# Final Exam <br> Economics 136 - Human Resources <br> Spring 2012 <br> Prof. Julian Betts 

June 12, 2012

Name: $\qquad$
Student ID $\qquad$
There are 6 written problems in this exam, worth a total of 83 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the graders.

If you use pencil, whiteout, or erasable pen on a question, then that question cannot be regraded. If you do submit your exam for regrading, you must do within the time and other guidelines listed in the syllabus.

## SHOW ALL YOUR WORK!

You have 2 hours 50 minutes. Good luck.
For the graders:

| 1. | 18 |
| :---: | :---: |
| 2. | /10 |
| 3. | /23 |
| 4. | /17 |
| 5. | /11 |
| 6. | /14 |
| SUM | /83 |

## STUDENT CONSENT FOR RELEASE OF STUDENT INFORMATION <br> (Buckley Waiver) <br> (VOLUNTARY)

I hereby authorize the UCSD Economics Department to return my graded final examination/research paper by placing it in a location accessible to all students in the course. I understand that the return of my examination/research paper as described above may result in disclosure of personally identifiable information, that is not public information as defined in UCSD PPM 160-2, and I hereby consent to the disclosure of such information.

Quarter $\qquad$ Course $\qquad$ Date $\qquad$

Instructor $\qquad$

Student ID\# $\qquad$

Print Name $\qquad$

Signature $\qquad$

1. (8 points) There are five types of workers in the labor market, each accounting for $20 \%$ of workers. Their productivities per hour (VMP's) are $\$ 2, \$ 5, \$ 7, \$ 9$, and $\$ 11$.

There are two firms in the economy. The first firm, Firm A, pays a fixed hourly wage of W. The second firm, firm B, pays actual Value Marginal Product, minus $\$ 2$ per hour, or VMP-2. (The $\$ 2$ charge occurs because this firm hires many more managers than the first firm to measure the workers' value marginal product per hour.) a) There is one hourly wage $\mathrm{W}^{*}$ that the Firm A can offer so that it maximizes the proportion of workers who want to work at that firm, and for which expected profits are zero. What is that wage $\mathrm{W}^{*}$ ? Which types of workers work at firm A and at firm B. Show your work. (You should show the one scenario that fits the above description, and if it helps you feel free to show why other scenarios do not work.) (5)
b) Which types of workers choose the fixed wage firm (Firm A)? What economic concept is this an example of? (3)
2. (10 points) In the local economy, there is a minimum wage law stating that firms cannot pay workers less than $\$ 2$ per hour. A firm interviews two applicants for a job. Worker 1 seems like a sure thing, and has probability 1 of having VMP of $\$ 2$ per hour. The firm decides after interviewing Worker 2 that he or she is a bit more risky, with a $1 / 3$ probability of having VMP=\$3 per hour and a $2 / 3$ probability of having VMP=\$1 per hour.

The firm needs to hire a worker for 2000 hours per year, for up to 5 years, after which time it will shut down.
a) Calculate the expected VMP (per hour) of workers 1 and 2. Who has the higher expected productivity per hour? (3)
b) What will the expected profit be from hiring worker 1 over 5 years? (1)
c) If it hires worker 2, after one year it will know whether he or she is productivity 1 or 3 . If the worker has productivity 1 he or she will be fired, and the firm won't bother hiring anybody else for the remaining 4 years. But if the worker has productivity 3 , the firm can
continue to pay the worker $\$ 2$ per hour for the remaining four years. What are the expected profits over the next 5 years from hiring worker 2? Which worker should the firm hire? (4)
d) What are two real-world reasons why a firm may have trouble firing an unproductive worker at the end of one year of employment? (2)
3. (23 points) A firm is trying to establish a wage $=a+b E$ where $E$ is worker effort and $a$ and b are to be chosen by the firm.

Each unit of output produces $\$ 200$ of sales revenue. But additional output of 1 unit also leads to additional material costs for your firm of \$5, and extra energy costs of $\$ 3$.

Q is our symbol for output, and the production function is given by $\mathrm{Q}=\mathrm{E}^{1 / 3}$.
The worker maximizes utility, which the firm knows to be:

$$
\mathrm{U}=\text { earnings }-\mathrm{E}^{2}
$$

Workers must receive a utility of at least $\mathrm{U}_{\text {res }}$ which is the worker's reservation utility.
a) Solve the worker's problem. (4 points)
b) Calculate the profit maximizing values of a and b , and the optimal effort E and profit $\pi$ that result. (12 points)
c) In the problems we have typically done before, what is the relation between profits and "a"? What do you find now? What is the economic intuition for any difference you find? (Hint: You may want to do a rough plot of net revenues, the disutility of effort and the optimal wage $\mathrm{a}+\mathrm{bE}$, against E.) (3)
d) (2 points) What happens to the optimal values of profits, $\mathrm{a}, \mathrm{b}$ and E if $\mathrm{U}_{\text {res }}$ rises by 1 ? In part $b$ ) we defined $U_{\text {res }}$ as the reservation utility. Give an example of something that might cause all workers' $U_{\text {res }}$ to rise.
4. (17 points) Senior management asks you to devise a promotion tournament for two workers, where the worker who produces more earns $\mathrm{W}_{1}$ and the worker who produces less earns a smaller amount $\mathrm{W}_{2}$. The two workers have identical productivity (on average) and the same utility function:
$U=E\left(\right.$ wage $\left._{i}\right)-m_{i}^{2}$ for workers $i=k, j$ where $m_{i}$ is effort.
Each worker's output is given by
$q_{i}=m_{i}+e_{i}$ where $m$ is effort and $e$ is a random luck factor with mean 0 . Each unit of output brings in $\$ 20$ of revenues to your company. There are no costs apart from labor.
$x=e_{k}-e_{j}$ takes on values between -2 and +2 with a uniform probability distribution.
To ensure that both workers accept the job, you must pay an expected wage for a given level of effort to ensure that expected utility equals 0 .
a) Solve the workers' problems and the firm's problem. Calculate the firm's expected profits. DEFINE ALL NEW VARIABLES THAT YOU USE THAT ARE NOT MENTIONED IN THIS QUESTION. (12 points)
b) (3 points) Suppose that your workers both have a really bad shock due to marketing your company's product in an area in which a little known local company is beginning to steal your market share. Thus, both $\mathrm{e}_{\mathrm{k}}$ and $\mathrm{e}_{\mathrm{j}}$ turn out to be quite negative. Do the workers suffer lower pay on average because of this negative shock? How does this compare to the situation in which the firm had instead offered the workers a piece rate? In which situation is the risk of negative shocks like the one described here borne mainly or entirely by the firm?
c) Suppose that your firm's workforce is quite heterogeneous in ability. If you decide to promote people through tournaments, what does this high degree of variation in worker ability imply about the number of tiers (that is, levels) there should be in your firm's hierarchy? Why? (2)
5. (11 points) It is often thought that a worker's value marginal product (VMP) and best alternative to working at her current company, Alt, take the general forms shown in the graph below, where T is the "normal" age at which people retire:

a) Assume that the worker receives a wage equal to her VMP at each age. Who is more likely to goof off, that is, relax too much on the job, younger or older workers? Why? (3)
b) How could the firm alter the wages paid at each age to reduce the risk described in part a? Be specific about what mathematical requirements the new wage:age profile should obey for the firm to be willing to offer this wage (you can do this by using $\mathrm{W}_{\mathrm{t}}$ to stand for wages at time $t$, for example, and assume that the interest rate is $r$ ). (4)
c) Does your wage proposal create a risk that the worker will not want to retire at age T ? What is a common approach firms take to encourage older workers to retire "at the right time", that is, T? Hint: This is an approach that should not attract lawsuits. (4)
6) (14 points) Suppose that the CEO of your company is considering two investment projects and much choose between them.
Project A will produce a profit of $\$ 1$ billion with probability 0.5 , and $\$ 2$ billion with probability 0.5 . The stock price will be $\$ 50$ or $\$ 60$ depending on which outcome occurs (bad versus good outcome respectively).
Project B will produce a profit of - $\$ 3$ billion with probability 0.5 and a profit of $\$ 8$ billion with probability 0.5 . The stock price will be $\$ 10$ or $\$ 120$ depending on which outcome occurs (bad versus good outcome respectively). Right now, the stock price is $\$ 40$.
a) What is the expected profit from each project? Which looks like it is the riskier project? (3)
b) Assume that shareholders in this company are risk neutral and they care about company profit. Which project would they prefer that the CEO choose? (2)
c) Give an example of how the Board of Directors could issue call options to the CEO in a way that would incentivize the CEO to choose the investment project that shareholders would want. Be specific: should the firm issue call options, or require the CEO to grant put options to the firm? At what strike price should the options be issued? Back up your plan by showing the expected profits to the CEO from the stock options if the CEO chooses Project A, and if the CEO chooses Project B. (7)
d) Above we considered one of the reasons firms may use stock options as an incentive, which is related to how CEO's choose investment projects. What is a second reason why firms may use stock options as an incentive for top managers? (2)

