Econ 202a Spring 2000
Marc Muendler (TA)

Problem Set 1: Suggested Solutions

1 Question 1

We have to find the autocovariance function for the stationary AR(2) process

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t, \tag{1}$$

where ϵ_t obeys our usual assumptions $\mathbb{E}[\epsilon_t] = 0$, $\mathbb{E}[\epsilon_t^2] = \sigma^2 \ \forall t$, and $\mathbb{E}[\epsilon_t \epsilon_{t-s}] = 0 \ \forall t$ and $s \neq 0$. By definition, the autocovariance is

$$\gamma(s) \equiv \mathbb{E}\left[(y_t - \mathbb{E}[y_t])\,(y_{t-s} - \mathbb{E}[y_{t-s}])\right], \tag{2}$$

and can be viewed as a function of s. Taking expectations of (1) and using the fact that $\mathbb{E}[y_t] = const.$ by stationarity, we can conclude that $\mathbb{E}[y_t] = 0$ in the case of (1). So, the autocovariance function (2) simplifies to $\gamma(s) = \mathbb{E}[y_t y_{t-s}]$ here.

Multiplying (1) by y_{t-s} on both sides and taking expectations, we find that

$$\begin{aligned} \gamma(s) &= \mathbb{E}[y_t y_{t-s}] = \phi_1 \mathbb{E}[y_{t-1} y_{t-s}] + \phi_2 \mathbb{E}[y_{t-2} y_{t-s}] \\ &= \phi_1 \gamma(s-1) + \phi_2 \gamma(s-2) \quad \text{where } s \geq 1 \end{aligned} \tag{3}$$

for an AR(2) process. Thus, the autocovariance function of an AR(2) process follows a homogeneous second-order difference equation. To solve this difference equation, we could use the steps from section (1/25 and 1/27). (For a derivation, see section 1.3 at the end of the answer to this question.) But we actually need not go through all that to find the values of $\gamma(s)$ recursively.

No matter whether we solve the difference condition in general terms or simply want to use it recursively, we will always need two initial conditions to determine $\gamma(0)$ and $\gamma(1)$. There are many ways to obtain $\gamma(0)$ and $\gamma(1)$, three of them are considered here. First, the straight method of constructing an equation system. Second, a more elegant derivation that switches back and forth between the autocovariance and the autocorrelation function. And third, the precise solution to the second-order difference equation.

1.1 Equation system in $\gamma(0)$, $\gamma(1)$, and $\gamma(2)$

Start with $\gamma(0)$. Multiply both sides of (1) by y_t and take expectations to obtain

$$\begin{aligned} \gamma(0) &= \mathbb{E}\left[(y_t)^2\right] = \phi_1 \mathbb{E}\left[y_t y_{t-1}\right] + \phi_2 \mathbb{E}\left[y_t y_{t-2}\right] + \mathbb{E}\left[\epsilon_t y_t\right] \\ &= \phi_1 \gamma(-1) + \phi_2 \gamma(-2) + \mathbb{E}\left[(\epsilon_t)^2\right] \end{aligned}$$

or

$$\gamma(0) = \phi_1 \gamma(1) + \phi_2 \gamma(2) + \sigma^2. \tag{4}$$

For the last step, we made use of the symmetry property of the autocovariance function, $\gamma(-s) = \gamma(s)$. Applying (3) for $s = 1$ and $s = 2$ (and making use of the symmetry property again), we find

$$\gamma(1) = \phi_1 \gamma(0) + \phi_2 \gamma(1) \tag{5}$$

and

$$\gamma(2) = \phi_1 \gamma(1) + \phi_2 \gamma(0). \tag{6}$$

Together, (4), (5) and (6) form a system of three linear equations in three unknowns. Solving it yields the desired initial conditions.

1.2 Derivation of $\gamma(s)$ via $\rho(s)$

The second way is a little more elegant. It involves the autocorrelation function, too, which we are asked to derive anyway. By definition, the autocorrelation function is

$$\rho(0) \equiv 1 \quad \text{and} \quad \rho(s) \equiv \frac{\gamma(s)}{\gamma(0)} \text{ for } s \geq 1.$$

Thus, the difference equation (3) that applied to the autocovariance function, carries over to the autocorrelation function, and

$$\rho(s) = \phi_1 \rho(s-1) + \phi_2 \rho(s-2) \quad \text{where } s \geq 1. \tag{7}$$

For $s = 1$, this becomes

$$\rho(1) = \phi_1 \rho(0) + \phi_2 \rho(1) = \phi_1 + \phi_2 \rho(1) = \frac{\phi_1}{1-\phi_2}. \tag{8}$$

Once we have the two starting values $\rho(0) = 1$ and $\rho(1) = \frac{\phi_1}{1-\phi_2}$, it is quite straight forward to solve for $\rho(2)$, $\rho(3)$, and so forth. We can simply apply the difference equation of the autocorrelation function (7) again and again:

$$\begin{aligned} \rho(2) &= \phi_1\rho(1) + \phi_2\rho(0) = \frac{{\phi_1}^2 + \phi_2(1-\phi_2)}{1-\phi_2}, \\ \rho(3) &= \phi_1\rho(2) + \phi_2\rho(1) = \frac{{\phi_1}^3 + \phi_1\phi_2(2-\phi_2)}{1-\phi_2}, \\ &\vdots \end{aligned} \tag{9}$$

With this at hand, we can derive $\gamma(0)$ as

$$\begin{aligned} \gamma(0) &= \phi_1\gamma(1) + \phi_2\gamma(2) + \sigma^2 = \phi_1\rho(1)\gamma(0) + \phi_2\rho(2)\gamma(0) + \sigma^2 \\ &= \frac{1}{1-\phi_1\rho(1)-\phi_2\rho(2)}\sigma^2 = \frac{(1-\phi_2)}{\Phi}\sigma^2, \end{aligned} \tag{10}$$

where $\Phi \equiv (1+\phi_2)\left[(1-\phi_2)^2 - {\phi_1}^2\right]$. Then, by (5)

$$\gamma(1) = \frac{\phi_1}{\Phi}\sigma^2. \tag{11}$$

Finally, we can apply the difference equation of the autocovariance function (3) again and again, and find:

$$\begin{aligned} \gamma(2) &= \phi_1\gamma(1) + \phi_2\gamma(0) = \frac{{\phi_1}^2 + \phi_2(1-\phi_2)}{\Phi}\sigma^2, \\ \gamma(3) &= \phi_1\gamma(2) + \phi_2\gamma(1) = \frac{{\phi_1}^3 + \phi_1\phi_2(2-\phi_2)}{\Phi}\sigma^2, \\ &\vdots \end{aligned} \tag{12}$$

So, we have completely described the evolution of the autocovariance function without having to solve the difference equation explicitly.

1.3 Solving the difference equation $\gamma(s)$

If you love precision, you could make your life substantially more complicated and solve the second-order difference equation of the autocovariance function (3) explicitly. This was not required, but you may still find it interesting to see how the solution can be derived. (The solution itself will be ugly, so if you don't want to be disappointed, stop reading one paragraph before the final answer. If you don't care for the general solution, skip the rest of the answer to question 1 right now.) For the general solution, let's apply the three-step procedure as outlined in section.

First, transform the second-order difference equation into a two-variable system of order one:

$$\begin{pmatrix} \gamma_s \\ \gamma_{s-1} \end{pmatrix} = \begin{pmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{s-1} \\ \gamma_{s-2} \end{pmatrix} \equiv \mathbf{F} \cdot \begin{pmatrix} \gamma_{s-1} \\ \gamma_{s-2} \end{pmatrix}. \tag{13}$$

The two eigenvalues of $\mathbf{F}$ satisfy $\det(\mathbf{F} - \lambda \mathbf{I_2}) = 0$ and are thus

$$\lambda_{1,2} = \frac{\operatorname{tr}(\mathbf{F})}{2} \pm \frac{1}{2}\sqrt{\operatorname{tr}(\mathbf{F})^2 - 4\det(\mathbf{F})} = \frac{\phi_1}{2} \pm \sqrt{\left(\frac{\phi_1}{2}\right)^2 + \phi_2}, \tag{14}$$

or, written into a matrix,

$$\Lambda \equiv \begin{pmatrix} \frac{\phi_1}{2} + \sqrt{\left(\frac{\phi_1}{2}\right)^2 + \phi_2} & 0 \\ 0 & \frac{\phi_1}{2} - \sqrt{\left(\frac{\phi_1}{2}\right)^2 + \phi_2} \end{pmatrix}.$$

Second, the according eigenvectors must satisfy the relationship $\mathbf{F}\mathbf{e}_i = \lambda_i \mathbf{e}_i$ for each eigenvalue (by definition). Standardizing the eigenvectors to $\mathbf{e}_i = (e_i, 1)^T$ allows one to obtain

$$e_i = \frac{\lambda_i - \mathbf{F}_{22}}{\mathbf{F}_{21}} = \lambda_i \quad \text{and} \quad \mathbf{P} \equiv (\mathbf{e_1}, \mathbf{e_2}) = \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix}.$$

Third, using the fact that $\mathbf{FP} = \mathbf{P}\Lambda$ or $\mathbf{F} = \mathbf{P}\Lambda\mathbf{P^{-1}}$, we can substitute $\mathbf{P}\Lambda\mathbf{P^{-1}}$ for $\mathbf{F}$ in (13), pre-multiply both sides of (13) by $\mathbf{P^{-1}}$ and obtain a 'decoupled' system of two independent first-order difference equations. We know the solution of such homogeneous first-order difference equations to be of the form $x_t = c(\lambda)^t$ for a certain initial value $x_0 = c$ or a boundary condition. Hence, pre-multiplying our decoupled system with $\mathbf{P}$ again, the solution to (13) must be

$$\begin{pmatrix} \gamma_s \\ \gamma_{s-1} \end{pmatrix} = \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} c_1(\lambda_1)^s \\ c_2(\lambda_2)^s \end{pmatrix}$$

or

$$\gamma_s = c_1(\lambda_1)^{s+1} + c_2(\lambda_2)^{s+1}. \tag{15}$$

Everything in (15) is known except for the two coefficients c_1 and c_2. To find the exact solution that satisfies all of our assumptions on the autocovariance function, we must formulate two boundary conditions. These boundary conditions will determine the two coefficients that are left over, c_1 and c_2. One boundary condition comes from the general requirement that $\gamma_s = \gamma_{-s}$. In particular, the difference equation of the autocovariance function must 'turn around' at γ_0, so that $\gamma_1 = \gamma_{-1}$. Let's work with this. By (15), $\gamma_1 = c_1(\lambda_1)^2 + c_2(\lambda_2)^2$ and $\gamma_{-1} = c_1 + c_2$. Hence, the first boundary condition requires that

$$c_1(\lambda_1)^2 + c_2(\lambda_2)^2 = c_1 + c_2. \tag{16}$$

The other boundary condition comes from the special case of γ_0 which must equal $\gamma_0 = \phi_1\gamma_1 + \phi_2\gamma_2 + \sigma^2$ as we saw above in (4). From (15) we know that $\gamma_0 = c_1\lambda_1 + c_2\lambda_2$, $\gamma_1 = \gamma_{-1} = c_1 + c_2$, and $\gamma_2 = c_1(\lambda_1)^3 + c_2(\lambda_2)^3$. So, the second boundary condition requires that

$$c_1\lambda_1 + c_2\lambda_2 = \phi_1(c_1 + c_2) + \phi_2(c_1\lambda_1^3 + c_2\lambda_2^3) + \sigma^2. \tag{17}$$

Taken together, the two initial conditions are a system of two linear equations in the two unknowns c_1 and c_2. (Remember that we know the two eigenvalues λ_1 and λ_2 already from above, (14), and remember to stop reading now if you don't want to be disappointed with the ugliness of the solution.)

Solving the two equations for c_1 and c_2 yields

$$c_1 = -\frac{(1 - (\lambda_2)^2)\,\sigma^2}{\Gamma} \quad \text{and} \quad c_2 = \frac{(1 - (\lambda_1)^2)\,\sigma^2}{\Gamma}, \tag{18}$$

where $\Gamma \equiv (1 - {\lambda_1}^2)(\lambda_2 - \phi_1 - \phi_2{\lambda_2}^3) - (1 - {\lambda_2}^2)(\lambda_1 - \phi_1 - \phi_2{\lambda_1}^3)$. These are quite unpleasant expressions. Handing the problem over to a mathematical software package at this point reduces our frustration, resolves the simplification of terms, and helps us find some even more unfriendly terms after plugging the eigenvalues λ_1 and λ_2 (14) into (18). Since the resulting terms for c_1 and c_2 are all ugly, ugly, ugly, the general solution does not become much prettier, but we obtain it! It is, after a round of simplifications (and a movie would play a muted fanfare at this point):

$$\gamma_s = \frac{1}{\Phi}\frac{X}{\sqrt{\left(\frac{\phi_1}{2}\right)^2+\phi_2}}\sigma^2 \quad \text{for } s \geq 0,$$

where $\Phi \equiv (1+\phi_2)\left[(1-\phi_2)^2 - {\phi_1}^2\right]$ as in the quasi-solution (12) before (see section 1.2) and

$$\begin{aligned} X \;\equiv\;& (1-\phi_2)\sqrt{\phi_1^2+4\phi_2}\,\times \\ & \times\left[\left(\frac{\phi_1}{2}-\sqrt{\left(\frac{\phi_1}{2}\right)^2+\phi_2}\right)^s + \left(\frac{\phi_1}{2}+\sqrt{\left(\frac{\phi_1}{2}\right)^2+\phi_2}\right)^s\right] \\ & -\phi_1(1+\phi_2)\left[\left(\frac{\phi_1}{2}-\sqrt{\left(\frac{\phi_1}{2}\right)^2+\phi_2}\right)^s - \left(\frac{\phi_1}{2}+\sqrt{\left(\frac{\phi_1}{2}\right)^2+\phi_2}\right)^s\right]. \end{aligned}$$

2 Question 2

An MA(2) process takes the form

$$y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}, \tag{19}$$

with the usual conditions on ϵ_t. Before we proceed to specific values for the coefficients, let's derive the autocorrelation function $\rho(s) \equiv \gamma(s)/\gamma(0)$ for an MA(2) process in general terms. For this, it is most convenient to first find the autocovariance function. Note that $\mathbb{E}[y_t] = \mu$. So, the autocovariance of y_t becomes

$$\begin{aligned} \gamma(s) &\equiv \mathbb{E}\left[(y_t - \mathbb{E}[y_t])\,(y_{t-s} - \mathbb{E}[y_{t-s}])\right] \\ &= \mathbb{E}\left[(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2})\,(\epsilon_t + \theta_1\epsilon_{t-s-1} + \theta_2\epsilon_{t-s-2})\right] \end{aligned} \tag{20}$$

in the case of an MA(2) process.

First apply this definition (20) to the variance of y_t, i.e. to $s = 0$:

$$\begin{aligned} \gamma(0) &= \mathbb{E}\left[(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2})^2\right] \\ &= \mathbb{E}\left[\epsilon_t^2 + \theta_1^2\epsilon_{t-1}^2 + \theta_2^2\epsilon_{t-2}^2 + 2\theta_1\epsilon_t\epsilon_{t-1} + 2\theta_1\theta_2\epsilon_{t-1}\epsilon_{t-2} + 2\theta_2\epsilon_t\epsilon_{t-2}\right] \\ &= \left(1+\theta_1^2+\theta_2^2\right)\mathbb{E}\left[\epsilon_t^2\right] = \left(1+\theta_1^2+\theta_2^2\right)\sigma^2. \end{aligned}$$

The cancellations are possible because all covariances of the ϵ_t's with their own lagged realizations are zero by assumption. Similarly, for $s = 1$ and $s = 2$ we obtain

$$\begin{aligned}
\gamma(1) &= \mathbb{E}\left[\left(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}\right)\left(\epsilon_{t-1} + \theta_1\epsilon_{t-2} + \theta_2\epsilon_{t-3}\right)\right] \\
&= \theta_1\mathbb{E}\left[\epsilon_{t-1}^2\right] + \theta_1\theta_2\mathbb{E}\left[\epsilon_{t-2}^2\right] = \theta_1\left(1+\theta_2\right)\sigma^2
\end{aligned}$$

and

$$\begin{aligned}
\gamma(2) &= \mathbb{E}\left[\left(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}\right)\left(\epsilon_{t-2} + \theta_1\epsilon_{t-3} + \theta_2\epsilon_{t-4}\right)\right] \\
&= \theta_2\mathbb{E}\left[\epsilon_{t-2}^2\right] = \theta_2\sigma^2,
\end{aligned}$$

respectively. From then on, the autocovariance function remains absolutely inactive: $\gamma(s) = 0$ for $s \geq 3$. Equipped with these insights, the autocorrelation function is easily derived:

$$\begin{aligned}
\rho(0) &= \gamma(0)/\gamma(0) = 1, \\
\rho(1) &= \frac{\gamma(1)}{\gamma(0)} = \frac{\theta_1\left(1+\theta_2\right)}{1+\theta_1^2+\theta_2^2} = \frac{\frac{2}{5}\frac{4}{5}}{1+\left(\frac{2}{5}\right)^2+\left(\frac{1}{5}\right)^2} = \frac{4}{15}, \\
\rho(2) &= \frac{\gamma(2)}{\gamma(0)} = \frac{\theta_2}{1+\theta_1^2+\theta_2^2} = \frac{-\frac{1}{5}}{1+\left(\frac{2}{5}\right)^2+\left(\frac{1}{5}\right)^2} = -\frac{1}{6}, \\
\rho(s) &= 0 \quad \text{for } s \geq 3,
\end{aligned}$$

where $\theta_1 = \frac{2}{5}$ and $\theta_2 = -\frac{1}{5}$.

3 Question 3

From the solution in question 1 (section 1.2) we know several facts about the autocorrelation function of an AR(2) process. For one, the correlation of any y_t with itself must always be equal to one, $\rho(0) = 1$. Then, as we derived in section 1.2, the autocorrelation of y_t with its first lag y_{t-1} is $\rho(1) = \phi_1/(1-\phi_2)$ by (8). Together with the second-order difference equation (7)

$$\rho(s) = \phi_1\rho(s-1) + \phi_2\rho(s-2) \quad \text{where } s \geq 1$$

we can calculate $\rho(s)$ for every lag s once we know ϕ_1 and ϕ_2. (We need not know the variance of the underlying white-noise process ϵ_t because we got rid of the variance by standardizing $\rho(s) = \gamma(s)/\gamma(0)$.) We can now calculate $\rho(s)$ step by step, starting with $s = 2$ and gradually proceeding to $s = 5$. The following table summarizes the results.

$s =$	$(1 - \phi_2)\rho(s) =$	$\rho(s)$ for $\varphi_1 = .6$, $\varphi_2 = -.2$	$\rho(s)$ for $\varphi_1 = -.6$, $\varphi_2 = .2$
0	$1 - \phi_2$	1.0000	1.0000
1	ϕ_1	0.5000	−0.7500
2	$\phi_1^2 + \phi_2(1 - \phi_2)$	0.1000	0.6500
3	$\phi_1^3 + \phi_1\phi_2(2 - \phi_2)$	−0.0400	−0.5400
4	$\phi_1^4 + \phi_1^2\phi_2(3 - \phi_2) + \phi_2^2(1 - \phi_2)$	−0.0440	0.4540
5	$\phi_1^5 + \phi_1^3\phi_2(4 - \phi_2) + \phi_1\phi_2^2(3 - 2\phi_2)$	−0.0184	−0.3804

The autocorrelation dies out faster when φ_1 is positive. That is, a negative φ_1 causes more persistence in correlation than a positive φ_1 of the same magnitude. The cycling is 'less rapid' in the case of a positive φ_1, whereas the sign alters every period when φ_1 is negative. (These patterns prevail for longer lags than 5, too.)

4 Question 4

Any ARMA(p,q) process can be rewritten in lag operators as

$$\left(1 - \phi_1\mathbb{L} - \phi_2\mathbb{L}^2 - \ldots - \phi_p\mathbb{L}^p\right) y_t = \left(1 + \theta_1\mathbb{L} + \theta_2\mathbb{L}^2 + \ldots + \theta_q\mathbb{L}^q\right) \epsilon_t. \quad (21)$$

Each of the two sums in lag-operators can be viewed as closely related to a certain polynomial. For an AR(p) process, this polynomial is

$$1 - \phi_1 z - \phi_2 z^{-2} - \ldots - \phi_p z^{-p} = 0,$$

and for an MA(q) the polynomial is

$$1 + \theta_1 z + \theta_2 z^{-1} + \ldots + \theta_q z^{-q} = 0.$$

Multiplying the former polynomial by z^p and the latter by z^q, we obtain the so-called *associated* polynomials. The *associated* polynomial for an AR(p) process thus becomes

$$z^p - \phi_1 z^{p-1} - \phi_2 z^{p-2} - \ldots - \phi_p = 0. \quad (22)$$

It is of order p. Similarly, for an MA(q) the associated polynomial is of order q:

$$z^q + \theta_1 z^{q-1} + \theta_2 z^{q-2} + \ldots + \theta_q = 0. \tag{23}$$

By the Fundamental Theorem of Algebra, any finite polynomial of order n can be factored into (up to) n factors that involve the n, possibly not distinct roots of the polynomial. As shown in section (1/25 and 1/27), the p characteristic roots of an AR(p) process are equivalent to the eigenvalues of the corresponding difference equation. Similarly, the q characteristic roots of an MA(q) process are equivalent to the eigenvalues of the difference equation that they obey. Therefore, the ARMA(p,q) process in (21) can be rewritten as

$$(1-\lambda_1\mathbb{L})(1-\lambda_2\mathbb{L})\cdots(1-\lambda_p\mathbb{L})\, y_t = (1-\omega_1\mathbb{L})(1-\omega_2\mathbb{L})\cdots(1-\omega_q\mathbb{L})\, \epsilon_t, \tag{24}$$

where $\lambda_1, \ldots, \lambda_p$ are the p (possibly non-distinct) eigenvalues of the AR(p) process and the $\omega_1, \ldots, \omega_q$ are the q (possibly non-distinct) eigenvalues of the MA(q) process. If there is any pair of identical eigenvalues in the two processes, $\omega_i = \lambda_j$ say, then the ARMA(p,q) process is *overparametrized*. By simply cancelling the according factors on both sides of (24) we can reduce it to an ARMA ($p-1$,$q-1$) process.

Let's consider the concrete example of the question where an ARMA(2,2) process has parameters $\phi_1 = -\frac{1}{5}$, $\phi_2 = \frac{12}{25}$, $\theta_1 = \frac{3}{5}$, $\theta_2 = -\frac{4}{25}$. So, the ARMA(2,2) process can be written as

$$\left(1 + \frac{1}{5}\mathbb{L} - \frac{12}{25}\mathbb{L}^2\right) y_t = \left(1 + \frac{3}{5}\mathbb{L} - \frac{4}{25}\mathbb{L}^2\right) \epsilon_t.$$

The eigenvalues of the AR(2) process are equal to the roots of the associated polynomial $z^2 + \frac{1}{5}z - \frac{12}{25} = 0$, which are $z_1 = \frac{3}{5}$ and $z_2 = -\frac{4}{5}$. Similarly, the eigenvalues of the MA(2) process are nothing but the roots of the associated polynomial $z^2 + \frac{3}{5}z - \frac{4}{25} = 0$, which are $z_1 = \frac{1}{5}$ and $z_2 = -\frac{4}{5}$. Therefore, the process can be rewritten in factored form as

$$\left(1 - \frac{3}{5}\mathbb{L}\right)\left(1 + \frac{4}{5}\right)\mathbb{L} y_t = \left(1 - \frac{1}{5}\mathbb{L}\right)\left(1 + \frac{4}{5}\mathbb{L}\right)\epsilon_t$$

or

$$\left(1 - \frac{3}{5}\mathbb{L}\right) y_t = \left(1 - \frac{1}{5}\mathbb{L}\right) \epsilon_t.$$

This is an ARMA(1,1) process with $y_t = \frac{3}{5}y_{t-1} + \epsilon_t - \frac{1}{5}\epsilon_{t-1}$. The process was overparametrized as an ARMA(2,2) process, and is now correctly parametrized.