvantages later…)

An Example of How Piece Rates Can Screen Out Less Productive Workers

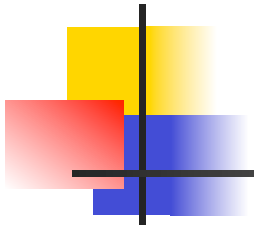
- From 2000 Test 1
- Shows how workers self-select between firms with fixed salary and those that pay a piece rate (essentially, paying workers their actual VMP minus other costs of production)

- 3. (25 points) Suppose that the labor force in your region consists of one-third each of workers with productivity of $3, $6, and $9 per hour. There are two types of firms, those that offer a fixed salary of $4.50 per hour and those that pay a piece rate. Because it is costly to monitor output, these latter firms deduct $2 per hour for the costs of monitoring. Thus, workers of the three types can earn the following per hour:

Worker type (productivity per hour)	Piece rate (output-$2)/hour	Fixed salary per hour
$3	$1	$4.50
$6	$4	$4.50
$9	$7	$4.50

- a) Which workers will work at the fixed salary firms? Which will work at the piece rate firms? (10 points)

- b) Ignoring monitoring costs, at which firms are workers more productive on average? Explain. (6 points)
- c) Calculate the profit per hour per worker for the two types of firms. (6 points)



- d) Is there an incentive for fixed-salary firms to switch to piece rates, or for piece-rate firms to switch to fixed salaries? (3 points)

- The point: Paying piece rate attracts more productive workers!

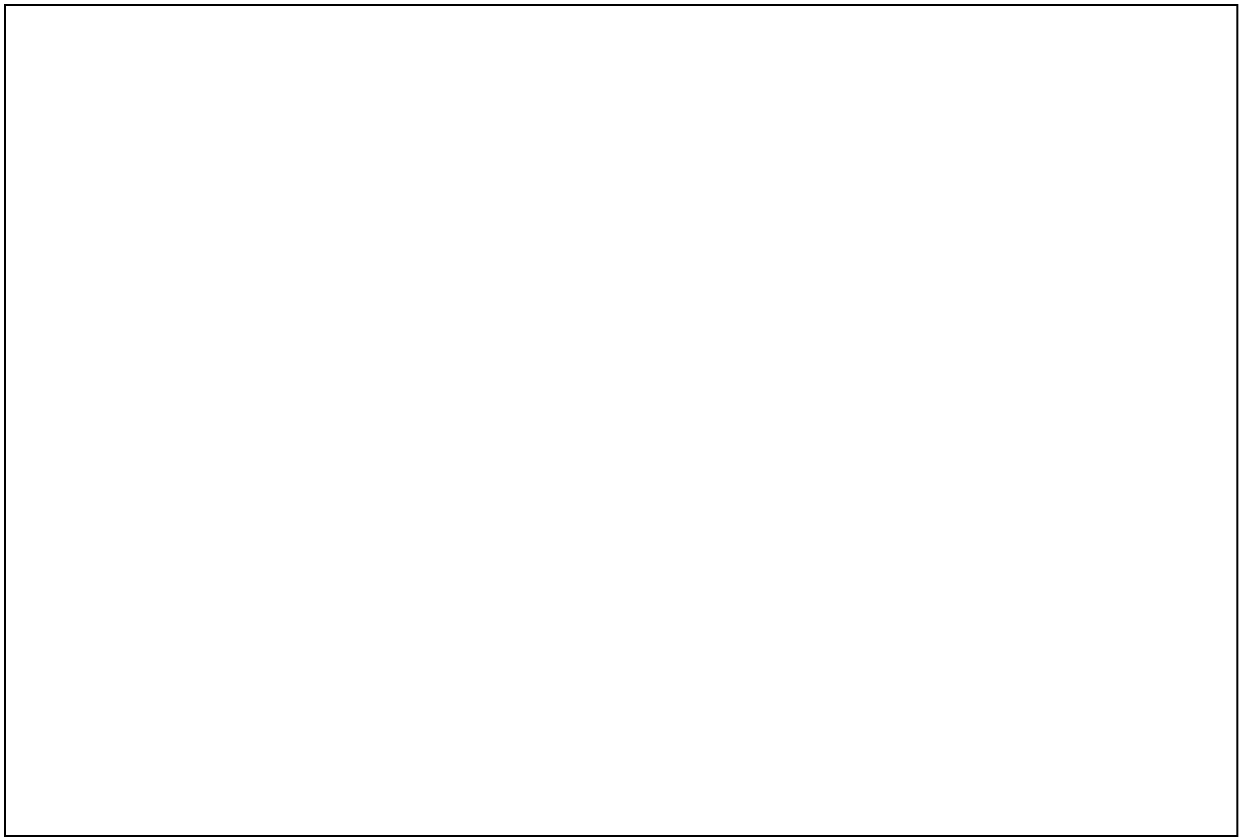
Piece Rates: Advantages

- 1) Screen out less productive workers
  - (Similar to probation idea in chapter 2)

- 2) Provides incentive to work harder

Workers are very willing to increase effort 1 unit from low levels; less willing at high levels

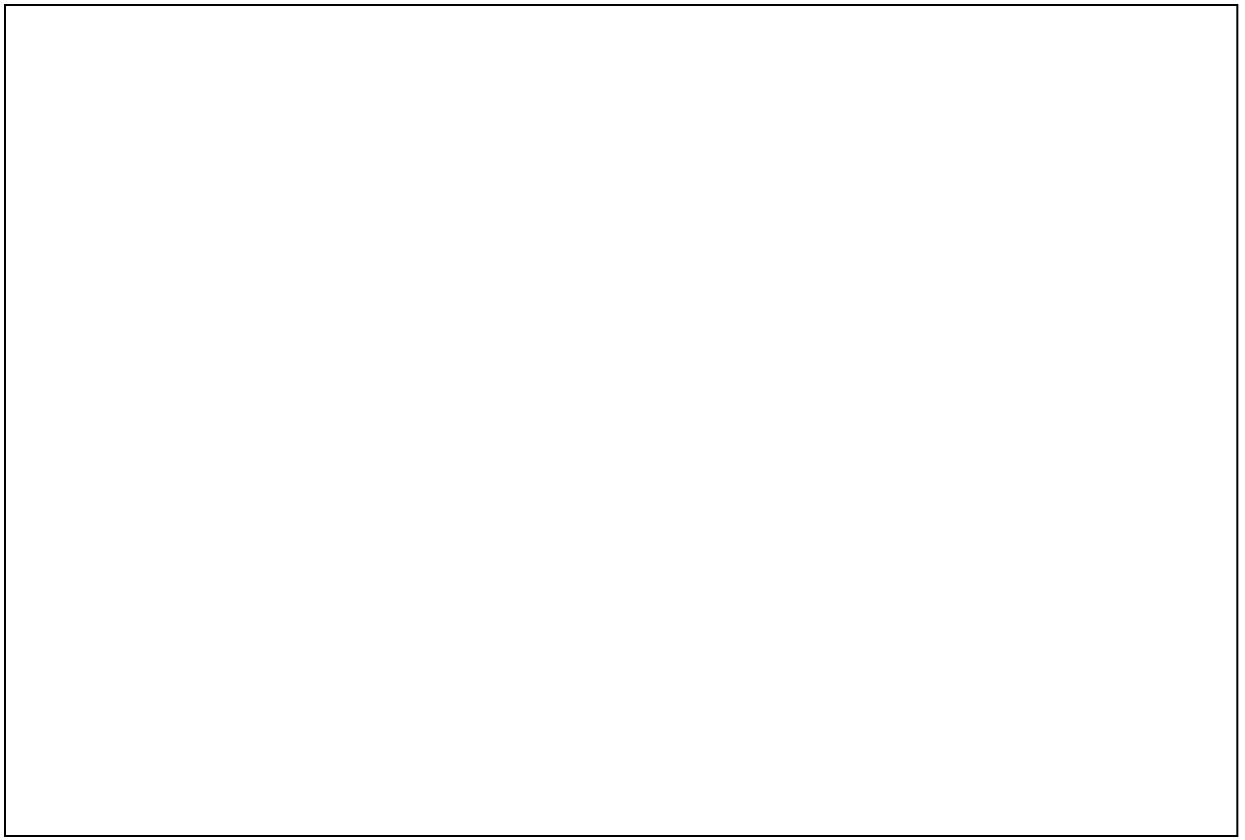
$/hour



Hours worked

Paying workers a greater share of (revenue – non-labor costs) will increase effort

$/hour

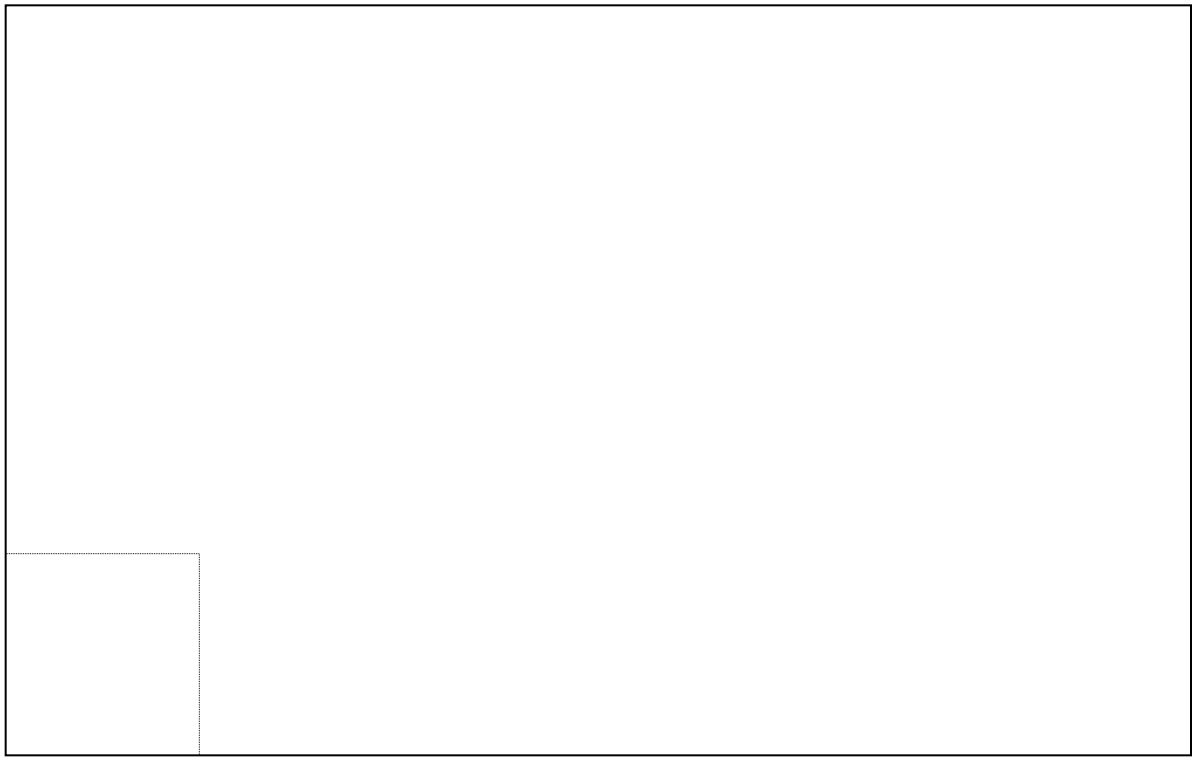


Hours worked

Real World Case Study: Safelite Glass

- Old system was fixed salary subject to firing if fixed too few windshields per day

New Pay Structure: Combine Flat Hourly Wage with Bonuses (as long as meet minimum level)



Effects on Worker Earnings and Productivity

- Average worker earnings increased 9%
- Average worker productivity increased 36%
  - So profits rose too!
- Why the increase in productivity?
  - 1) 2/3 due to increased effort
  - 2) 1/3 due to more productive workers staying and perhaps, less productive workers quitting

2) For piece rates, what is the best commission rate to pay?

- Should you pay < 100% commission on net revenues, 100% or more?
- Answer:

Mathematical derivation

- Appendix to chapter covers a fairly general case. Here is one version (we start with a simpler case of the risk neutral worker).
- Assumptions:
  - Utility: U = earnings – C(e) where e=effort, C' >0, C' ' >0
  - Firm: Sets two-part piece rate, paying wage of $a + be$
  - Revenues from worker's effort are Ve where V>0
  - Therefore worker' s problem is simply:
  - Max $a + be - C(e)$
    e

Math derivation: Worker's Problem

- Worker's FOC
- $b - C'(e^*) = 0$ (1)
- This is e^* as an implicit function of b. Thus $de^*/db = -1/\{-C''(e^*)\} = 1/C''(e^*)$ (2)
- The firm, knowing this is how workers will react, then solves for optimal a and b, subject to the constraint that worker's utility is equal to that of best alternative U_{alt}

Math derivation: Continued

So need $a + be - C(e) = U_{alt}$ (3)

Firm has two constraints, (3) and "Effort Supply Function":

$b - C'(e^*) = 0$ (1)

Firm's goal

Where substituted for wage from (3).

Note that from (1) e is a function of b only!!

Firm's F.O.C.

$$\max_{b} Ve - (U_{alt} + C(e))$$

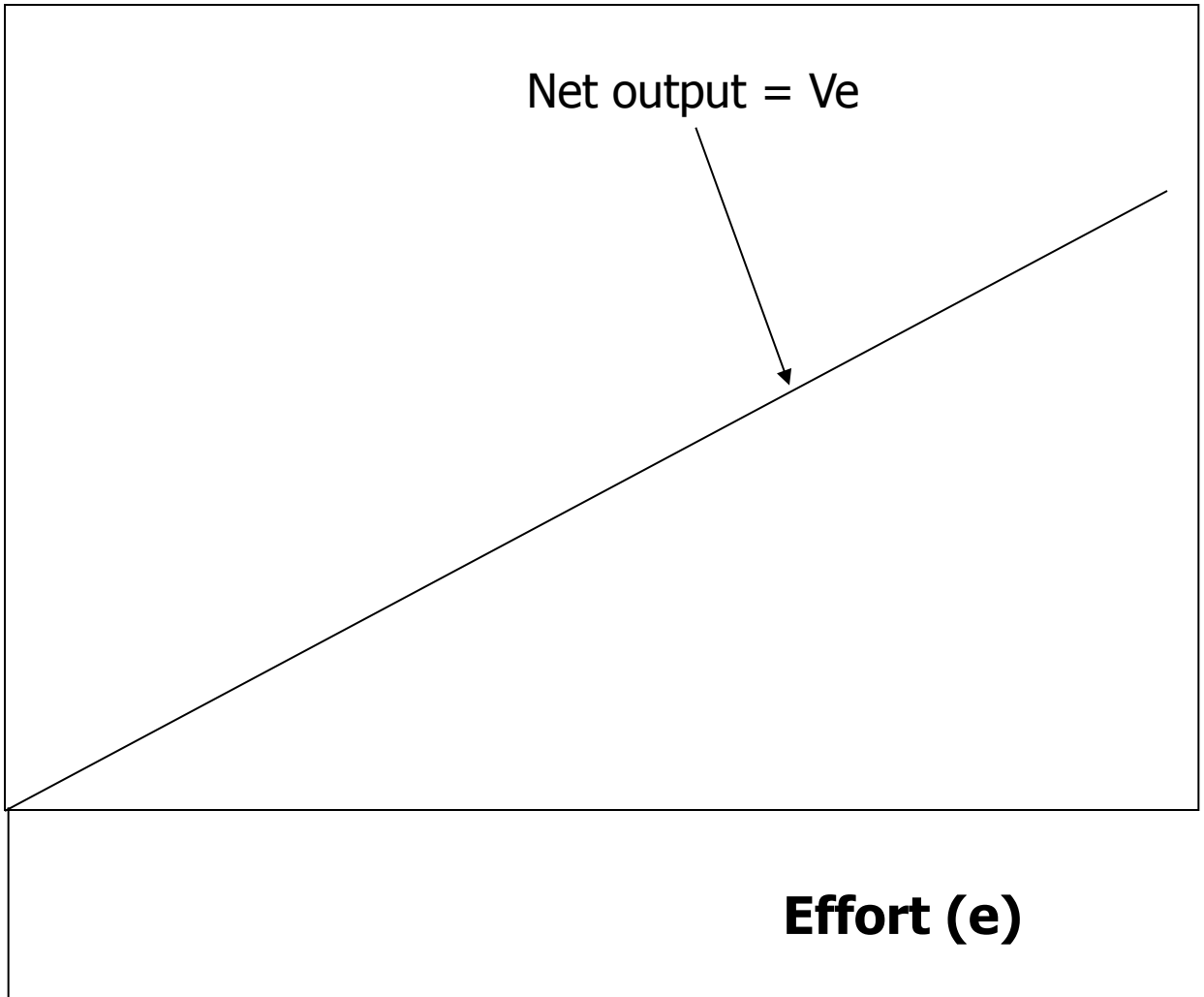
$V = C'(e) \therefore$ from (1): b=C'(e*)

$V = b^*$ (100% commission!)

Substitute into (3)

$a^* = U_{alt} + C(e^*) - Ve^*$

Graph of solution for case where output = Ve (and e=Effort)



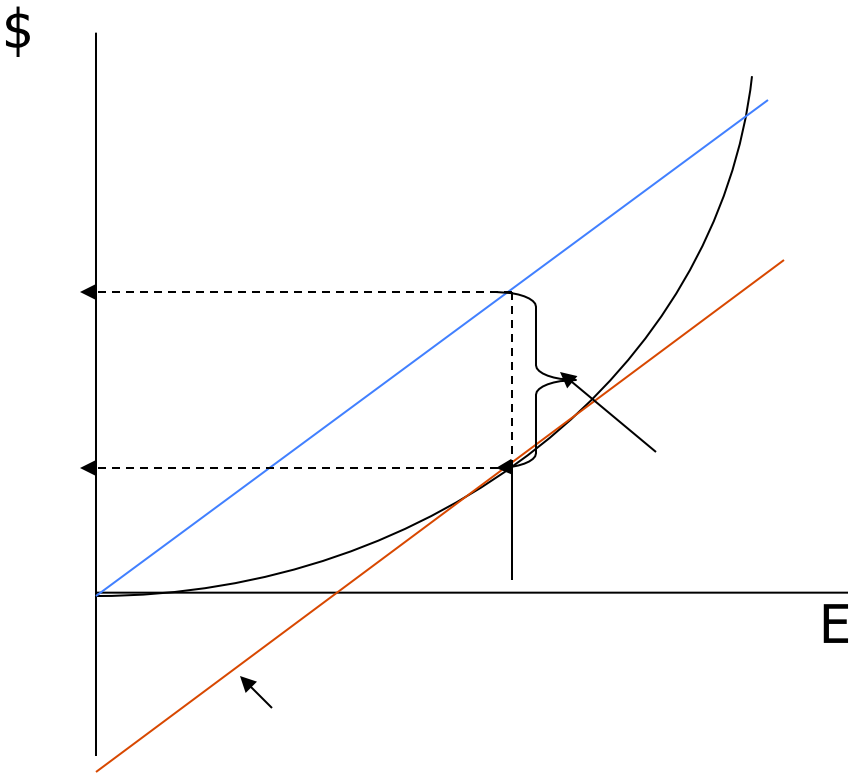
Are There Any Real-World Examples of Paying 100% Commission on Net Revenues, After Extracting a Payment a

- a) Taxi drivers.

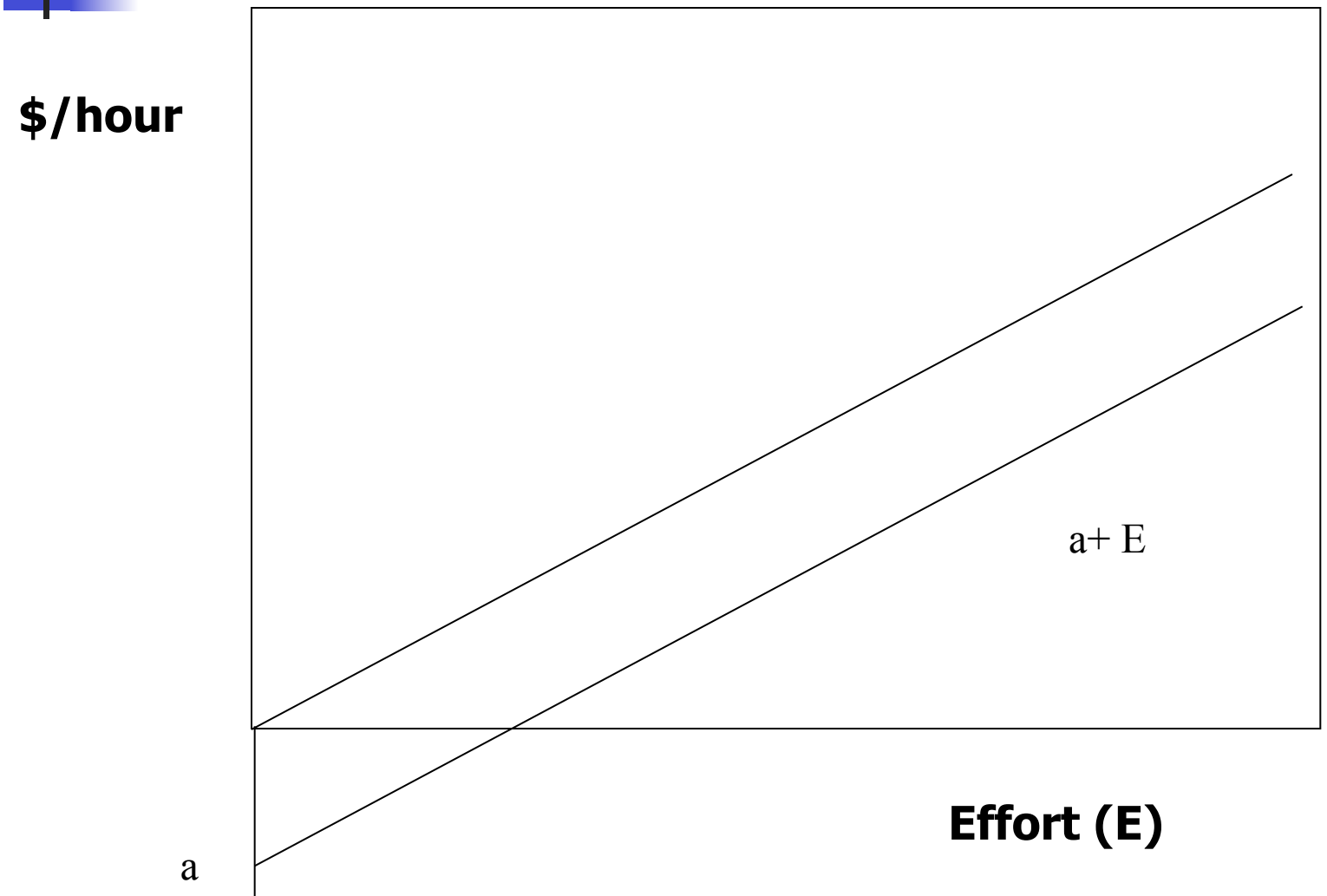
- b) Other examples (not quite as close a fit)
  - Farm workers
  - Garment workers
  - Safelite Glass workers
  - Lots of salespeople paid modified version of a + E (with firing rather than negative wage if low productivity)
  - Securities traders have to buy a seat on the exchange
  - Waiters in restaurants earn most of their money from tips
  - See pp 282-293: Similar to franchisees paying for a franchise

Numerical Example

- Firm wants to pay $w = \alpha+\beta E$
- Effort cost: $C(E)=E^2$
- Each unit effort gives **2 units** output sold for **$3/unit,** and there are no non-labor costs, so NR=6E.
- **Find E*, α, β**
- Worker maximizes $w-C(E)= \alpha+\beta E-E^2$
  - FOC: $\beta-2E=0 \rightarrow \beta=2E^*$
- Firm chooses β to max NetRev – Cost =

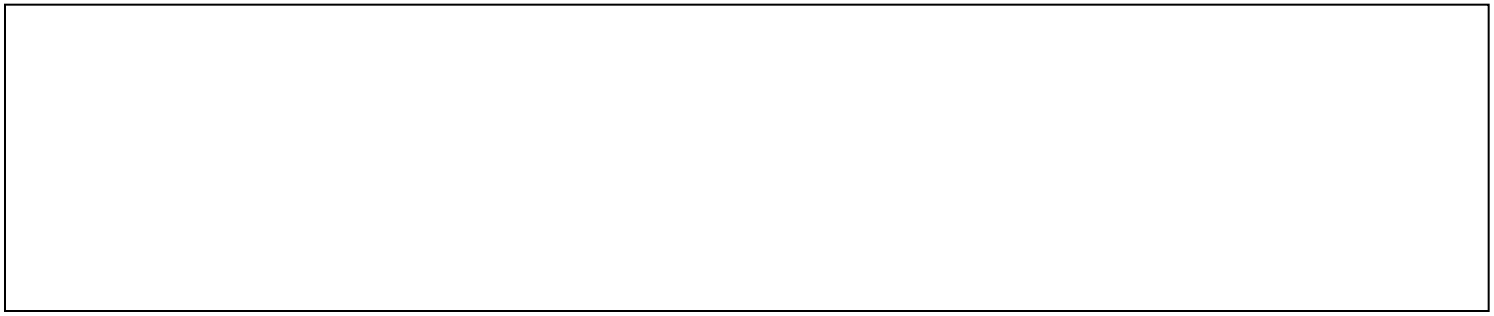


Typical Salesperson Payment: Salary Floor Plus Incentives



3) Should commission be based on total revenues or revenues net of costs (profits)?

- In theory, should be based on net revenues (profits). In practice often based on sales. WHY?



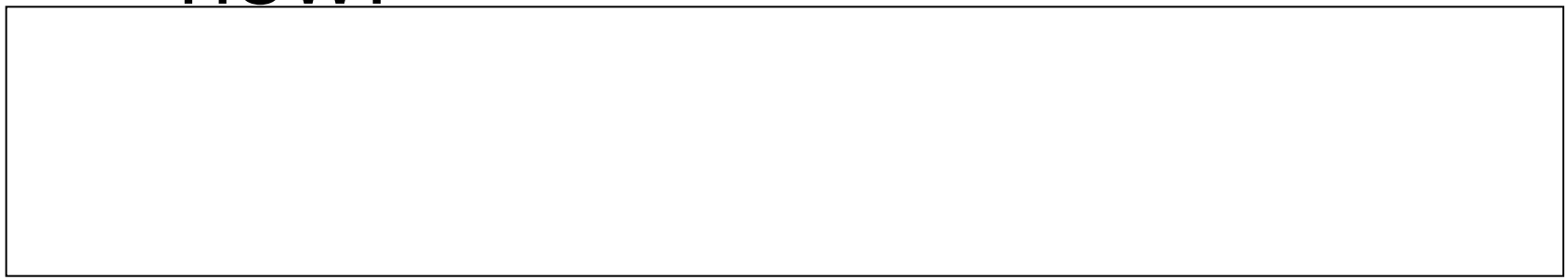
- In practice, a 100% commission on net revenues translates to a much smaller % commission on sales

4) 7 factors affecting how strong incentives should be

- Make incentives stronger if:
  - 1) Effort matters a lot
  - 2) Firm profits would fall a lot if did not attract most productive workers
  - 3) Can measure output precisely
  - 4) Workers are not very risk averse
  - 5) Workers trust employer to evaluate subjectively
  - 6) Does not distort incentive for workers to work on unrewarded tasks
  - 7) Worker cannot "cheat" or manipulate outcome

What to do if workers at your firm are always paid per hour or day worked?

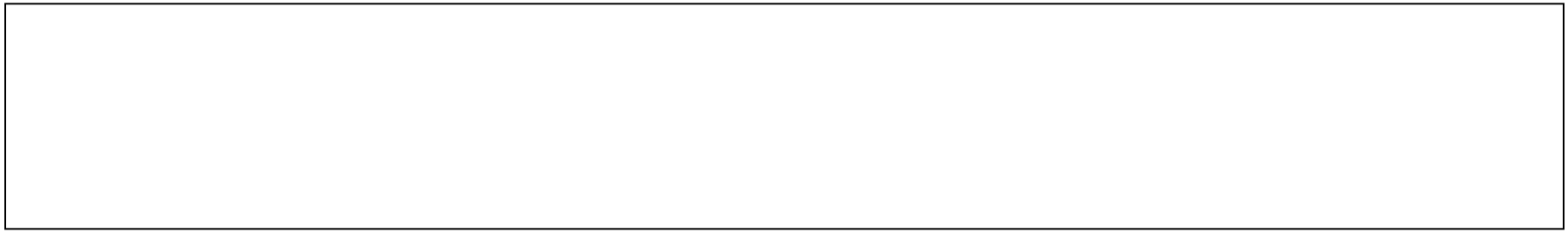
- Firm could implicitly pay piece rates. HOW?



- Now, back to question 1 on relative advantages/disadvantages of fixed salary vs piece rates….

Advantages of Fixed Salaries over Piece Rates

- A) Piece rates encourage workers to run down the capital stock
  - Examples:



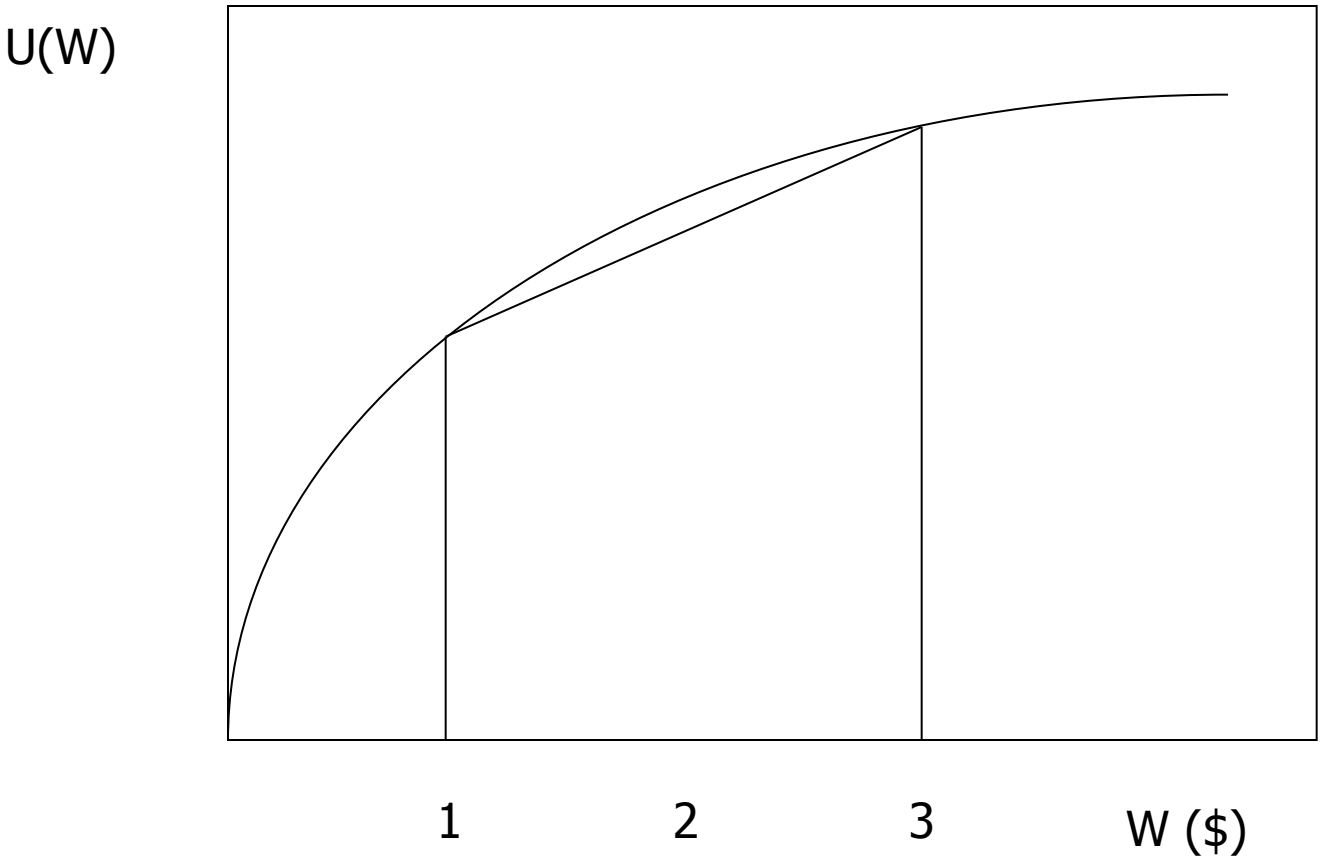
- B) With fixed salary don't need to measure individual output. Saves money
- C) Piece rates encourage quantity over quality
- D) Unpredictable events beyond worker's control makes piece rate risky.
  - Risk averse worker will prefer fixed salary to piece rate **if** both jobs have same expected pay

What is Risk Aversion?

- Simple example in which worker's utility depends only on wages W (because s/he spends wages on consumption)
- $U = U(W)$, $U' > 0$
- Risk averse if $U'' < 0$ e.g. $d^2U/dW^2 < 0$
- Suppose worker will be paid \$1 or \$3 each with probability ½
- E(payment) = \$2
- Expected utility E(U) =

- In words: the worker would strictly prefer \$2 for sure to a risky situation with E(payment) of \$2

Here is why:



Fixed Wage Viewed as Insurance

- A worker getting paid $2 for sure is better off in utility terms
- The firm bears the risk; the worker is insured
- Generally speaking the less a worker earns the better it is to pay a fixed wage
- Reason:

Incorporating Risk Aversion (a version of Appendix p. 289)

- Now assume utility is
- $U=pay - C(e) - 1/2*R\sigma^2_{PAY}$
- σ^2_{PAY} is the variance of pay. R>0 if risk averse, R=0 if risk neutral
- Let's assume variance occurs because employer does not observe actual effort e but only $(e+\varepsilon)$ where ε is noise
- ε has mean 0 and variance σ^2_{ε}

What is variance of pay σ^2_{PAY}?

- If firm pays piece rate of a+b(e+ ε), then variance of pay, given a, b and effort e, is as follows (I write the expectation operator as " "E" – not the same as effort e!)
- σ^2_{PAY} =E[a+b(e+ ε) – E{a+b(e+ ε)}]^2

Worker's Optimization

- Chooses optimal effort e* to maximize
- $a+b(e+\varepsilon) - C(e) - 1/2*Rb^2\sigma_\varepsilon^2$
- F.O.C. is
- $b - C'(e^*) = 0$
- This is e* as an implicit function of b. Thus

- Note: Same solution as risk neutral case!

Firm Must Give Worker the Same Utility Could Get Elsewhere

- Suppose worker could obtain U=0 in next best job
- If firm's revenues from worker's efforts e are given by Ve, where V>0, and it must offer U=0 to worker, its problem becomes:

$$\max_{a,b} Ve - (wage|U = 0)$$

Assuming worker's best alternative is U=0

- U=0 means firm sets a+b(e+ ε)

  $=C(e) +1/2*R\sigma^2_{PAY}$

- So firm's objective function becomes:

$$\max_b Ve - C(e) - \frac{1}{2} Rb^2 \sigma_\varepsilon^2$$

- F.O.C.:

So optimal value of b is given by….

$$\left(V - C'(e^*)\right)\frac{de^*}{db} - Rb\sigma_\varepsilon^2 = 0$$

Substituting C′(e*)=b from worker's FOC, and using value of de*/db=1/C″(e)….

$$\left(V - b^*\right)\frac{1}{C''(e^*)} - Rb^*\sigma_\varepsilon^2 = 0$$

- Or $$b^* = V \Big/ \left(1 + R\sigma_\varepsilon^2 C''(e^*)\right)$$

So if worker risk averse and is measurement error…

- If R>0 then b* < V
- That is, commission rate < 100%
- Reason: as b rises U falls due to aversion to the higher risk created:

- $U = pay - C(e) - 1/2 * R b^2 \sigma^2_{\varepsilon}$

4) How to reduce problems with variable pay?

- Can reduce neglect of capital and low quality by making pay partly dependent on these things
  - Cost:
- Alternative solution to neglect of capital:
- Reduce temptation of workers to cheat or manipulate by capping reward

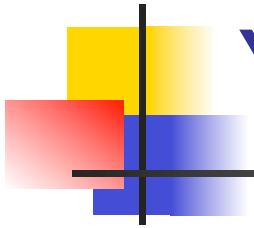
Aligning Workers' Interests with those of Owners

- Piece rates might increase worker's focus on short run if he discounts future $ more than do shareholders
- One partial solution: make workers' earnings dependent on market value of the company's stock. How?

  - But this increases worker's risk.
    - This is why senior managers often receive combination of salary plus stock options

#2 from Sample Exercises for Chapters 2, 3 and 10

- A firm is trying to establish a wage =a+bE where E is worker effort and a and b are to be chosen by the firm. The worker maximizes utility which is given by
- wage – $5E^2$
- where the latter term reflects the cost of effort to the worker.
- Each unit of effort E produces 4 units of output, which can be sold for $3 per unit. But additional worker effort of 1 unit also leads to additional material costs for your firm of $2. Assume that the worker's utility must be at least zero for him or her to accept the job. Calculate the profit maximizing values of a and b, and the optimal effort E that results.



Your Answer

Miscellaneous Real World Concerns and Applications

- See pp. 276-278 for lumpsum awards or promotions and benefits/weaknesses
  - A lumpsum award or promotion is in some ways a capped piece rate
  - Capping the reward can cause best employees to leave
    - Ross Perot left IBM for this reason

Profit Sharing and Employee Stock Ownership Plans

- Pay depends on performance of company
- Not nearly as effective as piece rate as 1 employee typically has little effect on profits
- Similarly, the reward for your efforts is diluted across all employees/stockholders. But maybe firms do this because it will:

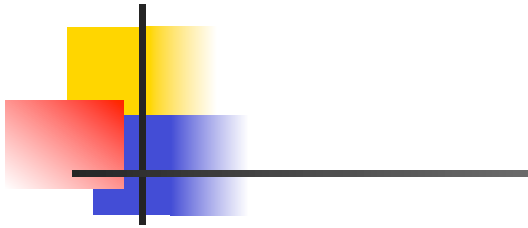
What happens to profits if there are diminishing returns to effort?!

- 2007 test 2 #1

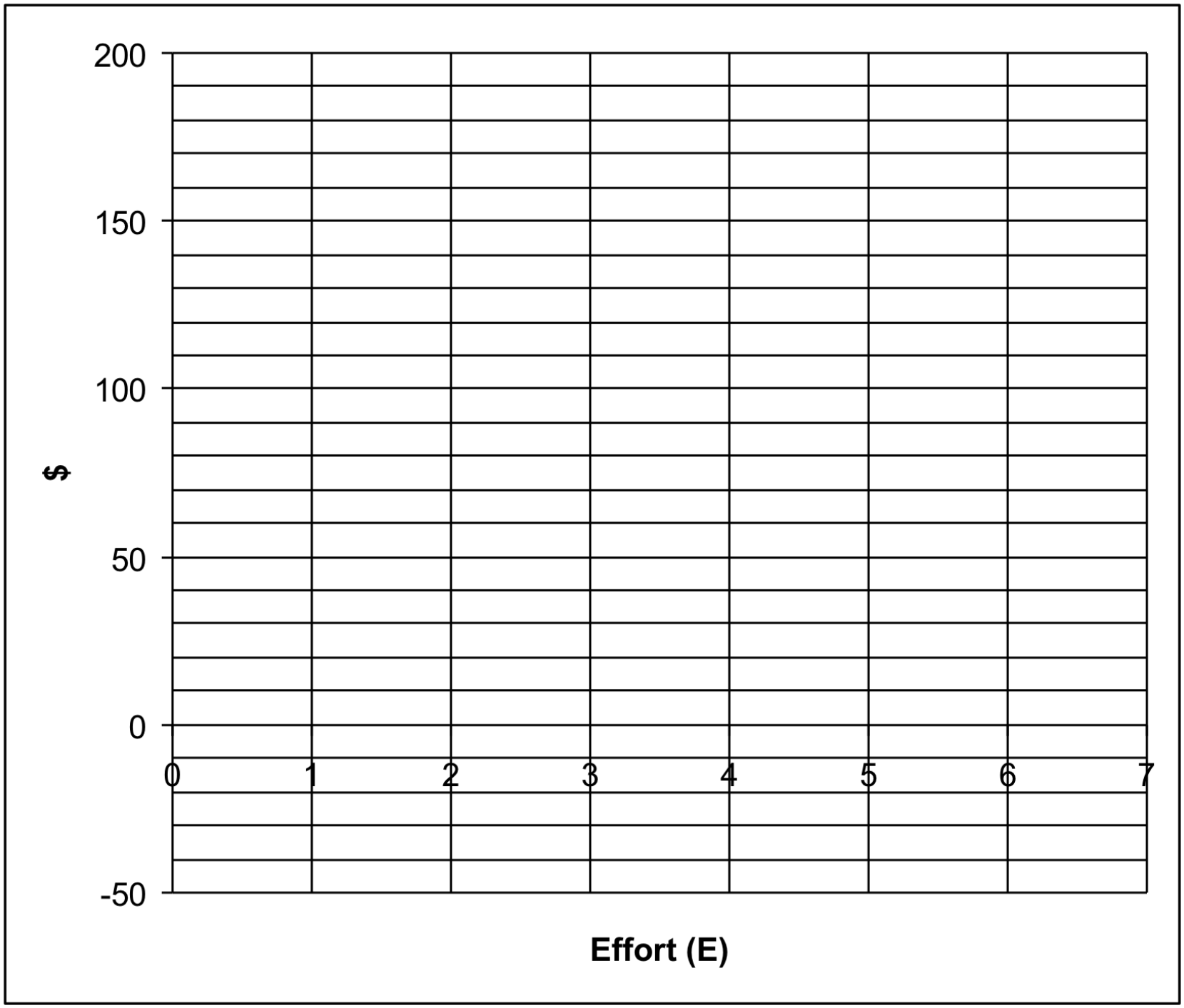
- 1. You have been asked to design a compensation scheme where earnings = a + bE where E is worker effort. The utility of workers is
- U = earnings - $2E^2$
- where the second term reflects the disutility of effort.
- Currently, given the strength of the labor market, workers will quit unless U >= 0.
- Each worker will be assigned an area of the state in which to sell the company's product. Each salesperson will know from your database which areas of his or her region are going to have the most people likely to buy your product. You tell your salesperson to start selling in those most promising areas, and then after, if there is time left, try to sell the product in the less promising areas of his or her assigned sales region.
- Because of this, there are diminishing returns to each worker's effort. Thus, production per worker per unit of time at your company is given by $Q=E^{1/2}$.
- Output sells at $64 per unit. There are no other marginal costs.

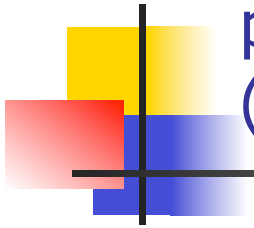
a) Calculate the value of net revenues and the total disutility of effort ($2E^2$) for values of E between 0 and 7 and enter them into the table below. (4 points)

E	0	1	2	3	4	5	6	7
Net revenues								
Disutility of effort $2E^2$								



- b) In the graph on the next page use your answers to a) to draw at least roughly (and label clearly) the lines for net revenues and the disutility of effort. Indicate with E* what appears to you to be the level of effort that will maximize the firm's profits. Then draw in, at least roughly, a line for the optimal wage a + bE your firm should pay to induce the level of effort E*. Label this line "wages". **I do not expect you to solve for E* exactly or for optimal a and b. But I do want you to draw at least a rough solution.** (4 points)

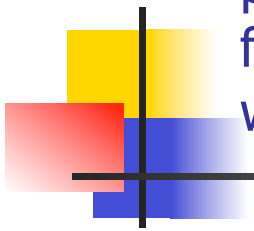

200
150
100
50
0
-50
$
0
1
2
3
4
5
6
7
Effort (E)



c) Now, let's do this using calculus. Solve the worker's problem of maximizing utility, given earnings of a + bE. (2 points)

d) Write down the firm's profit maximization problem, and then solve for optimal values of a, and b, and profits per worker. (4)

e) What is the *overall* commission bE (calculated at the optimal effort level E*) as a percentage of net revenues? What is the commission *rate* b as a percentage of *marginal* revenues calculated at the optimal effort level E*? (3)



f) In the problems we have typically done before, with linear production functions, what was the relation between “a” and the firm’s profits? Is that same relation obtained here? Explain why or why not. (Hint: What did you learn from your answer to part e?)