Instrumental Variables II
Weak Instruments

1. Review: The attraction of IV
2. IV vs. Heterogeneity Bias: Compulsory Schooling, birth quarter and earnings – Angrist & Krueger (1991)
3. Weak instrument bias in two flavors
4. Protection against weak instrument bias

1. Review: The Attraction of IV

Solution Problem	Add the omitted var.	experiment	instrument
1. Forgot X_2			
2. Selection			
3. Meas. Err.			
4. Misspecification			
5. Heterogeneity			
6. Endogeneity/ Simultaneity			

Good omitted variables, experimental data and instruments are all hard to find.

1. Review: The attraction of IV

Sample	Population
	1. CEF
$y = Xb + e$, $x'e=0$	2. BLP
b ols	3. Causal Effect
	4. Linear Causal Effect
	5. Perfectly specified equation model including all relevant variables

- $b^{IV}=(z'x)^{-1}z'y$ has no interpretation as a predictor

2. IV vs. Heterogeneity Bias: Compulsory Schooling, birth quarter and earnings – Angrist & Krueger (1991)

- School boards have age at entry requirements.
- States have compulsory schooling laws according to age.
- So a one-day difference in birthdate can create a one year difference in lifetime schooling.

TABLE II
PERCENTAGE OF AGE GROUP ENROLLED IN SCHOOL BY BIRTHDAY AND LEGAL DROPOUT AGE[a]

Date of birth	Type of state law[b]: School-leaving age: 16 (1)	Type of state law[b]: School-leaving age: 17 or 18 (2)	Column (1) − (2)
	Percent enrolled April 1, 1960		
1. Jan 1–Mar 31, 1944	87.6	91.0	−3.4
(age 16)	(0.6)	(0.9)	(1.1)
2. Apr 1–Dec 31, 1944	92.1	91.6	0.5
(age 15)	(0.3)	(0.5)	(0.6)
3. Within-state diff.	−4.5	−0.6	−4.0
(row 1 − row 2)	(0.7)	(1.0)	(1.2)
	Percent enrolled April 1, 1970		
4. Jan 1–Mar 31, 1954	94.2	95.8	−1.6
(age 16)	(0.3)	(0.5)	(0.6)
5. Apr 1–Dec 31, 1954	96.1	95.7	0.4
(age 15)	(0.1)	(0.3)	(0.3)
6. Within-state diff.	−1.9	0.1	−2.0
(row 1 − row 2)	(0.3)	(0.6)	(0.6)
	Percent enrolled April 1, 1980		
7. Jan 1–Mar 31, 1964	95.0	96.2	−1.2
(age 16)	(0.1)	(0.2)	(0.2)
8. Apr 1–Dec 31, 1964	97.0	97.7	−0.7
(age 15)	(0.1)	(0.1)	(0.1)
9. Within-state diff.	−2.0	−1.5	0.5
(row 1 − row 2)	(0.1)	(0.2)	(0.3)

a. Standard errors are in parentheses.
b. Data set used to compute rows 1–3 is the 1960 Census, 1 percent Public Use Sample; data set used to compute rows 4–6 is 1970 Census, 1 percent State Public Use Sample (15 percent form); data set used to compute rows 7–9 is the 1980 Census, 5 percent Public Use Sample. Each sample contains both boys and girls. Sample sizes are 4,153 for row 1; 12,512 for row 2; 7,758 for row 4; 24,636 for row 6; 42,740 for row 7; and 131,038 for row 8.

And it works..

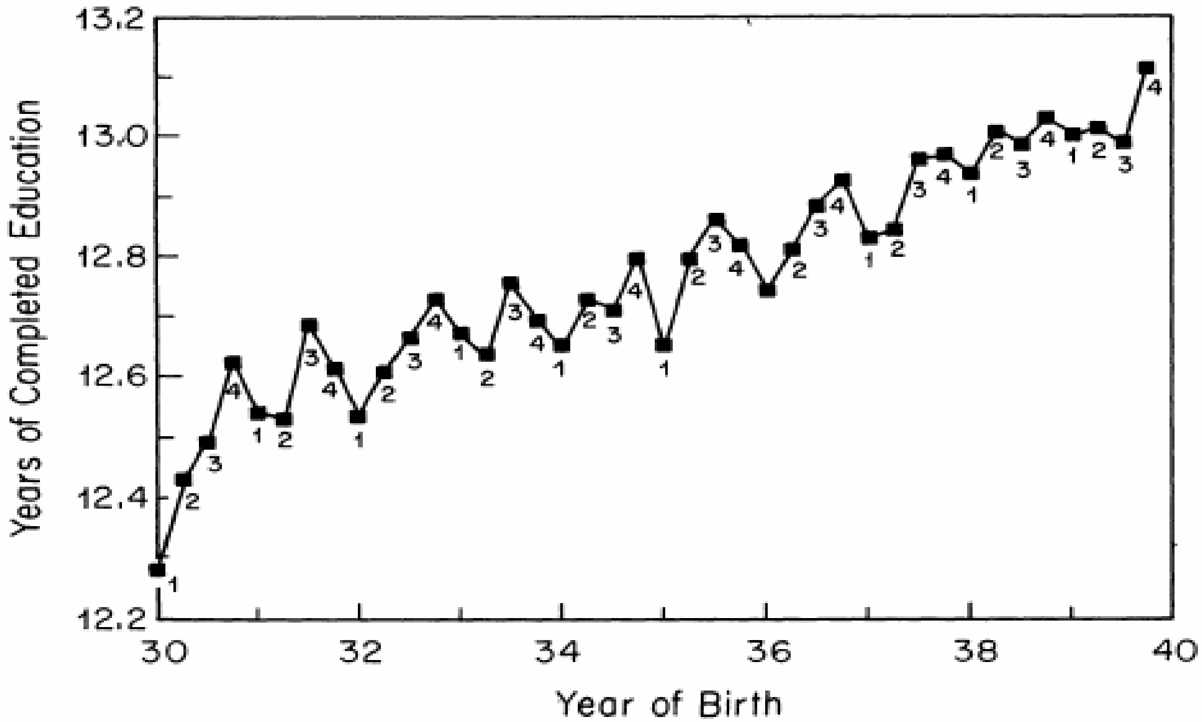

FIGURE I
Years of Education and Season of Birth
1980 Census
Note. Quarter of birth is listed below each observation.

Quarter of birth and schooling completed

TABLE I
THE EFFECT OF QUARTER OF BIRTH ON VARIOUS EDUCATIONAL OUTCOME VARIABLES

Outcome variable	Birth cohort	Mean	Quarter-of-birth effect[a] I	II	III	F-test[b] [P-value]
Total years of education	1930–1939	12.79	−0.124	−0.086	−0.015	24.9
			(0.017)	(0.017)	(0.016)	[0.0001]
	1940–1949	13.56	−0.085	−0.035	−0.017	18.6
			(0.012)	(0.012)	(0.011)	[0.0001]
High school graduate	1930–1939	0.77	−0.019	−0.020	−0.004	46.4
			(0.002)	(0.002)	(0.002)	[0.0001]
	1940–1949	0.86	−0.015	−0.012	−0.002	54.4
			(0.001)	(0.001)	(0.001)	[0.0001]
Years of educ. for high school graduates	1930–1939	13.99	−0.004	0.051	0.012	5.9
			(0.014)	(0.014)	(0.014)	[0.0006]
	1940–1949	14.28	0.005	0.043	−0.003	7.8
			(0.011)	(0.011)	(0.010)	[0.0017]
College graduate	1930–1939	0.24	−0.005	0.003	0.002	5.0
			(0.002)	(0.002)	(0.002)	[0.0021]
	1940–1949	0.30	−0.003	0.004	0.000	5.0
			(0.002)	(0.002)	(0.002)	[0.0018]
Completed master's degree	1930–1939	0.09	−0.001	0.002	−0.001	1.7
			(0.001)	(0.001)	(0.001)	[0.1599]
	1940–1949	0.11	0.000	0.004	0.001	3.9
			(0.001)	(0.001)	(0.001)	[0.0091]
Completed doctoral degree	1930–1939	0.03	0.002	0.003	0.000	2.9
			(0.001)	(0.001)	(0.001)	[0.0332]
	1940–1949	0.04	−0.002	0.001	−0.001	4.3
			(0.001)	(0.001)	(0.001)	[0.0050]

a. Standard errors are in parentheses. An *MA*(+2, −2) trend term was subtracted from each dependent variable. The data set contains men from the 1980 Census, 5 percent Public Use Sample. Sample size is 312,718 for 1930–1939 cohort and is 457,181 for 1940–1949 cohort.

b. *F*-statistic is for a test of the hypothesis that the quarter-of-birth dummies jointly have no effect.

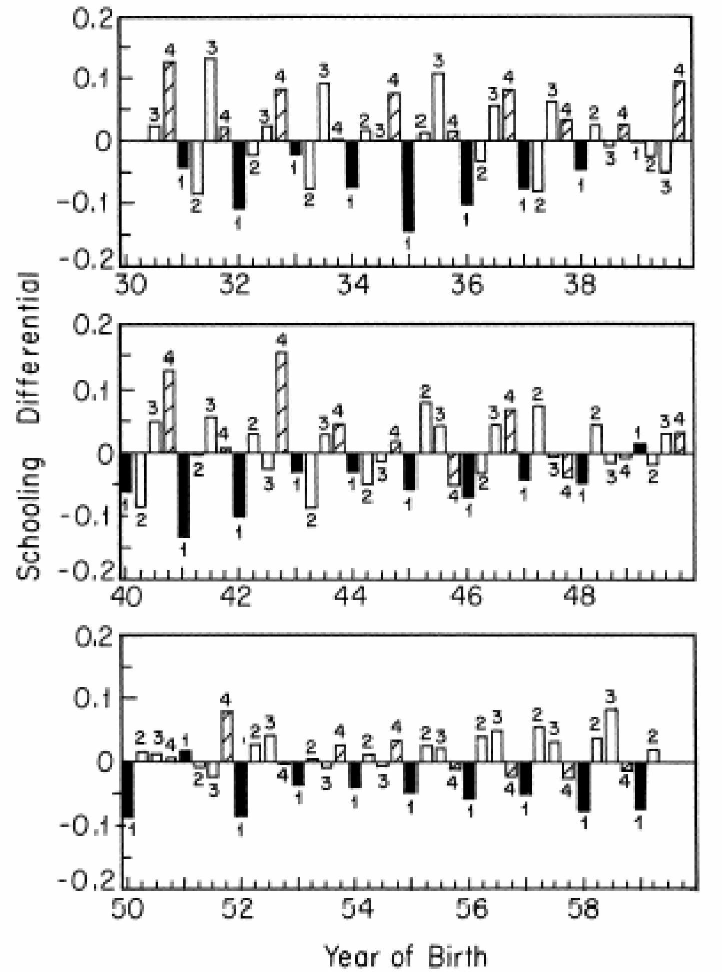

FIGURE IV
Season of Birth and Years of Schooling
Deviations from *MA*(+2,−2)

So here's an instrument for ability in the "Mincer" regression

- $y_i = \beta_0 + \beta_1 x_i + X_2 \beta_2 + a_i + \varepsilon_i$
- x_1 – schooling, y – log(earnings)
- The human capital wage regression ("Mincer" regression) is the foundation of human capital theory.
- Yet we worry about bias due to unobserved ability, which is potentially correlated with schooling, $Cov(x_1,a)$
- z – quarter of birth, is a valid instrument if $Cov(z, \varepsilon) = 0$, i.e., quarter of birth affects earnings only through its' effect on schooling. From Figure I we know that it's relevant.

Reduced form: Do 1st quarter babies have lower earnings (as adults)?

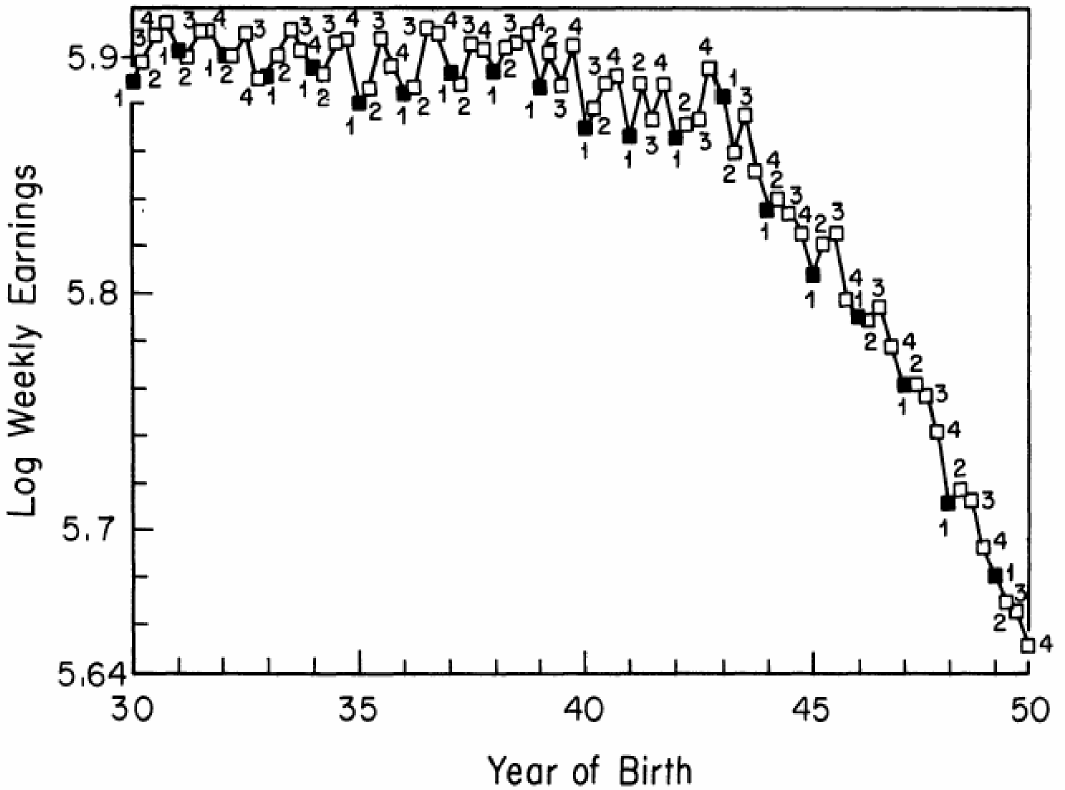

FIGURE V
Mean Log Weekly Wage, by Quarter of Birth
All Men Born 1930–1949; 1980 Census

Wald Estimates

TABLE III

PANEL A: WALD ESTIMATES FOR 1970 CENSUS—MEN BORN 1920–1929[a]

	(1) Born in 1st quarter of year	(2) Born in 2nd, 3rd, or 4th quarter of year	(3) Difference (std. error) (1) − (2)
ln (wkly. wage)	5.1484	5.1574	−0.00898
			(0.00301)
Education	11.3996	11.5252	−0.1256
			(0.0155)
Wald est. of return to education			0.0715
			(0.0219)
OLS return to education[b]			0.0801
			(0.0004)

Panel B: Wald Estimates for 1980 Census—Men Born 1930–1939

	(1) Born in 1st quarter of year	(2) Born in 2nd, 3rd, or 4th quarter of year	(3) Difference (std. error) (1) − (2)
ln (wkly. wage)	5.8916	5.9027	−0.01110
			(0.00274)
Education	12.6881	12.7969	−0.1088
			(0.0132)
Wald est. of return to education			0.1020
			(0.0239)
OLS return to education			0.0709
			(0.0003)

a. The sample size is 247,199 in Panel A, and 327,509 in Panel B. Each sample consists of males born in the United States who had positive earnings in the year preceding the survey. The 1980 Census sample is drawn from the 5 percent sample, and the 1970 Census sample is from the State, County, and Neighborhoods 1 percent samples.

b. The OLS return to education was estimated from a bivariate regression of log weekly earnings on years of education.

Two stage least squares

A. TSLS Estimates

To improve efficiency of the estimates and control for age-related trends in earnings, we estimated the following TSLS model:

$$(1) \qquad E_i = X_i\pi + \Sigma_c Y_{ic}\delta_c + \Sigma_c \Sigma_j Y_{ic} Q_{ij} \theta_{jc} + \epsilon_i$$

$$(2) \qquad \ln W_i = X_i\beta + \Sigma_c Y_{ic}\xi_c + \rho E_i + \mu_i,$$

where E_i is the education of the ith individual, X_i is a vector of covariates, Q_{ij} is a dummy variable indicating whether the individual was born in quarter j ($j = 1,2,3$), and Y_{ic} is a dummy variable indicating whether the individual was born in year c ($c = 1, \ldots, 10$), and W_i is the weekly wage. The coefficient ρ is the return to education. If the residual in the wage equation, μ, is correlated with years of education due to, say, omitted variables, OLS estimates of the return to education will be biased.

TSLS estimates:

TABLE IV
OLS AND TSLS ESTIMATES OF THE RETURN TO EDUCATION FOR MEN BORN 1920–1929: 1970 CENSUS[a]

Independent variable	(1) OLS	(2) TSLS	(3) OLS	(4) TSLS	(5) OLS	(6) TSLS	(7) OLS	(8) TSLS
Years of education	0.0802	0.0769	0.0802	0.1310	0.0701	0.0669	0.0701	0.1007
	(0.0004)	(0.0150)	(0.0004)	(0.0334)	(0.0004)	(0.0151)	(0.0004)	(0.0334)
Race (1 = black)	—	—	—	—	0.2980	−0.3055	−0.2980	−0.2271
					(0.0043)	(0.0353)	(0.0043)	(0.0776)
SMSA (1 = center city)	—	—	—	—	0.1343	0.1362	0.1343	0.1163
					(0.0026)	(0.0092)	(0.0026)	(0.0198)
Married (1 = married)	—	—	—	—	0.2928	0.2941	0.2928	0.2804
					(0.0037)	(0.0072)	(0.0037)	(0.0141)
9 Year-of-birth dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
8 Region of residence dummies	No	No	No	No	Yes	Yes	Yes	Yes
Age	—	—	0.1446	0.1409	—	—	0.1162	0.1170
			(0.0676)	(0.0704)			(0.0652)	(0.0662)
Age-squared	—	—	−0.0015	−0.0014	—	—	−0.0013	−0.0012
			(0.0007)	(0.0008)			(0.0007)	(0.0007)
χ^2 [dof]	—	36.0 [29]	—	25.6 [27]	—	34.2 [29]	—	28.8 [27]

a. Standard errors are in parentheses. Sample size is 247,199. Instruments are a full set of quarter-of-birth times year-of-birth interactions. The sample consists of males born in the United States. The sample is drawn from the State, County, and Neighborhoods 1 percent samples of the 1970 Census (15 percent form). The dependent variable is the log of weekly earnings. Age and age-squared are measured in quarters of years. Each equation also includes an intercept.

Possible Validity Problems:

- Why might quarter of birth be correlated with the residual in the earnings equation?

- Age at entry and earnings
- Season of birth and earnings

- These seem like 2nd order problems,

- OID tests don't raise any red flags

.. so we can stop worrying about ability bias in earnings equations and proudly claim that estimated returns to education are causal, right?

3. Weak instrument bias in IV estimators

- The graduate labor class at the University of Michigan does replication exercises. (Moderately short papers).
- Regina Baker and David Jaeger manage to replicate the results (Angrist and Krueger shared the data).
- But two things bother them and Prof. Bound: (Tables 1 and 2).

Small Sample Bias of IV Estimators

Table 1. *Estimated Effect of Completed Years of Education on Men's Log Weekly Earnings (standard errors of coefficients in parentheses)*

	(1) OLS	(2) IV	(3) OLS	(4) IV	(5) OLS	(6) IV
Coefficient	.063 (.000)	.142 (.033)	.063 (.000)	.081 (.016)	.063 (.000)	.060 (.029)
F (excluded instruments)		13.486		4.747		1.613
Partial R^2 (excluded instruments, ×100)		.012		.043		.014
F (overidentification)		.932		.775		.725
Age Control Variables						
Age, Age^2	x	x			x	x
9 Year of birth dummies			x	x	x	x
Excluded Instruments						
Quarter of birth		x		x		x
Quarter of birth × year of birth				x		x
Number of excluded instruments		3		30		28

NOTE: Calculated from the 5% Public-Use Sample of the 1980 U.S. Census for men born 1930–1939. Sample size is 329,509. All specifications include Race (1 = black), SMSA (1 = central city), Married (1 = married, living with spouse), and 8 Regional dummies as control variables. F (first stage) and partial R^2 are for the instruments in the first stage of IV estimation. F (overidentification) is that suggested by Basmann (1960).

Worry #1: The results are imprecise and unstable when the controls and instrument sets change.

Small Sample Bias of IV Estimators

Table 2. *Estimated Effect of Completed Years of Education on Men's Log Weekly Earnings, Controlling for State of Birth (standard errors of coefficients in parentheses)*

	(1) OLS	(2) IV	(3) OLS	(4) IV
Coefficient	.063	.083	.063	.081
	(.000)	(.009)	(.000)	(.011)
F (excluded instruments)		2.428		1.869
Partial R^2 (excluded instruments, ×100)		.133		.101
F (overidentification)		.919		.917
Age Control Variables				
Age, Age^2			x	x
9 Year of birth dummies	x	x	x	x
Excluded Instruments				
Quarter of birth		x		x
Quarter of birth × year of birth		x		x
Quarter of birth × state of birth		x		x
Number of excluded instruments		180		178

NOTE: Calculated from the 5% Public-Use Sample of the 1980 U.S. Census for men born 1930–1939. Sample size is 329,509. All specifications include Race (1 = black), SMSA (1 = central city), Married (1 = married, living with spouse), 8 Regional dummies, and 50 State of Birth dummies as control variables. F (first stage) and partial R^2 are for the instruments in the first stage of IV estimation. F (overidentification) is that suggested by Basmann (1960).

Worry #2:
The results become precise and stable only when the first stage F tests cannot reject coefficients which are jointly zero.

Small (finite) sample bias

- Consider the first stage:
  $x = z\delta + \omega$.
  Even if $\delta=0$ in the population, as the number of instruments increases the R^2 of the first stage regression in the sample can only increase.
- As we add instruments, x hat approximates x better and better, so that the 2nd stage IV estimate converges to the OLS estimate.

Simulation with a random instrument

Table 3. *Estimated Effect of Completed Years of Education on Men's Log Weekly Earnings, Using Simulated Quarter of Birth (500 replications)*

Table (column)	*1 (4)*	*1 (6)*	*2 (2)*	*2 (4)*
Estimated Coefficient				
Mean	.062	.061	.060	.060
Standard deviation of mean	.038	.039	.015	.015
5th percentile	−.001	−.002	.034	.035
Median	.061	.061	.060	.060
95th percentile	.119	.127	.083	.082
Estimated Standard Error				
Mean	.037	.039	.015	.015

NOTE: Calculated from the 5% Public-Use Sample of the 1980 U.S. Census for men born 1930–1939. Sample size is 329,509.

As an illustration, B,B and J estimated the IV coefficient with a randomly assigned Z so that $\delta=0$ by construction.

They did a great job reproducing the OLS estimate.

Flavor #2:
Weak Instruments when the IV is almost, but not quite, valid

- Is the cure worse than the disease?

- OLS bias vs. IV bias

- What looks like a second order Cov(z, ε) can create a first order inconsistency if Cov(z,x) is small.

4. What to do about weak instruments?

- First Stage F tests on the marginal excluded instrument or sets of instruments
- First Stage R^2