

The Possibility of Informationally Efficient Markets*

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Abstract

A rational-expectations equilibrium with positive demand for financial information does exist under fully revealing asset price—contrary to a wide-held conjecture. Whereas a continuum of investors is inconsistent with fully revealing equilibrium, finitely many investors with average portfolios demand information in equilibrium if they can adjust portfolio size in an additive-signal return model. More information diminishes the expected excess return of a risky asset so that investors who only have a choice of portfolio composition or whose asset endowments strongly differ from the average portfolio are worse off. Under fully revealing price, information market equilibria both with and without information acquisition are Pareto efficient. JEL D82, D83, G14

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Contrary to a commonplace conjecture, a unique equilibrium with strictly positive demand for financial information does exist under fully revealing asset price—just as demand for public goods in other economic contexts is positive albeit not necessarily socially optimal. Existence merely requires a finite number of investors.

An equilibrium is called fully revealing if every investor can infer a sufficient statistic of all other investors' information from asset price. Similar to other benchmarks in economics—such as perfect competition, complete asset markets, the welfare theorems, or perfect foresight—, the benchmark case of a fully revealing asset price may not be applicable in general but it is instructive.

General equilibrium theory often benefits from the use of a continuum of agents to establish equilibrium existence. In contrast, a continuum of agents is irreconcilable with information acquisition in a fully revealing financial market equilibrium. Fully revealing equilibrium means by its statistical definition that every investor's information enters a sufficient statistic inferrable from price, irrespective of the total number of agents. The presence of infinitely many investors, however, conflicts with the fully revealing nature of price in that the marginal investor's information has no, rather than the statistically required full, impact on asset price.

This article establishes conditions for information acquisition when asset price is fully revealing. An intertemporal decision is crucial for the value of information. When private information turns public under fully revealing price, it diminishes the expected excess return and results in a welfare loss. The information market equilibrium is efficient in an *ex ante* Pareto sense, however, both when investors choose to acquire information and when they don't.

Consider an *additive signal-return model*.

Assumption 1 (Additive signal-return model). *The gross asset return θ of a risky asset is the sum of a fundamental S , which can become fully known through the signal realization s , and independent noise ε :*

$$\theta = S + \varepsilon. \tag{1}$$

Strands of research that explicitly use the additive signal-return model include, for instance, those on information acquisition (Grossman and Stiglitz 1980), disclosure (Diamond 1985), delegated portfolio management (Bhattacharya and Pfleiderer 1985), or currency attacks (Morris and Shin 1998).

Together with assumption 1, let two further assumptions completely characterize the class of exchange economies.

Assumption 2 (Common CARA). *Investors evaluate portfolios with intertemporally additive von Neumann-Morgenstern utility and share an identical degree of constant absolute risk aversion (CARA).*

Assumption 3 (Resilience of risk free interest rate). *The equilibrium return of risk free bonds responds negligibly little to signal realizations on risky asset returns.*

Assumption 3 ensures that the economy's general equilibrium has a tractable closed-form solution. The assumption is equivalent to the limiting case where markets for individual stocks are small relative to the overall market for risk free bonds. This article considers both an infinite number of investors and an arbitrarily large finite number of investors. An information equilibrium exists only if there are finitely many individuals.

For simplicity, let signal S and noise ε be normally distributed as in Grossman and Stiglitz (1980) and let ε have zero mean. Normality does not compromise generality in additive signal-return models under CARA.¹

An Illustration. Consider two assets, a bond B with certain gross return R and a stock X with risky return θ . The stock sells at price P . CARA utility gives rise to an asset demand function $X(RP)$. So, it is convenient to refer to RP as the asset price in opportunity cost terms (of holding a bond). In additive signal-return models with CARA utility, demand for the stock is zero if opportunity cost RP equals the expected return $\mathbb{E}[\theta]$. As opportunity cost RP falls below $\mathbb{E}[\theta]$, an investor demands more and more of the stock. Figure 1 depicts the resulting demand schedule $X(RP)$ with a dashed curve.

An informed investor gets to observe realization s of the signal S . With fully revealing price, there can only be two cases. Either no one acquires the signal S . Or one investor acquires the signal S and everyone gets to know the signal realization s by the statistical definition of fully revealing price. An investor who anticipated not to act on realization s would not acquire signal S (the entitlement to receive s) in the first place. By analogy, fully revealing

¹Theorems in this article generalize to distribution functions with a CARA-consistent moment generating function (Muendler 2004). Appendix C proves Theorem 4 in general terms, of which Theorem 2 is a special case for $R = 0$.

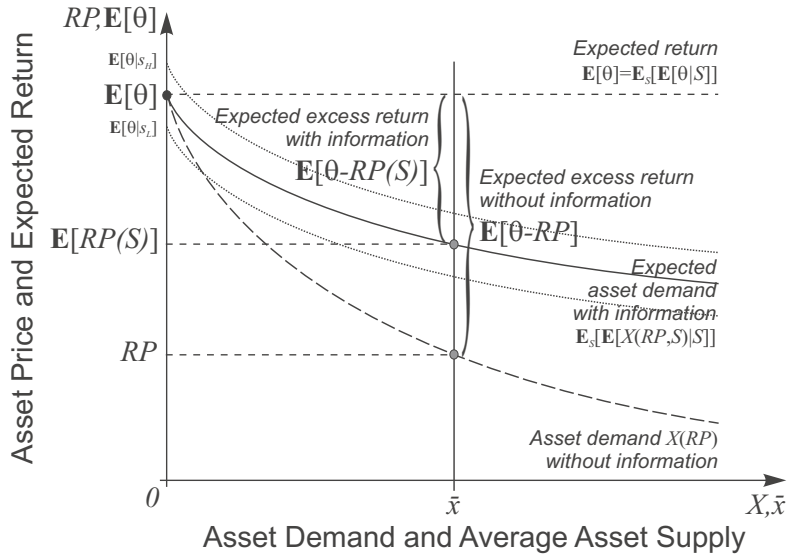


Figure 1: **Information, Asset Demand and Expected Excess Return**

price gives every investor the choice to either push an information button and broadcast the signal realization to everyone or, alternatively, to keep everyone uninformed.² So, equilibrium price $P(s)$ depends on the signal realization if at least one investor acquires the signal, otherwise P is independent of S .

In additive signal-return models with CARA utility and identical risk preferences, the unique equilibrium allocation of the stock is symmetric irrespective of the investor's wealth. No matter whether or not information is acquired, every investor ends up with the average amount of the stock \bar{x} in her portfolio. Suppose no investor acquires the signal. Then the unique asset-market equilibrium results in an opportunity cost RP as depicted in figure 1.

When, on the other hand, one investor acquires the signal S , a message with the signal realization s goes out to everyone before portfolios are chosen. Should an investor acquire the signal S in the first place? *Post notitiam* (after the signal realization), price may go up in response to a good realization (s_H) or go down in response to bad news (s_L). Figure 1 shows these possibilities as dotted curves around the expected demand schedule (the solid curve). *Ante notitiam* (before the signal realization), however, the intercept of the demand

²Joel Watson pointed out this analogy.

curve stays put since, by the law of iterated expectations, the expected return remains at $\mathbb{E}_S [\mathbb{E} [\theta|S]] = \mathbb{E} [\theta]$.

Information (the ability to condition on s) reduces the risk *post notitiam*. So, for any given opportunity cost $RP(s)$, asset demand will be higher. This results in an upward turn of the demand curve. Figure 1 depicts the resulting expected demand schedule $\mathbb{E}_S [\mathbb{E} [X(RP(S), S)|S]]$ as a solid curve. The expected new equilibrium price is $\mathbb{E} [RP(S)] > RP$.³ The asset's expected excess return over opportunity cost drops, falling from $\mathbb{E} [\theta - RP(S)]$ to $\mathbb{E} [\theta - RP]$. Although the stock loses attractiveness relative to the bond, the investor will still have to put \bar{x} in her portfolio since the unique equilibrium is symmetric both with and without information. This strictly worsens her utility *ante notitiam*. But there are two benefits of information.

First, an investor who has information scheduled to arrive anticipates to reshuffle the asset composition of her portfolio (of given size). This reduces the expected variance of her future consumption *ante notitiam* (by variance decomposition $\mathbb{E}_S [\mathbb{V} (\theta|S)] = \mathbb{V} (\theta) - \mathbb{V}_S (\mathbb{E} [\theta|S])$). As it turns out in additive signal-return models with CARA utility, the diminishing expected excess return wipes out the benefits from improved asset composition. So, no signal will be acquired in equilibrium, and the absence of information is efficient in a Pareto sense.

Second, an investor who has information scheduled to arrive also anticipates to adjust her consumption path and the size of her portfolio. This benefit from improved intertemporal choice outweighs the costs from diminishing expected excess returns for an investor with a 'market endowment' of stocks. So, when investors are allowed to change their portfolio size in response to information, in addition to their portfolio composition, then there is a joint competitive equilibrium in asset and information markets under fully revealing price in which one, and only one, investor with close-to-average initial stock holdings acquires the signal. This equilibrium too is efficient in a Pareto sense.⁴

The remainder of this paper is organized as follows. Section 1 reviews equilibrium conjectures for information demand under fully revealing asset

³Veldkamp (2004) shows that information also raises the asset price in the Grossman and Stiglitz (1980) model with exogenous noise in price.

⁴Froot, Scharfstein and Stein (1992) find for a market with liquidity traders that rational investors can make themselves better off by inducing others in the market to act on the same information as they have. The present model shows that these incentives exist more generally for rational investors.

price. Section 2 elaborates the model and establishes its fully revealing financial market equilibrium (under assumptions 1 through 3). In following Grossman and Stiglitz (1980), section 3 derives the information market equilibrium when investors only have a choice between assets. A unique equilibrium exists but is one with no information. Section 4 presents the reason for lacking information demand: more information diminishes the excess return of the risky asset over its opportunity costs. Section 5 revisits the information market equilibrium when investors can condition their intertemporal savings decision on the signal realization and shows that the unique type of equilibrium is either one with or one without information acquisition. Section 6 concludes.

1 Equilibrium Conjectures in the Literature

There is an extensive literature on the generic existence of a rational expectations equilibrium with fully revealing price (e.g. Radner 1979, Jordan 1983, Citanna and Villanacci 2000, Reny and Perry 2003). Even when investors are not fully rational and update conditional on past price, or when they follow inaccurate prediction rules, price can converge to a fully revealing state over time (e.g. Vives 1995, Sandroni 2000). These strands of the literature consider the arrival of information as exogenous, however, and stop short of investigating the incentives for investors to acquire information in the first place.

Grossman and Stiglitz (1976) first outlined and in Grossman and Stiglitz (1980) later formulated the following no-equilibrium paradox for financial markets (Grossman and Stiglitz 1980, conjecture 6):

In the limit, when there is no [exogenous] noise [in prices], prices convey all information, and there is no incentive to purchase information. Hence, the only possible equilibrium is one with no information. But if everybody is uninformed, it clearly pays some individual to become informed. Thus, there does not exist a competitive equilibrium.

There are numerous more instances of this ‘no-equilibrium conjecture’ in the literature. Froot et al. (1992) argue, for instance, that in the absence of liquidity traders “prices would reveal all the information in the economy, so there would be no return to becoming informed.” Barlevy and Veronesi

(2000) remark: “Finally, as Grossman and Stiglitz point out, we need to prevent prices from being fully revealing; otherwise an equilibrium will fail to exist.” O’Hara (2003) states: “If the equilibrium is fully revealing, then the uninformed learn the information from the equilibrium price ... [T]he informed have no incentive to gather information.”

Approaches to overcome the no-equilibrium paradox under fully revealing prices include Jackson (1991) with price setting investors, or Jackson and Peck (1999) with investors who submit demand functions in a Shapley-Shubik fashion. Routledge (1999) considers adaptive learning from past price so that investors cannot condition on current price. Pesendorfer and Swinkels (2000) show for commodity auctions that equilibrium price is fully revealing when the number of bidders is finite. To my knowledge, the no-equilibrium conjecture has so far not been reconsidered in the original Walrasian financial market equilibrium with rational expectations. Grossman and Stiglitz (1980) posed the ‘no-equilibrium’ conjecture in the context of infinitely many investors. An arbitrarily large but finite number of investors suffices for existence of a fully revealing equilibrium with well-defined information demand.

The ‘no-equilibrium conjecture’ lent support to the claim that financial markets could, by their mere logic, never be informationally efficient in a fully revealing sense. Admati (1991) summarizes this view succinctly: “[T]his impossibility result is important since it examines the theoretical and conceptual underpinnings of the frequently used notion of *efficient financial markets*. It ... shows that under some conditions it is logically impossible for financial markets to be efficient in the ‘strong’ sense that they reflect all the information in the market” (Admati’s emphasis).

While there are empirical reasons why financial markets may not be informationally inefficient, this paper argues that the sources of these inefficiencies are more subtle than outright theoretical impossibility. If the unique equilibrium is one with no information as in section 3 (choice of assets given portfolio size), the outcome is efficient in the sense that a social planner would also allocate no information. Conversely, when the unique equilibrium entails positive information as in section 5 (choice of portfolio size and assets), the information equilibrium is efficient even though some investors could be better off without information. A social planner obeying the Pareto criterion cannot take the indivisible signal from the acquiring investor since that would leave the investor worse off by revealed preference. In presenting information demand under fully revealing asset price, this paper revisits the overlooked benchmark case of informational efficiency.

2 Fully Revealing Equilibrium

This section shows that an asset-market equilibrium in an additive signal-return model with CARA utility is symmetric and fully reveals the signal realization s . It is unique if the share of informed investors is known at the time of the portfolio choice. Then the information equilibrium too is symmetric, given an indivisible signal S , in the sense that either all investors are informed or no investor is informed.

There is a finite number I of investors with arbitrary time preferences and arbitrary initial wealth. Investor i holds initial wealth W_0^i and chooses consumption C_0^i today along with a portfolio (B^i, X^i) to secure consumption C_1^i tomorrow. There is no income in period 1 other than asset returns. So, $C_1^i = RB^i + \theta X^i$ and $C_0^i = W_0^i(s) - (B^i + P(s)X^i)$. Initial wealth is $W_0^i(s) = B_0^i + P(s)X_0^i$ given asset price $P(s)$, which is known at the time of these choices. Under CARA, investor i 's period utility becomes $v(C) = -\exp\{-AC\}$, where $A > 0$ is the Pratt-Arrow measure of absolute risk aversion. So, under assumption 2,

$$V^i \equiv -\alpha \exp\{-AC_0^i\} - \beta^i \exp\{-AC_1^i\},$$

where either $\alpha = 0$ or $\alpha = 1$, and $\beta^i \in (0, 1)$ is the time discount factor. As Grossman and Stiglitz (1980) do, sections 3 and 4 consider a terminal consumption maximization problem with $\alpha = 0$. Section 5 will consider the intertemporal consumption problem with $\alpha = 1$.

Irrespective of whether investor i has only a choice of the asset allocation ($\alpha = 0$) or an intertemporal choice in addition ($\alpha = 1$), investor i 's demand for the stock must satisfy the same first order condition. Using the moment generating function of the normal asset return $M_{\theta|s}(t) = \exp\{st + \sigma_\varepsilon^2 t^2/2\}$ (conditional on signal realization s), expected utility *post notitiam* becomes

$$\mathbb{E}[V^i|s] = -\beta^i \exp\{-ARW_0^i(s)\} \exp\{ARP(s)X^i\} M_{\theta|s}(-AX^i) \quad (2)$$

when investor i only has an inter-asset choice ($\alpha = 0$ so $C_0^i = 0$), and

$$\mathbb{E}[V^i|s] = -\exp\{-A[W_0^i(s) - B^i - P(s)X^i]\} - \beta^i \exp\{-ARB^i\} M_{\theta|s}(-AX^i) \quad (3)$$

when investor i also has an intertemporal choice ($\alpha = 1$). The Walrasian auctioneer presents $P(s)$ to every investor at the time of portfolio choice.

Maximizing expected utility—i.e. maximizing either (2) over X^i , or (3) over X^i and B^i —establishes the first order condition

$$RP(s) = \frac{\mathbb{E}[\theta \exp\{-AX^i \theta\} | s]}{\mathbb{E}[\exp\{-AX^i \theta\} | s]} = s - A \sigma_\varepsilon^2 X^i. \quad (4)$$

for an informed investor who knows s . In contrast, stock demand of an uninformed investor must satisfy

$$RP = \frac{\mathbb{E}[\theta \exp\{-AX^i \theta\}]}{\mathbb{E}[\exp\{-AX^i \theta\}]} = \mu_S - A(\sigma_S^2 + \sigma_\varepsilon^2)X^i. \quad (5)$$

First-order conditions (4) and (5) are the inverse demand functions for the risky asset. Asset demand strictly decreases in asset price and can be shown to satisfy second-order conditions for utility maximum. Bond demand B^i varies to satisfy the wealth constraint. Figure 1 depicts examples of stock demand. Note that $RP = \mathbb{E}[RP(s)] = \mathbb{E}[\theta]$ at $X^i = 0$ so that the stock demand schedules share the same intercept *ante notitiam* (before the signal realization is observed) as depicted in figure 1.

If the share of informed investors $\lambda \in [0, 1]$ is known to all investors at the time of their portfolio choice, assumptions 1 and 2 suffice to make the fully revealing financial market equilibrium unique. Then, either $\lambda = 0$, or $\lambda = 1$ as soon as one investor acquires the signal. Let $\bar{x} \equiv \sum_{i=1}^I X_0^i / I > 0$ denote average asset supply per investor. A financial market equilibrium requires that the market for the risky asset clears

$$\lambda X_{\text{inf.}} + (1 - \lambda) X_{\text{uninf.}} = \bar{x}, \quad (6)$$

where $X_{\text{inf.}}$ and $X_{\text{uninf.}}$ denote demands of informed and uninformed investors, respectively.

Theorem 1 (Unique Fully Revealing Financial Market Equilibrium). *In additive signal-return models with CARA utility (assumptions 1 and 2), a symmetric and fully revealing financial market equilibrium exists. It is unique if the share of informed investors λ is common knowledge at the time of the Walrasian auctioning process.*

Proof. Using $\lambda = 1$ and $X^i = \bar{x}$ in (6), (5) and (4) shows that a symmetric financial market equilibrium exists. Equilibrium price is fully revealing because $RP(s)$ is invertible in the signal realization by (4): if $\lambda \in (0, 1]$ is

common knowledge, every uninformed investor can infer informed investors' demand X_{inf} from market clearing (6) and thus infer $s = A\sigma_\varepsilon^2 X^i + RP$ from (4). If $\lambda = 0$, the equilibrium is unique, symmetric, and fully revealing in the degenerate sense that nothing can be revealed. ■

Known risk aversion is a necessary condition for price to be fully revealing since the realization of s cannot be inferred from an informed investor's demand otherwise. Jordan (1983) shows in addition that constant (absolute or relative) risk aversion, of which risk neutrality is a limiting case, is *necessary* for a fully revealing equilibrium to exist under regularity conditions. With the additive signal-return model (assumption 1), Theorem 1 provides a *sufficient* condition for existence (and for uniqueness if λ is known). Assumption 1 does not require the state space to be finite and is in this regard more general than other sufficient conditions (e.g. Citanna and Villanacci 2000). If the share of informed investors λ is not known at the time of portfolio choice, or if there are different degrees of being informed in the presence of more than one signal, partially revealing equilibria could be supported alongside the fully revealing equilibrium depending on the beliefs that investors are allowed to hold. The focus of this paper, however, lies on fully revealing equilibrium.

3 No-information Equilibrium

Does any investor i have an incentive to acquire the signal under a fully revealing asset price? To investigate the answer, first consider the case where investors only face an *inter-asset* decision but have no intertemporal choice. Section 5 will extend the problem to an intertemporal setting.

Under fully revealing price, information is a public commodity. Two equilibrium definitions are commonly applied to public commodities: (i) Nash equilibria, or (ii) public goods equilibria in the style of Samuelson (1954). Both concepts rest on the following principle.

Definition 1 (Competitive REE). *A competitive rational expectations equilibrium (REE) in an exchange economy is an allocation of commodities and assets to agents, along with a price for each unit of the commodities and assets, so that no agent wants to acquire amounts that differ from this allocation subject to the observed choice of other agents and a wealth constraint.*

The financial market equilibrium in Theorem 1 satisfies this equilibrium definition for asset demand. It remains to establish the competitive equilibrium for the signal S . If at least one investor buys the signal S , everyone becomes fully informed of s after its transmission and it is not rational for any other investor to acquire the signal again. So, there can be at most one investor to whom the indivisible signal S is allocated in a competitive equilibrium under fully revealing price. Will there be one investor to acquire the signal?

Grossman and Stiglitz' (1980) concise solution strategy uses investors' indirect utility in financial equilibrium to determine the information equilibrium. Applying (5) and (4) to (2), we obtain an investor i 's indirect utility *post notitiam* in a financial market equilibrium with no information ($\lambda=0$) and with full information ($\lambda=1$).

In the absence of an informed investor, indirect utility $\mathbb{E}[V^i]$ is

$$\mathbb{E}[V^i] = -\beta^i \exp\{-AR B_0^i - ARP(X_0^i - \bar{x})\} M_S(-A\bar{x}) M_\varepsilon(-A\bar{x}), \quad (7)$$

where $M_S(t) = \exp\{\mu_S t + \sigma_S^2 t^2/2\}$, $M_\varepsilon(t) = \exp\{\sigma_\varepsilon^2 t^2/2\}$, and $RP = \mu_S - A(\sigma_S^2 + \sigma_\varepsilon^2)X^i$ by (5). *Post notitiam*, an informed investor's utility $\mathbb{E}[V^i|s]$ is

$$\mathbb{E}[V^i|s] = -\beta^i \exp\{-AR B_0^i - ARP(s)(X_0^i - \bar{x})\} \exp\{-A\bar{x}s\} M_\varepsilon(-A\bar{x}), \quad (8)$$

where $RP(s) = s - A\sigma_\varepsilon^2 X^i$ by (4). Note that the symmetry of the fully revealing equilibrium allows investors to condition their expectations on the anticipated asset position \bar{x} , which does not depend on the signal realization. So, the value of the position $RP(s)\bar{x}$ only depends on the response of $RP(s)$ to the signal realization.

At the time when investor i chooses whether or not to acquire a costly signal S , its realization s must still be unknown. So, the investor bases information demand on a comparison between the *ante notitiam* indirect utilities with and without the expected receipt of a signal realization. If the indirect utility ratio satisfies

$$\frac{\mathbb{E}_S[\mathbb{E}[V^i|S]]}{\mathbb{E}[V^i]} < 1, \quad (9)$$

information acquisition is worthwhile for investor i . Recall that $V^i < 0$ under CARA utility so that this ratio must fall below unity. For costly signals, (9) must hold with strict inequality.

Condition (9) translates into a restriction on the signal distribution. As it turns out, this restriction is never satisfied in an additive signal-return

model with CARA utility. So, no investor has an incentive to acquire the signal if the only choice is an *inter-asset* decision. The unique information equilibrium is one with zero information.

Theorem 2 (Unique No-information Equilibrium under Inter-asset Choice). *In an additive signal-return model with CARA utility (assumptions 1 and 2), when a finite number of investors has an inter-asset choice, price is fully revealing and information demand criterion (9) fails.*

Proof. Investors' signal choices are known under equilibrium definition 1, so price is fully revealing by Theorem 1. Divide the *ante notitiam* expectation of (8) by (7) to find $\mathbb{E}_S [\mathbb{E}[V^i|S]]/\mathbb{E}[V^i] = \exp\{A^2\sigma_S^2(\bar{x} - X_0^i)^2/2\} \geq 1$. ■

The unique equilibrium entails no information acquisition. Worse, investors who are endowed with more or less of the risky asset (X_0^i) than the average market participant (\bar{x}) would pay not to receive information. But the *ante notitiam* variance of the asset return falls: $\mathbb{E}_S [\mathbb{V}(\theta|S)] = \mathbb{V}(\theta) - \mathbb{V}_S(\mathbb{E}[\theta|S])$ by a common decomposition result. Why can signal acquisition be undesirable for every investor although *ante notitiam* indirect utility should increase with reduced risk?

4 Diminishing Expected Excess Return

There is no demand for information when investors merely have an *inter-asset* choice because, *ante notitiam*, information diminishes the expected excess return of the asset

$$\mathbb{E}_S [\mathbb{E}[\theta - RP(S) | S]] = \mathbb{E}_{ante} [\theta] - R \mathbb{E}_{ante} [P(S)]$$

over its opportunity cost. The expected excess return falls because investors will bid up the asset price when they face less uncertainty *post notitiam*. While information does not affect $\mathbb{E}_S [\mathbb{E}[\theta|S]] = \mathbb{E}_{ante} [\theta]$ by the law of iterated expectations, the anticipated asset price $\mathbb{E}_{ante} [P(S)]$ is higher in the equilibrium with information than $\mathbb{E}_{ante} [P]$ without information. Information lowers risk, increases asset demand and raises asset price. Figure 1 illustrates this effect with an upward turn in the asset demand schedule.

Theorem 3 (Diminished Expected Excess Return). *In additive signal-return models with CARA utility (assumptions 1 and 2), when asset price is*

anticipated to fully reveal a signal realization s , anticipated receipt of the signal S strictly reduces the ante notitiam excess return of the risky asset $\mathbb{E}_S[\mathbb{E}[\theta - RP(S) | S]]$.

Proof. By the law of iterated expectations, the difference between the excess returns with and without information acquisition is $-\mathbb{E}_S[RP(S) - RP]$. Using the first-order conditions for informed investors (4) and uninformed investors (5) and taking prior expectations, the difference becomes $-\mathbb{E}[S] + \mu_S - A\sigma_S^2\bar{x}$, which is strictly negative for $\bar{x} > 0$. ■

The diminishing excess return decreases expected consumption tomorrow because C_1^i depends positively on the excess return of the stock ($\theta - RP$). So, investors with a larger stock endowment than market average ($X_0^i > \bar{x}$) get to lay off their initial risk at a better relative price but have to accept lower expected consumption. Similarly, investors with a smaller initial stock endowment than average ($X_0^i < \bar{x}$) do not take on as much risk in asset trades but also have to accept a lower expected consumption. The resulting effect on the value of information is symmetric for both types of investors in an additive signal-return model with CARA utility. The proof to Theorem 2 shows that an investor's value of information drops further the more the investor's risky asset endowment differs from the market endowment in absolute value ($|\bar{x} - X_0^i|$).

The diminishing excess return may also be interpreted as a reflection of the Hirshleifer (1971) effect: information reduces risk but, in resolving uncertainty, removes risk sharing opportunities so that final consumption depends more on initial wealth. “[P]ublic information ... in advance of trading adds a significant distributive risk” (Hirshleifer 1971, p. 568). An investor whose endowment X_0^i differs strongly from the market average \bar{x} loses more trading opportunities with the receipt of information and therefore values information less. The less risk sharing opportunities an asset provides compared to the bond, the closer its price to the bond price. Put differently, the stronger the Hirshleifer effect, the further diminished the excess return.

The information equilibrium exists but entails *zero information demand* if investors only have an *inter-asset* choice in a portfolio of given size. This resolves one part of Grossman and Stiglitz' no-equilibrium paradox in additive signal-return models under CARA. Grossman and Stiglitz (1980, conjecture 6) write (my emphasis):

... *But if everybody is uninformed, it clearly pays some individual*

to become informed. Thus, there does not exist a competitive equilibrium.

The emphasized part of this conjecture can fail. It may never pay any investor to become informed in an additive signal-return model with CARA utility. A competitive equilibrium does exist. It is unique and entails zero demand for information by Theorem 2 when investors only have an inter-asset choice. Furthermore, the equilibrium is efficient in the sense that a benevolent social planner would not want any investor to acquire information.

If there were infinitely many investors, each investor with a measure zero, then individual demand would not affect the Walrasian price finding process and individual information would not be revealed. The expected excess returns would remain unaltered. A full measure of investors, however, has an incentive to acquire the information so that information would be revealed. So, the assumption of infinitely many investors, where each individual investor has no price impact while the full measure has the full impact, lies behind the no-equilibrium paradox. With a finite number of investors, the value of information is well defined.

The value of information is strictly negative in additive signal-return models when investors only have an *inter-asset* choice. The diminishing excess return, however, does not prevent information acquisition under all circumstances: if investors have an *intertemporal* consumption choice in addition to the mere inter-asset choice, they demand information.

5 Information Equilibrium

This section shows that investors demand a strictly positive amount of information in fully revealing equilibrium when information arrives before they take their *intertemporal* consumption decision. With an intertemporal choice, investors can adjust the size of their portfolio in response to the signal realization. The anticipation of information raises the *ante notitiam* utility of investors further than when their response to information is limited to an inter-asset choice. Loosely speaking, investors anticipated ability to condition both $C_0^i(s)$ and $C_1^i(s)$ on the signal realization s (and not only $C_1^i(s)$) presents enough benefits to acquire the signal S . One, and only one, investor with close-to-average initial stock holdings has the incentive to acquire the signal in a joint asset and information market equilibrium under fully revealing price. Consequently, everybody becomes informed.

When investors have an intertemporal consumption choice, the first order conditions for the bond and the stock imply that *post notitiam* indirect utility (3) becomes

$$\mathbb{E}[V^i|s] = -\delta^i \exp\{-ARB_0^i - ARP(s)(X_0^i - \bar{x})\}^{\frac{1}{1+R}} M_{\theta|s}(-A\bar{x})^{\frac{1}{1+R}} \quad (10)$$

in financial market equilibrium for $\delta^i \equiv [(1+R)/R](\beta^i R)^{1/(1+R)}$ and $M_{\theta|s}(t) = \exp\{st + \sigma_\varepsilon^2 t^2/2\}$ (see appendix A).

Whereas the bond return R was a parameter in the final consumption maximization problem, R now serves to clear the bond market. *Ante notitiam*, R is given and independent of signal acquisition by the law of iterated expectations. The bond return, however, can respond to the signal realization *post notitiam* and correlate with other payoffs in indirect utility. To keep the analysis to closed-form solutions, I impose assumption 3 that the signal realization s alters R negligibly little. This assumption is justified for an economy with small individual stock volumes compared to the size of the market for risk free bonds (see appendix B for a formal derivation). Small open economies and economies with government debt are examples. Assumption 3 also makes the intertemporal model more closely comparable to Grossman and Stiglitz' (1980) inter-asset choice under deterministic R .

Applying (5) and (4) to (10), we obtain an investor i 's *post notitiam* indirect utility in a financial market equilibrium with no information ($\lambda=0$) and with full information ($\lambda=1$). In the absence of an informed investor, expected indirect utility $\mathbb{E}[V^i]$ is

$$\mathbb{E}[V^i] = -\delta^i \exp\{-ARB_0^i - ARP(X_0^i - \bar{x})\}^{\frac{1}{1+R}} M_S(-A\bar{x})^{\frac{1}{1+R}} M_\varepsilon(-A\bar{x})^{\frac{1}{1+R}}, \quad (11)$$

where $M_S(t) = \exp\{\mu_S t + \sigma_S^2 t^2/2\}$, $M_\varepsilon(t) = \exp\{\sigma_\varepsilon^2 t^2/2\}$, and $RP = \mu_S - A(\sigma_S^2 + \sigma_\varepsilon^2)X^i$ by (5). *Post notitiam*, an informed investor's expected indirect utility $\mathbb{E}[V^i|s]$ is

$$\mathbb{E}[V^i|s] = -\delta^i \exp\{-ARB_0^i - ARP(s)(X_0^i - \bar{x}) - A\bar{x}s\}^{\frac{1}{1+R}} M_\varepsilon(-A\bar{x})^{\frac{1}{1+R}}, \quad (12)$$

where $RP(s) = s - A\sigma_\varepsilon^2 X^i$ by (4).

As discussed in section 3 before, investor i bases information choice on a comparison between the *ante notitiam* indirect utilities with and without the expected receipt of a signal realization. If the indirect utility ratio satisfies (9)

$$\frac{\mathbb{E}_S[\mathbb{E}[V^i|S]]}{\mathbb{E}[V^i]} < 1,$$

then information acquisition is worthwhile.

Under CARA and an additive signal-return distribution, condition (9) translates into a restriction on the signal distribution and investors' initial risky asset endowments. Contrary to the earlier finding in section 3, the restriction can be satisfied: an investor with close-to-average endowments of the risky asset acquires the signal on the stock return if she can take the *intertemporal* consumption decision after observing the signal realization. At most one investor will optimally acquire the indivisible signal in a competitive equilibrium (definition 1). Then the unique information equilibrium is one with full information.

Theorem 4 (Unique Information Equilibrium under Intertemporal Choice). *In an additive signal-return model with CARA utility and a resilient interest rate to signal realizations (assumptions 1 through 3), when a finite number of investors has an intertemporal choice in addition to the inter-asset choice, price is fully revealing and information demand criterion (9) is satisfied only if*

$$\frac{R}{1+R} \sigma^2 A^2 (X_0^i)^2 - \sigma^2 A^2 (X_0^i - \bar{x})^2 > 0. \quad (13)$$

If the signal cost is sufficiently low, the unique type of equilibrium is one in which one and only one investor with a strictly positive risky asset endowment, sufficiently close to market average, acquires the costly signal. Otherwise the unique equilibrium is one in which no investor acquires the signal.

Proof. Since investors' signal choices are known under equilibrium definition 1, price is fully revealing by Theorem 1. Appendix C derives information demand criterion (13), using (11) and (12) in (9). The remaining statements follow immediately under normality (and are also proven for CARA-consistent signal distributions with a moment generating function in appendix C). ■

The average investor with mean endowment $X_0^i = \bar{x}$ has a strict incentive to acquire the signal for $R > 0$. Other investors with endowments X_0^i close to \bar{x} may also demand information. So, multiple equilibria can exist in the sense that it is indeterminate who exactly acquires the signal. The signal allocation is asymmetric. The information level is unique, however, and revealed information is public and symmetric. It does not matter who bears the cost of acquiring the indivisible signal, there will be full information. Note that condition (13) cannot be satisfied for $R = 0$, similar to Theorem 2.

The intertemporal choice allows investors to adjust their portfolio size in response to the signal realization. Anticipating this additional choice, investors value information more than they do if they only have a choice between assets. The expected portfolio size is larger in the presence of information. Optimal portfolio size *post notitiam* is $\pi^i(s) = (1+R)^{-1}[B_0^i + RP(s)(X_0^i/R + X^i) + A^{-1} \ln \beta^i RM_{\theta|s}(-AX^i)]$ (see (16) in the appendix), so the *ante notitiam* difference between the expected portfolio sizes with and without information becomes

$$\mathbb{E}_S [\pi_{\lambda=1}^i(S) - \pi_{\lambda=0}^i] = (X_0^i/R + \bar{x}) [\mu_s + A\sigma_\varepsilon^2 \bar{x}/2] / (1+R) > 0.$$

The difference is strictly positive, a corollary of diminishing-excess-return Theorem 3. Better information leads every investor to save more. In fact, the portfolio size increases more strongly than the stock price due to the wealth effect of the price increase (reflected in the term X_0^i/R).

Would a social planner implement this equilibrium outcome? An equilibrium with full information must be Pareto optimal in an additive signal-return model since the acquiring investor is better off with the signal by revealed preference. While any equilibrium with information must be Pareto optimal in this sense, signal acquisition can reduce overall welfare.

Suppose, for instance, that investor i initially holds the entire endowment of stocks while the $I-1$ remaining investors have their initial wealth in bonds only. The single stock owner acquires the signal because the expected price increase awards her with the wealth effect of a more valuable initial portfolio. In particular, for a total of three investors $R > 5/4$ satisfies criterion (13); for a total of four investors $R > 9/7$ is needed. A social planner who follows the Pareto criterion cannot improve on this equilibrium outcome since taking away the signal would make investor i worse off. Overall welfare, however, may fall with signal acquisition. In the example, the unweighted sum of the logs of *ante notitiam* indirect utilities is equal to the sum of criterion (13) over all investors. Summing up criterion (13) yields $R/(1+R) - (I-1)$, which is negative for three investors and any $R \geq 0$ (it exceeds $-13/9$ for $R > 5/4$). Although the Pareto criterion judges information acquisition in additive signal-return models necessarily as socially optimal, there can yet be cases when overall welfare drops with more information as it diminishes the excess return for everyone.

Theorem 4 addresses the first part of Grossman and Stiglitz' no-equilibrium conjecture. Grossman and Stiglitz (1980, conjecture 6) write (my emphasis):

In the limit, when there is no [exogenous] noise [in prices], prices convey all information, and *there is no incentive to purchase information* . . .

Theorem 4 refutes this part of the conjecture. It does pay an investor with an average endowment of the risky asset to become informed in an additive signal-return model with CARA utility. For the individual decision to acquire a public good, given other agents' choice of zero, only individual incentives matter. (The definition of a competitive equilibrium does not specify by what mechanism it comes about.) At most one investor, however, will find it optimal to acquire the signal.

If there were infinitely many investors, each investor with a measure zero, then a strictly positive measure of investors would be needed to acquire the same signal so that price can reveal information. Individual demand would not affect the Walrasian price finding process, however, and every single investor who acquired a duplicate of the signal would be better off not acquiring it. This non-concavity from the assumption of infinitely many investors, where each individual investor has no price impact while an arbitrarily small but strictly positive measure has the full impact on revealing price, lies behind the no-equilibrium paradox.

When there is a finite number of investors, an allocation of the indivisible signal to any of the investors with positive information demand is a competitive equilibrium (no matter how many investors want to acquire the signal): given the signal allocation, the paying investor would be worse off without the signal; no other investor with positive information demand wants to pay for the duplicate of fully revealed information; and those investors who prefer no information have no choice in competitive equilibrium.

6 Conclusion

Contrary to a common no-equilibrium conjecture, an information market equilibrium does exist in additive signal-return models with CARA utility. Investors acquire information to a Pareto efficient degree under fully revealing price and, when no one demands information, a social planner also agrees with that market outcome. But information is not beneficial to every investor. Although equilibrium is Pareto efficient, information may reduce overall social welfare: there may be a majority of investors who would prefer

that there were less information in the market because information diminishes the excess return of a risky asset.

Information diminishes the expected excess return since it resolves uncertainty so that the risky asset's price approaches the bond price. This is the counterpart to a Hirshleifer (1971) effect: in resolving uncertainty, information also erodes risk sharing opportunities when it is publicly revealed before trading. Recent research into the Hirshleifer effect in financial markets includes Marin and Rahi (2000), who relate the informational consequences of market completeness to the Hirshleifer effect, and Dow and Rahi (2003), who discern the Hirshleifer effect and a spanning effect by source of uncertainty. Investors with a choice of information, as in the approach of the present paper, partly internalize such welfare effects of information. It remains a task for research to investigate the welfare implications of the intricate relationships between market completeness, adverse selection, and risk sharing when investors have a choice of information.

Additive signal-return models are common among many strands of research into information effects in financial markets. While investors' receipt of information is often treated as exogenous, results of the present article are reassuring. Rational investors demand financial information to a Pareto efficient extent even in the extreme benchmark case of a fully revealing asset price. The model has several testable implications. More information diminishes the excess return of a risky asset. Investors with close-to-average asset endowments are more likely to acquire information. Portfolios are larger in the presence of more information. Information acquisition occurs even in perfectly efficient markets.

Appendix

A Intertemporal choice

Maximizing expected utility (3) over X^i and B^i yields the first-order conditions

$$\frac{1}{\beta^i R} = \mathbb{E} [\exp\{-A(C_1^i - C_0^i)\}] = H^i(s) M_{\theta|s}(-AX^i) \quad (14)$$

and

$$\frac{P(s)}{\beta^i} = \mathbb{E} [\theta \exp\{-A(C_1^i(s) - C_0^i(s))\}] = H^i(s) M'_{\theta|s}(-AX^i), \quad (15)$$

where $H^i(s) \equiv \exp\{-A[(1+R)B^i + P(s)X^i - W_0^i(s)]\}$ (and $M_{\theta|s}(t) = \exp\{st + \sigma_\varepsilon^2 t^2/2\}$ under normality).

The optimal portfolio size $\pi^i(s)$ can be written as

$$\begin{aligned} \pi^i(s) \equiv B^i + P(s)X^i &= \frac{1}{1+R} [W_0^i(s) + RP(s)X^i - \frac{1}{A} \ln H^i(s)] \\ &= \frac{1}{1+R} [B_0^i + RP(s)(X_0^i/R + X^i) + \frac{1}{A} \ln \beta^i R M_{\theta|s}(-AX^i)], \end{aligned} \quad (16)$$

where the second line follows from (14). Bond income can be written as

$$RB^i = \frac{R}{1+R} (W_0^i(s) - P(s)X^i - \frac{1}{A} \ln H^i(s)).$$

Note that H^i and W_0^i are functions of s since $RP(s)$ is. Using this fact along with (16) in (3) yields

$$\begin{aligned} \mathbb{E} [V^i|s] &= -\exp\left\{-\frac{A}{1+R} [RW_0^i - RP(s)X^i + \frac{1}{A} \ln H^i]\right\} (1 + \beta^i H^i M_{\theta|s}(-AX^i)) \\ &= -\exp\left\{-A\frac{R}{1+R} W_0^i\right\} (\exp\{ARP(s)X^i\}/H^i)^{\frac{1}{1+R}} (1 + \frac{1}{R}) \end{aligned}$$

for $\alpha=1$. The second step follows from the first order condition (14) for the bond, substituting it for $H^i(s)M_{\theta|s}(-AX^i)$. Indirect utility (10) in the text follows using (14) once more. Function (10) is a proper indirect utility function since the asset price $P(s)$ in equilibrium reflects the first order condition (15) for the risky asset.

B Bond return response to stock information

Taking logs of both sides of first order condition (14) for bond demand yields

$$A(1+R)B^i - AB_0^i + AP(s)(X^i - X_0^i) = \ln \beta^i R M_{\theta|s}(-AX^i),$$

a permissible operation since $\beta^i, R, M_{\theta|s}(\cdot) > 0$ by their definitions. Summing up both sides over investors i and dividing by their total number (measure) yields

$$\exp\{AR\bar{b}\}/\beta^i R = M_{\theta|s}(-AX^i), \quad (17)$$

after exponentiating both sides, where $\bar{b} \equiv \sum_{i=1}^I B_0^i/I$ is the average initial bond endowment per investor. Equation (17) implicitly determines the gross bond return R .

Post notitiam, R responds to the signal realization. Apply the implicit function theorem to the logarithm of (17) to find

$$\frac{\partial R}{\partial s} = \frac{AR\bar{x}}{AR\bar{b} - 1}.$$

The bond return strictly increases in response to a favorable signal realization s if $\bar{b} > 1/(AR)$ and falls otherwise. In principle, R too is a function of the signal realization s . For large initial bond endowments \bar{b} , however,

$$\lim_{\bar{b} \rightarrow \infty} \partial R / \partial s = 0.$$

So, $R \approx R_{\lambda=1} \approx R_{\lambda=0}$ in the presence of a small stock endowment relative to the bond endowment of the economy.

C General Proof of Theorem 4

Ante notitiam, expectations of (12) are

$$\mathbb{E}_S [\mathbb{E} [V^i | S]] = -\delta^i \exp \left\{ -ARB_0^i + (t_0^i - t) M'_\varepsilon(t) / M_\varepsilon(t) \right\}^{\frac{1}{1+R}} M_S \left(\frac{t_0^i}{1+R} \right) M_\varepsilon(t)^{\frac{1}{1+R}}$$

since $RP(s) = s + M'_\varepsilon(t) / M_\varepsilon(t)$ by (14) and (15), where $M_S(t)$ and $M_\varepsilon(t)$ are the moment generating functions for the S and ε distributions, $t \equiv -A\bar{x} < 0$ and $t_0^i \equiv -AX_0^i < 0$. Using this result and (11) in information demand criterion (9), rearranging and taking logs, yields the equivalent information demand criterion under *intertemporal* choice

$$Crit(t, t_0^i) := \ln M_S(t) - (1+R) \ln M_S \left(\frac{t_0^i}{1+R} \right) - (t - t_0^i) \frac{M'_S(t)}{M_S(t)} > 0. \quad (18)$$

where $R \in (0, \infty)$. Taking logs is permissible since $M_S(t) > 0$ for finite t . Normality implies $M_S(t) = \exp\{\mu_S t + \sigma_S^2 t^2 / 2\}$ and $M_\varepsilon(t) = \exp\{\sigma_\varepsilon^2 t^2 / 2\}$ and criterion (13) in the text follows.

The first derivative of (18) with respect to t ,

$$\frac{\partial \text{Crit}(t, t_0^i)}{\partial t} = -(t - t_0^i) \left(\frac{M_S''(t)}{M_S(t)} - \left(\frac{M_S'(t)}{M_S(t)} \right)^2 \right),$$

clarifies that criterion (18) is strictly increasing in t for $t < t_0^i$ and strictly decreasing in t for $t > t_0^i$ if $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$. So, criterion (18) attains a global maximum at $t = t_0^i$ if $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$. It can be shown that commonly known distribution functions satisfy $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$ and that the second-order condition of the intertemporal portfolio choice problem is equivalent to this condition for $\theta = S$ (called CARA consistency here; Muendler 2004, p. 22-23).

To prove that criterion (18) is satisfied for a sufficiently small difference $t - t_0^i$, it remains to establish that the maximum strictly exceeds zero. The fact that

$$\hat{t} M_S'(\hat{t}) / M_S(\hat{t}) > \ln M_S(\hat{t}) \quad (19)$$

for $\hat{t} < 0$ is a useful property for this purpose. Observe that both the left-hand and the right-hand side of (19) vanish for $\hat{t} = 0$. So, to establish (19), it suffices to show that its left-hand side increases faster than the right-hand side for all $\hat{t} < 0$ as \hat{t} falls. Taking the first derivative of either side with respect to \hat{t} shows that the increase in the left-hand side exceeds the increase in the right-hand side by $-\hat{t} \cdot [M_S''(\hat{t})/M_S(\hat{t}) - (M_S'(\hat{t})/M_S(\hat{t}))^2] > 0$ as \hat{t} falls, which is a positive amount because $\hat{t} < 0$ and because the second-order condition is equivalent to $M_S''(t)/M_S(t) > [M_S'(t)/M_S(t)]^2$.

If $R = 0$, criterion (18) attains a maximum value of zero. So, if we can show that (the maximum of) criterion (18) strictly increases in R , it is proven that criterion (18) is satisfied for a sufficiently small difference $t - t_0^i$. Taking the first derivative of (18) with respect to R yields

$$\partial \text{Crit}(t, t_0^i) / \partial R = -\ln M_S(\hat{t}) + \hat{t} M_S'(\hat{t}) / M_S(\hat{t}) > 0$$

for $\hat{t} \equiv t_0^i / (1+R) < 0$. The derivative is strictly positive by fact (19). So, if $R > 0$, criterion (18) holds for $t_0^i > 0$ in a neighborhood around t but fails otherwise. Uniqueness of the information and the no-information equilibrium follows since no more than one investor can optimally acquire an indivisible public commodity by definition 1.

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