Econ 202a Spring 2000
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Problem Set 3

1 Delegated control over monetary policy

(This question is based on K. Rogoff, QJE 1985; D. Romer, q. 9.12)

Suppose that output is given by the Lucas supply function

$$y = \bar{y} + b(\pi - \pi^e). \tag{1}$$

The social welfare function is

$$S = \tilde{\gamma} y - \frac{a}{2}\pi^2. \tag{2}$$

The coefficient $\tilde{\gamma}$ is a random variable with mean $\mathbb{E}[\gamma] = \bar{\tilde{\gamma}}$ and variance $\text{Var}(\tilde{\gamma}) = \sigma^2$. This reflects the fact that political objectives can vary, depending on the political process and a change of goals. It would be nice to have a flexible central banker who could quickly adjust to the needs of the situation, that is a central banker who would pursue a relatively more expansionary policy whenever the realization of $\tilde{\gamma}$ is high, and a relatively strict low-inflation policy whenever $\tilde{\gamma}$ is low. What is the best choice of a central banker?

Let the central banker follow the objective function

$$S^{CB} = c\tilde{\gamma} y - \frac{a}{2}\pi^2, \tag{3}$$

where $c \in R$.

The timing of the model is as follows. The private sector decides π^e without knowledge of $\tilde{\gamma}$. The central banker, however, chooses π after $\tilde{\gamma}$ is known.

a) What is the central banker's choice of π given π^e, $\tilde{\gamma}$, and c?

b) What expectation of inflation, π^e, does a rational private sector choose?

c) What is the expected value of the true social welfare function (2)?

d) What value of c maximizes expected social welfare? Is the 'optimal' central banker more or less conservative (inflation averse) than the private sector? Interpret your result.

2 A simple model of overshooting

Suppose money demand takes the Cagan form

$$m_t^d - p_t = \phi y_t^d - \eta i_{t+1}, \tag{4}$$

where m_t^d denotes the logarithm of nominal money demand, p_t the logarithm of the price level, y_t^d the logarithm of aggregate demand and i_{t+1} the nominal interest rate ($\approx \ln(1+i_{t+1})$). The coefficients ϕ and η are assumed to be positive. Money supply is constant,

$$m_t^s = \bar{m}. \tag{5}$$

In efficient international capital markets, uncovered interest parity (UIP) holds:

$$i_{t+1} = i_{t+1}^* + e_{t+1} - e_t, \tag{6}$$

where e_t denotes the logarithm of the *nominal* exchange rate. In this form, a high value of e_t means a weak currency. The logarithm of the *real* exchange rate is $q_t = e_t + p_t^* - p_t$, so that a high value coincides with a depreciated real exchange rate.

Suppose that aggregate demand depends on the real exchange rate. It is high, whenever the real exchange depreciates (because exports benefit):

$$y_t^d = \delta(e_t + p_t^* - p_t). \tag{7}$$

For the model to be consistent, we require $\delta \in (0, \frac{1}{\phi})$. The full-employment level of aggregate supply is

$$y_t^s = \bar{y}. \tag{8}$$

Finally, suppose in Keynesian style that prices are not immediately set to the expected equilibrium level, but adjusted slowly. In particular, let prices obey the response function

$$p_{t+1} - p_t = \pi(y_t^d - y_t^s). \tag{9}$$

In order to further simplify the model, let's standardize all foreign variables to constants $p_t^* = i_{t+1}^* = 0$, and let's suppose that money markets are always clearing immediately, $m_t^d = m_t^s = \bar{m}$. Then a simple Dornbusch model of overshooting can be built as a system in three equations:

$$\bar{m} - p_t = \phi y_t^d - \eta(e_{t+1} - e_t), \tag{10}$$

$$y_t^d = \delta(e_t - p_t) \qquad \delta \in (0, \frac{1}{\phi}), \tag{11}$$

$$p_{t+1} - p_t = \pi(y_t^d - \bar{y}). \tag{12}$$

a) Find the steady-state values of the exchange rate and the price level ($e_{t+1} = e_t = \bar{e}$, $p_{t+1} = p_t = \bar{p}$).

b) Express both $(e_{t+1} - e_t)$ and $(p_{t+1} - p_t)$ as functions of e_t, p_t and exogenous variables. Find the two functional relationships between p_t and e_t that satisfy $e_{t+1} - e_t = 0$ and $p_{t+1} - p_t = 0$. Draw them in a phase diagram with p_t on the y-axis and e_t on the x-axis. Complete the phase diagram indicating the motion of the system (using the conditions for $e_{t+1} - e_t \geq 0$ and $p_{t+1} - p_t \geq 0$).

Finally, add a line to the diagram that obeys a 'no-arbitrage condition' as mandated by UIP: $p_t - \bar{p} = -\hat{\theta}\,(e_t - \bar{e})$ for some $\hat{\theta} > 0$. (The $\hat{\theta}$ is not quite the same as in the original Dornbusch model since we have no uncertainty here.)

c) Is the steady-state stable? If not, what is the unique stable ("saddle") path given a steady-state of $\bar{p}$ and $\bar{e}$?

d) Suppose all variables except for p_t respond immediately to a monetary shock.

What is the new steady-state? Draw a new 'no-arbitrage' line.

What happens to e_t right after a reduction in the monetary base from $\bar{m}$ to $\bar{m}'$? How do e_{t+s} and p_{t+s} evolve over time?

3 Dynamics of the simple model of overshooting

Consider the simple model of overshooting from question 2 again:

$$\bar{m} - p_t = \phi y_t^d - \eta(e_{t+1} - e_t), \tag{13}$$

$$y_t^d = \delta(e_t - p_t) \qquad \delta \in (0, \frac{1}{\phi}), \tag{14}$$

$$p_{t+1} - p_t = \pi(y_t^d - \bar{y}). \tag{15}$$

We are interested in the behavior of the system around the steady-state.

a) Using (14) and the steady-states of $\bar{e}$ and $\bar{p}$ that you found in question 2a), express $y_t^d - \bar{y}$ as a function of $e_t - \bar{e}$ and $p_t - \bar{p}$.

b) Using (13) along with the results in 2a) and 3a), express $e_{t+1} - \bar{e}$ as a function of $\frac{1}{\eta}(e_t - \bar{e})$ and $\frac{1}{\eta}(p_t - \bar{p})$.

c) Using (14) and (15) along with the results in 2a), express $p_{t+1} - \bar{p}$ as a weighted sum of $e_t - \bar{e}$ and $p_t - \bar{p}$.

From now on, assume that $\pi = \frac{1}{\eta}$ and $\phi = 3$ for simplicity.

d) Write your findings from 3b) and 3c) into a system of two difference equations that takes the form

$$\begin{pmatrix} e_{t+1} - \bar{e} \\ p_{t+1} - \bar{p} \end{pmatrix} = \mathbf{A} \cdot \begin{pmatrix} e_t - \bar{e} \\ p_t - \bar{p} \end{pmatrix}. \tag{16}$$

Find the eigenvalues and eigenvectors of the system (you may, of course, simply plug your previous results into the formulas from section.)

[Hint: An intermediate result is $\mathrm{tr}(\mathbf{A}) = \frac{2(\delta+\eta)}{\eta}$ and $\det(\mathbf{A}) = 1 - \frac{(1-2\eta)\delta}{\eta^2}$.]

e) Is the system stable? That is, do the exchange rate and price levels converge to the steady-state?

If not, set the coefficient of the unstable root to zero. Then, using the simplified system (16) and the results from 2a), express p_{t+1} as a function of e_{t+1} and exogenous variables. Draw this function into the phase diagram from question 2 (Remember that $\delta < \frac{1}{\phi}$.)

What did you just find? What is the intuition for the fact that the economy obeys this relationship?