1 Productivity Shocks with Initially Unbalanced Current Accounts

There are two periods and two countries. Home produces with $Y = AF(K)$ and Foreign with $Y^* = A^* F(K^*)$, where $A$ and $A^*$ are productivity parameters. Both countries’ representative agents have the same period utility so that $U_1 = u(C_1) + \beta u(C_2)$ and $U_1^* = u(C_1^*) + \beta u(C_2^*)$. Assume period utility $u(\cdot)$ to be isoelastic. International financial market clearing $S + S^* = I + I^*$ (or $CA_1 = -CA_1^*$) determines the world interest rate $r$.

1. Use the intertemporal optimality conditions for production to show that an anticipated productivity shock $dA_2/A_2$ to production in Home changes equilibrium investment and the interest rate in the following way

$$r \frac{dA_2}{A_2} + \left( \frac{\partial I_1}{\partial r} \right)^{-1} dI_1 - dr = 0.$$  

Derive the according equation for Foreign and $dA_2^*/A_2^*$.

2. Use the intertemporal optimality conditions for consumption, along with your results in 1, to show that an anticipated productivity shock $dA_2/A_2$ to production in Home changes the equilibrium current account level and the interest rate in the following way

$$D \frac{dA_2}{A_2} - M dr + V dCA_1 = 0,$$

for some functions $V, D,$ and $M$. Derive the according equation for Foreign and $dA_2^*/A_2^*$ (with $V^*, D^*, M^*$ and using $CA_1 = -CA_1^*$). Derive the signs of $V, V^*, D$ and $D^*$.

3. Show that anticipated productivity shocks to Home and Foreign have the following effect on the equilibrium interest rate and the Home current account:

$$dr = \frac{1}{\frac{D}{M} + \frac{D^*}{M^*}} \left( \frac{D}{V} \frac{dA_2}{A_2} + \frac{D^*}{V^*} \frac{dA_2^*}{A_2^*} \right),$$

$$dCA_1 = -\frac{1}{\frac{D}{M} + \frac{D^*}{M^*}} \left( \frac{D}{M} \frac{dA_2}{A_2} - \frac{D^*}{M^*} \frac{dA_2^*}{A_2^*} \right).$$
4. Suppose Home runs a current account surplus \( CA_1 > 0 \) during period 1. How does an anticipated increase in Home productivity \( A_2 \) during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not?

5. Suppose again Home runs a current account surplus \( CA_1 > 0 \) during period 1. How does an anticipated increase in Foreign productivity \( A_2^* \) during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not?

6. Consider anticipated and equal proportional increases in Home and Foreign productivity \( \frac{dA_2}{A_2} = \frac{dA_2^*}{A_2^*} \). How does this change affect worldwide investment \( I_1 + I_1^* \) during period 1? Does the answer depend on the elasticity of intertemporal substitution? Is the effect unambiguous? Why or why not?

2 Exponential Period Utility

There are two periods. A country’s representative household has the exponential period utility function

\[ u(C) = -\gamma \exp\left(-\frac{C}{\gamma}\right) \]

with \( \gamma \in (0, \infty) \) and maximizes lifetime utility \( U_1 = u(C_1) + \beta u(C_2) \) subject to

\[ C_1 + RC_2 = Y_1 + RY_2 \equiv W, \]

where \( R \equiv 1/(1 + r) \) is the price of tomorrow’s consumption in terms of today’s consumption and \( W \) is initial wealth. The value of \( W \) depends on \( R \).

1. Derive the Euler equation and solve it for \( C_2 \) as a function of \( C_1 \), \( R \) and \( \beta \).

2. What is the optimal level of \( C_1 \) considering \( W \), \( R \) and \( \beta \) as given?

3. Differentiate this consumption function of \( C_1 \) with respect to \( R \) (differentiate \( W \) with respect to \( R \) too) and show that

\[ \frac{dC_1}{dR} = -\frac{C_1}{1 + R} + \frac{Y_2}{1 + R} + \frac{\gamma}{1 + R} \left( 1 - \ln(\beta/R) \right) \]

4. Derive the intertemporal elasticity of substitution of the exponential period utility \( (-u'(C)/Cu''(C)) \).

5. Use this result to show that the derivative \( dC_1/dR \) in part 3 can be expressed as

\[ \frac{dC_1}{dR} = \sigma(C_2) \frac{C_2}{1 + R} - \frac{C_2}{1 + R} + \frac{Y_2}{1 + R}. \]

Interpret the three additive terms in this derivative.
3 Stochastic Current Account Model

There are infinitely many periods. A country’s representative household has the linear-quadratic period utility function

$$u(C) = C - \frac{a_0}{2} C^2$$

with $a_0 \in (0, \infty)$ and maximizes lifetime utility

$$U_t = \mathbb{E}_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right]$$

subject to

$$CA_s = B_{s+1} - B_s = rB_s + \tilde{Z}_s - C_s \quad \forall s \geq t$$

where $R \equiv 1/(1 + r) = \beta$ and $\tilde{Z}_s(\equiv \tilde{Y}_s - \tilde{G}_s - \tilde{I}_s)$ is random net output.

1. Derive the stochastic Euler equations and show that $C_t$ satisfies

$$C_t = rR \left( (1 + r)B_t + \sum_{s=t}^{\infty} R^{s-t} \mathbb{E}_t[\tilde{Z}_s] \right)$$

2. Show that then $CA_t \equiv B_{t+1} - B_t = \tilde{Z}_t - \mathbb{E}_t[\tilde{Z}_t]$, where the hat denotes the permanent level of the variable. The permanent level $\hat{X}$ of a random variable $\tilde{X}$ is defined as $\sum_{s=t}^{\infty} R^{s-t} \hat{X} \equiv \mathbb{E}_t \left[ \sum_{s=t}^{\infty} R^{s-t} \tilde{X}_s \right]$.

3. Define $\Delta \tilde{Z}_s \equiv \tilde{Z}_s - \tilde{Z}_{s-1}$ and show that the current account can be rewritten as $CA_t = -\sum_{s=t+1}^{\infty} R^{s-t} \mathbb{E}_t \left[ \Delta \tilde{Z}_s \right]$.

4. Use $\Delta \tilde{Z}_t \equiv \tilde{Z}_t - \tilde{Z}_{t-1}$ to show that the quantity

$$CA_t - (1 + r)CA_{t-1} - \Delta \tilde{Z}_t$$

is stochastically independent of $CA_s$ and $\Delta \tilde{Z}_s$ for all $s < t$. How is this finding related to Hall’s (1978) result that consumption follows a random walk?

4 Current Account and Terms of Trade

In a small open economy, the representative individual maximizes the lifetime utility function

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left( X_s M_s^{1-\gamma} \right)^{1/\sigma} - 1,$$

where $X$ is consumption of an exported good and $M$ consumption of an imported good. The country completely specializes in production of the export good. The
endowment of this good is constant at $Y$. The representative individual faces the fixed world interest rate $r = (1 - \beta)/\beta$ in terms of the real consumption index $C = X^\gamma M^{1-\gamma}$ (so a loan of 1 real consumption unit today returns $1 + r$ real consumption units tomorrow). There is no investment or government spending.

1. Let $p$ bet the price of the export goods in terms of the import good. So, a rise in $p$ is an improvement in the terms of trade. Show that the consumption-based price index $P$ in terms of imports is

$$P = \frac{p \gamma}{\gamma (1 - \gamma)^{1-\gamma}}.$$ 

2. Show that the home country’s current account identity is

$$B_{t+1} - B_t = rB_t + \frac{p_t (Y - X_t)}{P_t} - \frac{M_t}{P_t}.$$ 

What is the corresponding intertemporal budget constraint for the representative consumer?

3. Derive the first-order conditions of the consumer’s problem. (Hint: Reformulate the utility function and budget constraint in terms of real consumption $C$.) What are the optimal paths for $X$ and $M$?

4. Suppose initial expectations are that $p$ remains constant over time. There is an unexpected temporary fall in the terms of trade from $p$ to $p' < p$. What is the effect on the current account $CA_t = B_{t+1} - B_t$ from part 2?

5. Now suppose foreign net wealth $B$ is indexed to the import good $M$ rather than to real consumption. Accordingly, let $r$ denote the own-rate of interest in imports but assume again that $r = (1 - \beta)/\beta$. How does a temporary drop in the terms of trade from $p$ to $p' < p$ affect the current account now? How do you explain differences, if any, to part 4?