1 Delegated control over monetary policy

(This question is based on K. Rogoff, QJE 1985; D. Romer, q. 9.12)

Suppose that output is given by the Lucas supply function
\[ y = \bar{y} + b(\pi - \pi^e). \]  
(1)

The social welfare function is
\[ S = \tilde{\gamma}y - \frac{a}{2}\pi^2. \]  
(2)

The coefficient \( \tilde{\gamma} \) is a random variable with mean \( \mathbb{E}[\gamma] = \bar{\gamma} \) and variance \( \text{Var}(\tilde{\gamma}) = \sigma^2 \). This reflects the fact that political objectives can vary, depending on the political process and a change of goals. It would be nice to have a flexible central banker who could quickly adjust to the needs of the situation, that is a central banker who would pursue a relatively more expansionary policy whenever the realization of \( \tilde{\gamma} \) is high, and a relatively strict low-inflation policy whenever \( \tilde{\gamma} \) is low. What is the best choice of a central banker?

Let the central banker follow the objective function
\[ S^{CB} = c\tilde{\gamma}y - \frac{a}{2}\pi^2, \]  
(3)

where \( c \in \mathbb{R} \).

The timing of the model is as follows. The private sector decides \( \pi^e \) without knowledge of \( \tilde{\gamma} \). The central banker, however, chooses \( \pi \) after \( \tilde{\gamma} \) is known.

a) What is the central banker’s choice of \( \pi \) given \( \pi^e, \tilde{\gamma}, \) and \( c \)?

b) What expectation of inflation, \( \pi^e \), does a rational private sector choose?

c) What is the expected value of the true social welfare function (2)?

d) What value of \( c \) maximizes expected social welfare? Is the ‘optimal’ central banker more or less conservative (inflation averse) than the private sector? Interpret your result.
2 A simple model of overshooting

Suppose money demand takes the Cagan form

\[ m^d_t - p_t = \phi y^d_t - \eta i_{t+1}, \]  

(4)

where \( m^d_t \) denotes the logarithm of nominal money demand, \( p_t \) the logarithm of the price level, \( y^d_t \) the logarithm of aggregate demand and \( i_{t+1} \) the nominal interest rate (\( \approx \ln(1 + i_{t+1}) \)). The coefficients \( \phi \) and \( \eta \) are assumed to be positive. Money supply is constant,

\[ m^s_t = \bar{m}. \]  

(5)

In efficient international capital markets, uncovered interest parity (UIP) holds:

\[ i_{t+1} = i^{*}_{t+1} + e_{t+1} - e_t, \]  

(6)

where \( e_t \) denotes the logarithm of the nominal exchange rate. In this form, a high value of \( e_t \) means a weak currency. The logarithm of the real exchange rate is \( q_t = e_t + p^{*}_t - p_t \), so that a high value coincides with a depreciated real exchange rate.

Suppose that aggregate demand depends on the real exchange rate. It is high, whenever the real exchange depreciates (because exports benefit):

\[ y^d_t = \delta(e_t + p^{*}_t - p_t). \]  

(7)

For the model to be consistent, we require \( \delta \in (0, \frac{1}{\phi}) \). The full-employment level of aggregate supply is

\[ y^a_t = \bar{y}. \]  

(8)

Finally, suppose in Keynesian style that prices are not immediately set to the expected equilibrium level, but adjusted slowly. In particular, let prices obey the response function

\[ p_{t+1} - p_t = \pi(y^d_t - y^a_t). \]  

(9)

In order to further simplify the model, let’s standardize all foreign variables to constants \( p^*_t = i^{*}_{t+1} = 0 \), and let’s suppose that money markets
are always clearing immediately, $m_t^d = m_t^s = \bar{m}$. Then a simple Dornbusch model of overshooting can be built as a system in three equations:

\[ \bar{m} - p_t = \phi y^d_t - \eta(e_{t+1} - e_t), \]  
\[ y^d_t = \delta(e_t - p_t) \quad \delta \in (0, \frac{1}{\phi}), \]  
\[ p_{t+1} - p_t = \pi(y^d_t - \bar{y}). \]

a) Find the steady-state values of the exchange rate and the price level ($e_{t+1} = e_t = \bar{e}$, $p_{t+1} = p_t = \bar{p}$).

b) Express both $(e_{t+1} - e_t)$ and $(p_{t+1} - p_t)$ as functions of $e_t$, $p_t$ and exogenous variables. Find the two functional relationships between $p_t$ and $e_t$ that satisfy $e_{t+1} - e_t = 0$ and $p_{t+1} - p_t = 0$. Draw them in a phase diagram with $p_t$ on the y-axis and $e_t$ on the x-axis. Complete the phase diagram indicating the motion of the system (using the conditions for $e_{t+1} - e_t \geq 0$ and $p_{t+1} - p_t \geq 0$).

Finally, add a line to the diagram that obeys a ‘no-arbitrage condition’ as mandated by UIP: $p_t - \bar{p} = -\hat{\theta}(e_t - \bar{e})$ for some $\hat{\theta} > 0$. (The $\hat{\theta}$ is not quite the same as in the original Dornbusch model since we have no uncertainty here.)

c) Is the steady-state stable? If not, what is the unique stable (“saddle”) path given a steady-state of $\bar{p}$ and $\bar{e}$?

d) Suppose all variables except for $p_t$ respond immediately to a monetary shock.

What is the new steady-state? Draw a new ‘no-arbitrage’ line.

What happens to $e_t$ right after a reduction in the monetary base from $\bar{m}$ to $\bar{m}'$? How do $e_{t+s}$ and $p_{t+s}$ evolve over time?

### 3 Dynamics of the simple model of overshooting

Consider the simple model of overshooting from question 2 again:

\[ \bar{m} - p_t = \phi y^d_t - \eta(e_{t+1} - e_t), \]
We are interested in the behavior of the system around the steady-state.

\[ y^d_t = \delta(e_t - p_t) \quad \delta \in (0, \frac{1}{\phi}), \quad (14) \]

\[ p_{t+1} - p_t = \pi(y^d_t - \bar{y}). \quad (15) \]

a) Using (14) and the steady-states of \( \bar{e} \) and \( \bar{p} \) that you found in question 2a), express \( y^d_t - \bar{y} \) as a function of \( e_t - \bar{e} \) and \( p_t - \bar{p} \).

b) Using (13) along with the results in 2a) and 3a), express \( e_{t+1} - \bar{e} \) as a function of \( \frac{1}{\eta}(e_t - \bar{e}) \) and \( \frac{1}{\eta}(p_t - \bar{p}) \).

c) Using (14) and (15) along with the results in 2a), express \( p_{t+1} - \bar{p} \) as a weighted sum of \( e_t - \bar{e} \) and \( p_t - \bar{p} \).

From now on, assume that \( \pi = \frac{1}{\eta} \) and \( \phi = 3 \) for simplicity.

d) Write your findings from 3b) and 3c) into a system of two difference equations that takes the form

\[
\begin{pmatrix}
    e_{t+1} - \bar{e} \\
    p_{t+1} - \bar{p}
\end{pmatrix} = A \cdot 
\begin{pmatrix}
    e_t - \bar{e} \\
    p_t - \bar{p}
\end{pmatrix}.
\]

(16)

Find the eigenvalues and eigenvectors of the system (you may, of course, simply plug your previous results into the formulas from section.)

[Hint: An intermediate result is \( \text{tr}(A) = \frac{2(\delta + \eta)}{\eta} \) and \( \det(A) = 1 - \frac{(1 - 2\eta)\delta}{\eta^2} \).]

e) Is the system stable? That is, do the exchange rate and price levels converge to the steady-state?

If not, set the coefficient of the unstable root to zero. Then, using the simplified system (16) and the results from 2a), express \( p_{t+1} \) as a function of \( e_{t+1} \) and exogenous variables. Draw this function into the phase diagram from question 2 (Remember that \( \delta < \frac{1}{\phi} \).)

What did you just find? What is the intuition for the fact that the economy obeys this relationship?