Rational Transparency Choice
in Financial Market Equilibrium[*]

Marc-Andreas Muendler[¶]

University of California, San Diego and CESifo

December 5, 2005

Abstract

Add a stage of signal acquisition to a canonical model of portfolio choice. Under fully revealing asset price, investors' information demand reflects their choice of transparency. In reducing uncertainty, financial transparency raises expected asset price and thus benefits holders of the risky asset. At a natural transparency limit, however, investors pay to inhibit further disclosure in order to forestall the erosion of the asset's expected excess return. The natural transparency limit varies with the portfolio position. There is a dominant investor with a risky asset endowment modestly above market average who single-handedly determines transparency in equilibrium. The dominant investor strictly improves welfare for investors with similar endowments but strictly reduces welfare for others when acquiring signals beyond their natural transparency limits. The welfare consequences of financial transparency are thus intricately linked to the wealth distribution.

Keywords: Information acquisition; rational expectations equilibrium; fully revealing asset price; financial transparency; disclosure; gamma distributed asset returns; Poisson distributed signals

JEL Classification: JEL D81, D83, G14

Financial transparency is widely regarded as a cornerstone of well-functioning capital markets and a remedy to financial distress. What is the welfare effect of financial transparency? This paper pays close attention to the importance of heterogeneous portfolios and financial market conditions for the economic value of transparency. The paper shows that there is a natural transparency limit at which rational investors pay to inhibit disclosure. The natural transparency limit varies with initial portfolio holdings.

A literature on learning and experimentation analyzes incentives for information acquisition and the public-goods character of information, but frequently treats markets in abstract terms and disregards equilibrium price effects of transparency. Many models of financial information, on the other hand, consider signal receipt as exogenous to investors. The framework of this paper permits an analysis of the utility value of financial transparency based on first principles. To determine the economic value of financial transparency, the paper examines fully revealing asset price. Under fully revealing price, the choice of individual information becomes a choice of public transparency. Finitely many rational investors have well-defined signal demands which, in turn, capture the marginal utility benefits and costs of transparency.

Natural transparency limits arise because, as information removes risk, it raises expected asset price and diminishes an asset's expected excess return. At an investor's transparency limit, the utility loss from diminished excess returns outweighs information benefits. To take an example, risk averse investors are compensated for default risk with returns that exceed the expected losses under default. More information strictly diminishes this excess return. At the transparency limit, rational lenders reject further information to keep the excess return. Prior to investors' natural transparency limits, financial information is a public good.

An investor's natural transparency limit crucially depends on initial portfolio holdings. When initial portfolio positions are heterogeneous, a dominant investor with an endowment close to market average emerges and dictates the transparency outcome.[1] Transparency makes the asset safer and thus raises the expected asset price. A higher expected asset price benefits the dominant investor most, who holds an asset endowment modestly above market average. Investors with smaller asset holdings experience only a smaller revaluation effect from more transparency, and investors with larger holdings suffer more from the higher expected variance of their endowment value because signal realizations may be favorable or unfavorable. The dominant investor acquires information to a degree that makes investors with smaller or larger initial asset holdings strictly

---

[1]This heterogeneity in endowments generalizes Muendler (2005) and stresses the importance of the wealth distribution for informational welfare.

worse off if the acquired number of signals exceeds those investors' transparency limits. As a consequence, additional information results in an unambiguous Pareto improvement only if investors are sufficiently homogeneous.

Information and transparency are matters of degree. Beyond binary information acquisition as in Grossman and Stiglitz (1980) or Diamond (1985), the present paper gives investors a choice of a number of signals in the spirit of experimentation. The present setup draws on Raiffa and Schlaifer (1961) whose conjugate-prior decision model provides a natural extension of the canonical portfolio choice model and gives rise to a law of demand for financial information—similar to the experimentation paradigm of repeated sampling (Moscarini and Smith 2001) but for risk averse, not risk neutral, agents. Most important, this approach allows rational investors to account for the equilibrium price impact of transparency. Demand for transparency is strong if many risky assets are in the market, or if investors are highly risk averse, or if the expected mean-variance ratio of the asset return is relatively low so that uncertainty matters strongly compared to expected returns.

Recent financial-market or general-equilibrium models address transparency but often stop short of giving investors a rational choice of information (Morris and Shin 2002, Krebs 2005). Morris and Shin (2002) argue in a beauty-contest model, for instance, that more public information may reduce welfare. Their result has been widely interpreted as counsel against transparency but the conclusion has been challenged within the original setting (Svensson 2005) as well as in extensions (Angeletos and Pavan 2004). Experimentation models give rise to decreasing but strictly positive marginal benefits to information and show that the public-goods character of information induces free-riding (e.g. Moscarini and Smith 2001, Cripps, Keller and Rady 2005). The abstract experimentation framework, however, does not tie the value of information to financial primitives. When embedding experimentation in a financial market context, information turns into a public bad at the natural transparency limit.

Financial transparency can reduce welfare because information diminishes the asset's expected excess return by raising expected asset price—an effect that also occurs in partially revealing rational expectations equilibrium (e.g. Admati 1985, Easley and O'Hara 2004). Easley, Hvidkjaer and O'Hara (2002) empirically confirm the diminishing effect of public information on expected excess returns. They find for a set of NYSE listed stocks between 1983 and 1998 that assets exhibit a lower excess return if public information matters relatively more for their valuation.[2]

---

[2]The resolution of information risk as in Easley et al. (2002) does not need to be the cause of the diminishing excess return effect. Investors hold identical beliefs at all relevant stages in this paper; information nevertheless raises asset price and diminishes the excess return.

The joint equilibrium in signal and asset markets is called a *Rational Information Choice Equilibrium* (RICE) and builds on common equilibrium definitions: a Walrasian rational expectations equilibrium (REE) for assets and a Samuelson (1954) style public-goods equilibrium for signals, given rational Bayesian updating.[3] A fully revealing equilibrium exists for countably many investors (Muendler forthcoming). Conversely, an individual in a continuum of investors would have no price impact (Grossman and Stiglitz 1980, Kim and Verrecchia 1991) and there would be no well-defined marginal valuation for public information. In contrast to Grossman and Stiglitz (1980) or Diamond (1985), RICE does not require a grouping of agents into informed and uninformed investors (which would equate their *ex ante* utilities), and thus permits a welfare analysis of transparency in terms of individual expected utilities.

This paper employs the Poisson-gamma pair of signal-return distributions. Apart from its welcome tractability, the Poisson-gamma pair exhibits many realistic features. The normal-normal pair, in contrast, would result in an unreasonable natural transparency limit at no information for investors without initial risky asset holdings. Financial information often comes in discrete levels such as Standard & Poor's and Moody's investment grades or on a three-level buy-hold-sell scale; Poisson distributed signals are discrete. A gamma distribution of the asset payoff is the unique conjugate prior distribution to Poisson signals so that a closed-form solution of the financial market equilibrium ensues.[4] Special cases of the gamma distribution are the chi-squared, the Erlang, and the exponential distribution, for instance. The prominence and success of the Nelson (1991) exponential ARCH model in empirical finance suggests that the gamma family is a particularly relevant one for return distributions. Realistically, gamma distributed gross returns cannot be negative so that investors never lose more than their principal.

The existence of a natural transparency limit is largely invariant to distributional assumptions.[5] For price to be fully revealing, investors are given a common degree of constant absolute risk aversion (CARA). While CARA utility provides a generalization over risk neutrality in the experimentation and learning literatures, informational properties of utility functions other than CARA are little known in the financial information literature and beyond the scope of this paper.

---

[3] The public-goods character of signals is also common in models of experimentation (Bolton and Harris 1999, Cripps, Keller and Rady 2005). Under partially revealing price, information has public-goods character too because price permits belief updating (Admati 1985).

[4] Davis (1993) and Heston (1993) present earlier models in finance that employ the gamma distribution.

[5] When signals are a sum of the asset's fundamental plus noise and the fundamental and noise distributions have moment-generating functions, then the natural transparency limit is zero information for investors with no risky asset endowment (Muendler forthcoming).

In Muendler (2005) I argue that, to instill more transparency, cutting costs of information acquisition is superior to disclosure because disclosure crowds out private information acquisition and risks a violation of investors' transparency limits. The present paper shows that this recommendation carries over to the case of heterogeneous investors. Instilling more transparency is best achieved by cutting costs because it results in a Pareto improvement for all investors if they are sufficiently homogeneous so that no one's transparency limit is violated, whereas disclosure could breach transparency limits. If investors' endowments are strongly heterogenous, cutting information costs can inflict welfare losses on some investors with initial positions far from those of the dominant investor. But cutting information costs continues to be superior to disclosure because reduced information costs necessarily benefit investors with initial positions close to market average, whereas public disclosure risks violating the transparency limit even of investors with close-to-average endowments.

The remainder of this paper is organized as follows. Section 1 puts forth the modelling assumptions and adds a stage of information acquisition to a canonical portfolio choice model. The resulting financial market equilibrium is presented in Section 2. Section 3 derives the information market equilibrium and determines the level of financial transparency, while Section 4 analyzes the informational efficiency under a normative perspective. Section 5 concludes. Some proofs are relegated to the appendix.

1 Information and Portfolio Choice

There are two periods, today and tomorrow, and two assets: One safe bond b and one risky stock x. Assets are perfectly divisible. The safe bond sells at a price of unity today and pays a real interest rate $r \in (-1, \infty)$ tomorrow so that the gross interest factor is $R \equiv 1+r \in (0, \infty)$. The risky asset sells at a price P today and has a payoff of θ tomorrow.

Add an initial stage of signal acquisition to the standard expected utility model of portfolio choice. Investors can acquire signals to update their prior beliefs about the risky asset return. Think of signals as spy robots and of signal realizations as the spy robots' reports. Markets for spy robots (signals) S_n^i open at 9am today. Robot n, hired by investor i, reports back exclusively to investor i with a signal realization s_n^i before 10am. How many spy robots N^i should investor i hire?

Figure 1 illustrates the timing of decisions. Every investor i is endowed with initial wealth $W_0^i \equiv b_0^i + Px_0^i$. At 9am, investors choose the number of signals (spy robots) N^i. To do so, they maximize *ante notitias* expected utility based on their prior beliefs before signal realizations become known (*ante notitias*). Investors
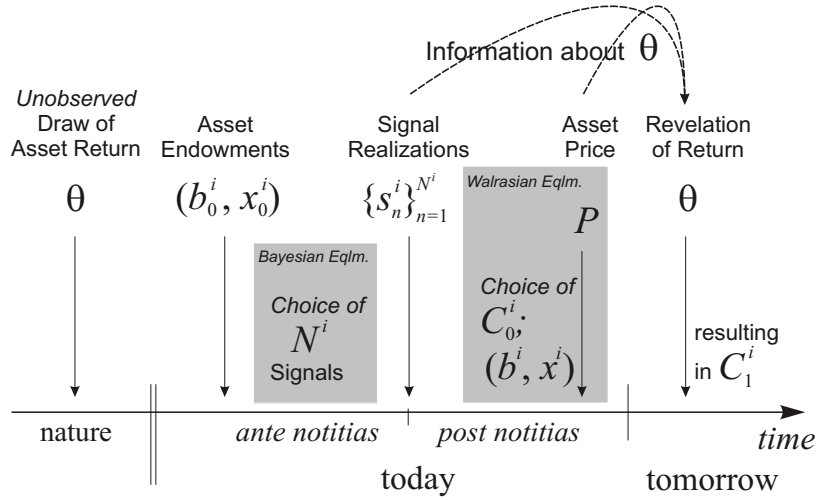
Figure 1: **Timing of decisions and information revelation**

then receive the realizations $\{s_1^i, ..., s_{N^i}^i\}$ of these N^i signals (they get to know the content of the spy robots' reports) and update their beliefs. When Wall Street opens at 10am today, investors choose consumption today and tomorrow, C_0^i and C_1^i, and decide how much of the risky asset to hold. At this stage, they maximize *post notitias* expected utility based on their posterior beliefs.[6] The Walrasian auctioneer in the financial market sets the price P for the risky asset such that the stock market clears. The bond market clears given the interest factor R.

The asset price at 10am will contain information. The reason is that each investor chooses her portfolio given her observations of signal realizations ($\{s_n^i\}_{n=1}^{N^i}$), and the Walrasian auctioneer at Wall Street clears the market by calling an equilibrium price. In the benchmark case of a fully revealing equilibrium, the asset price is invertible in a sufficient statistic of all investors' posterior beliefs and permits the rational extraction of all relevant market information. This is the case of analysis in the present paper.

In the spirit of competitive equilibrium, a rational expectations equilibrium (REE) that clears both the asset market and the market for signals can be defined as a Walrasian equilibrium at Wall Street preceded by a Bayesian public-goods equilibrium in the market for spy robot services. I call this extension of REE to a

---

[6]To clarify the timing of signal realizations, I distinguish between *ante notitias* and *post notitias* expected utility. *Ante notitias* expected utility is different from prior expected utility in that the arrival of N^i signals is rationally incorporated in *ante notitias* expected utility. Raiffa and Schlaifer (1961) favored the terms "prior analysis," "pre-posterior analysis" and "posterior analysis."

rational Bayesian public-goods equilibrium in the signal market and a subsequent Walrasian asset market equilibrium a Rational Information Choice Equilibrium, or RICE.

Definition 1 (RICE). *A Rational Information Choice Equilibrium (RICE) is an allocation of x^{i*} risky assets, b^{i*} safe bonds, and N^{i*} signals to investors $i = 1, ..., I$ and an asset price P along with consistent beliefs such that*

- *the portfolio (x^{i*}, b^{i*}) is optimal given RP and investors' post notitias beliefs for $i = 1, ..., I$,*

- *the market for the risky asset clears, $\sum_{i=1}^{I} x^{i*} = I\bar{x}$, and*

- *the choice of signals N^{i*} is optimal for investors $i = 1, ..., I$ given the sum of all other investors' signal choices $\sum_{k \neq i} N^{k,*}$ and a marginal signal cost c.*

$\bar{x}$ denotes the average risky asset supply per investor.

Rational Bayesian investors choose their demand for signals given the expected asset market REE at Wall Street under anticipated information revelation. The equilibrium in the market for signals is the benchmark public-goods equilibrium following Samuelson's (1954) definition, where agents know other agents' total demand for the public good at the time of their decision. When asset price fully reveals a sufficient statistic of all investors' signal realizations *post notitias*, signals are pure (non-rival and non-excludable) public goods *ante notitias*.

1.1 Conjugate updating

Financial information often comes in discrete levels such as Standard & Poor's or Moody's investment grades, or on a three-level buy-hold-sell scale. Poisson distributed signals in particular are discrete and exhibit several useful statistical properties. The sum of N^i conditionally independent Poisson signals, for instance, is itself Poisson distributed with mean and variance $N^i\theta$ (appendix B). For a large number of draws and small probabilities, Poisson probabilities approximate binomial signal distributions (many buy-sell signals) (Casella and Berger 1990, Example 2.3.6).

Assumption 1 (Poisson distributed signals and conjugate updating). *Signals are Poisson distributed and update the prior distribution of the asset return θ to a posterior distribution from the same family.*

A gamma distribution of the asset return, $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$, uniquely satisfies assumption 1 (Robert 1994, Proposition 3.3). The parameters α^i and β^i are specific to investors' beliefs in principle. The parameter α^i is sometimes referred to as the shape parameter and $1/\beta^i$ as the scale parameter.

Distributions that are closed under sampling so that prior and posterior distributions belong to the same family are called conjugate prior distributions. The gamma distribution is a conjugate prior to the Poisson distribution.[7] A gamma distributed asset return exhibits the additional advantage that its support is strictly positive so that, realistically, negative returns cannot occur. In contrast, a normal asset return would imply that stock holders must cover losses beyond the principal ($\theta < -P$) with a strictly positive probability.

Under assumption 1, signals $\{S_1^i, ..., S_{N^i}^i\}_{i=1}^I$ are conditionally independent given the realization of the asset return, $S_n^i | \theta \overset{i.i.d.}{\sim} f(s_n^i | \theta)$. While assets are assumed to be perfectly divisible, signals have to be acquired in discrete numbers.

Useful properties of the Poisson and gamma distributions are reported in appendix B. The most important property relates to the updating of beliefs.

Fact 1 (Conjugate updating). *Suppose the prior distribution of θ is a gamma distribution with parameters $\bar{\alpha} > 0$ and $\bar{\beta} > 0$. Signals $S_1^i, ..., S_{N^i}^i$ are independently drawn from a Poisson distribution with the realization of θ as parameter. Then the* post notitias *distribution of θ, given realizations $s_1^i, ..., s_{N^i}^i$ of the signals, is a gamma distribution with parameters $\alpha^i = \bar{\alpha} + \sum_{n=1}^{N^i} s_n^i$ and $\beta^i = \bar{\beta} + N^i$.*

Proof. See Robert (1994, Proposition 3.3). ∎

The mean of a gamma distributed return θ is α^i / β^i, and its variance $\alpha^i / (\beta^i)^2$. The mean-variance ratio will play a key role in particular: $\mathbb{E}^i[\theta] / \mathbb{V}^i(\theta) = \beta^i$.

1.2   Portfolio choice

A signal costs c. So, the intertemporal budget constraint of investor i becomes

$$b^i + Px^i = b_0^i + Px_0^i - C_0^i - cN^i \tag{1}$$

today.

$$C_1^i = Rb^i + \theta x^i \tag{2}$$

will be available for consumption tomorrow.

---

[7]The gamma distribution is also a conjugate prior distribution to itself and a normal distribution, for instance.

Assumption 2 (Expected CARA utility). *Investors $i = 1, \ldots, I$ evaluate consumption with additively separable utility U^i at constant individual discount rates ρ^i and under common constant absolute risk aversion:*

$$U^i = \mathbb{E}\left[u(C_0^i) + \rho^i u(C_1^i) \,\middle|\, \mathcal{F}^i\right], \tag{3}$$

where $u(C) = -\exp\{-AC\} < 0$, $A > 0$ is the Pratt-Arrow measure of absolute risk aversion, and $\mathcal{F}^i$ denotes investor i's information set.

For ease of notation, abbreviate investor i's conditional expectations with $\mathbb{E}^i[\cdot] \equiv \mathbb{E}[\cdot|\mathcal{F}^i]$ when they are based on *post notitias* beliefs, and with $\mathbb{E}_{ante}^i[\cdot] \equiv \mathbb{E}[\cdot|\mathcal{F}_{ante}^i]$ for *ante notitias* beliefs in anticipation of N^i signal receipts. *Post notitias* expectations will coincide for all investors under fully revealing price.

To analyze the utility benefit of signals, given expected price responses to signal realizations in general equilibrium, it is instructive to consider the case of investors who are identical in beliefs and risk aversion. This homogeneity will make price fully revealing (common priors are not necessary for fully revealing price but convenient). Investors' initial portfolios, however, are heterogeneous.

Assumption 3 (Common priors and risk aversion). *Investors hold identical prior beliefs about the joint signal-return distribution and share the same degree of risk aversion.*

If investors also know market size, asset price becomes fully revealing.

Assumption 4 (Known market size). *The average supply of the risky asset $\bar{x}$ and the total number of investors I are certain and known.*

Assumptions 2 through 4 provide a closed-form solution for financial market equilibrium in a RICE. The prior information market equilibrium in a RICE, however, has no closed form. Its analysis becomes tractable under the following additional assumption.

Assumption 5 (Single-price responses to signal realizations). *The equilibrium price of the safe asset does not respond to signal realizations on other assets' returns.*

The assumption is equivalent to the limiting case where markets for single risky assets are small relative to the overall market for safe bonds so that single signal realizations alter R negligibly little (see appendix D for a formal derivation). Economies with large government debt and small open economies are leading examples.

On the second stage (Wall Street at 10am), investor i maximizes expected utility (3) with respect to consumption C_0^i today and stock holdings x^i, given (1) and (2) and the asset price P, and after having received the realizations of her N^i signals $\{s_j^i\}_{j=1}^{N^i}$. For a gamma distributed asset return, demand for the risky asset becomes

$$x^{i*} = \frac{\beta^i}{A} \frac{\mathbb{E}^i[\theta] - RP}{RP} \equiv \frac{\beta^i}{A} \cdot \xi^i \tag{4}$$

(plug the moment-generating function of the gamma distribution in appendix B (fact 4) into the first-order condition (A.1) in appendix A). Demand for the risky asset decreases in price and the safe asset's return; demand is the higher the less risk averse investors become (lower A) or the higher the expected mean-variance ratio β^i of the asset is. Investors go short in the risky asset whenever their return expectations fall short of opportunity cost, $\mathbb{E}^i[\theta] < RP$, and go long otherwise. Under CARA, demand for the risky asset is independent of wealth W_0^i.

The term $\mathbb{E}^i[\theta - RP]/RP$ in (4) is an individual investor i's expected relative excess return over opportunity cost. Risk averse investors demand this premium. For later reference, define the *expected relative excess return* as

$$\xi^i \equiv \frac{\mathbb{E}^i[\theta] - RP}{RP}. \tag{5}$$

The expected relative excess return ξ^i has important informational properties that crucially affect incentives for information acquisition.

2 Financial Market Equilibrium

To solve for a RICE backwards, this section establishes financial market equilibrium given any market equilibrium for spy robots.[8] Investors $i = 1, ..., I$ have received the realizations of their conditionally independent $N^i \geq 0$ signals. It is 10am, and investors choose portfolios (x^{i*}, b^{i*}) given their *post notitias* information.

In REE, rational investors not only consider their own signal realizations. They extract information also from price. Because RP and $\sum_{n=1}^{N^i} s_n^i$ are correlated in equilibrium, the *post notitias* distribution of the asset return, based on this information set, can be complicated. If price P is fully revealing, however, the information sets of all investors coincide. This gives the rational beliefs in REE a closed and linear form analogous to fact 1.

---

[8]The equilibrium at Wall Street is analogous to that in Muendler (2005) because risky asset demand is independent of endowments under CARA.

Lemma 1 (Unique asset market REE). *Under assumptions 1 through 4, the asset market REE in RICE is unique and symmetric with*

$$\alpha^i = \bar{\alpha} + \sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k \equiv \alpha, \tag{6}$$

$$\beta^i = \bar{\beta} + \sum_{k=1}^{I} N^k \equiv \beta, \tag{7}$$

$$RP = \frac{\alpha}{\beta} \frac{1}{1+\xi}, \tag{8}$$

where $x^{i*} = \bar{x}$ and $\xi^i = \xi \equiv A\bar{x}/\beta$.

Proof. By (4) and for beliefs (6) and (7), $x^{i*} = \alpha/(A\,RP) - \beta/A$ for all i. So, market clearing $x^{i*} = \bar{x}$ under definition 1 of RICE implies (8).

Uniqueness of beliefs (6) and (7) follows by construction. By (4) and market clearing, RP can be expressed as a linear affine function of $\Sigma_{k=1}^{I}\Sigma_{n=1}^{N^k}s_n^k$ with known parameters, because risk aversion A is common to all investors. But then, every investor i can infer $\Sigma_{k\neq i}\Sigma_{n=1}^{N^k}s_n^k$ from her knowledge of own signal realizations. Since the random variables $\Sigma_{k\neq i}\Sigma_{n=1}^{N^k}s_n^k$ and $\Sigma_{n=1}^{N^i}s_n^i$ are Poisson distributed by fact 3 (appendix B) and conditionally independent given θ, a rational investor applies Bayesian updating following fact 1. Hence, $\alpha^i = \bar{\alpha} + \Sigma_{n=1}^{N^i}s_n^i + \Sigma_{k\neq i}^{I}\Sigma_{n=1}^{N^k}s_n^k$ and $\beta^i = \bar{\beta} + N^i + \Sigma_{k\neq i}^{I}N^k$. $\Sigma_{k\neq i}^{I}N^k$ is known by definition 1 of RICE.

No less than $\Sigma_{k=1}^{I}\Sigma_{n=1}^{N^k}s_n^k$ signals can get revealed in REE because, if some α^i did not include some s_n^i, investor i would violate Bayesian updating (fact 1). ∎

The equilibrium price P fully reveals aggregate information of all market participants $\sum_{i=1}^{I}\sum_{n=1}^{N^i}s_n^i$. This is a sufficient statistic for every moment of θ given $\sum_{i=1}^{I}N^i$ (which is known by definition 1 of RICE). Price is also fully revealing if investors' individual prior beliefs differ and investors merely know average prior beliefs. In general, the equilibrium price is fully revealing if and only if assumptions 1, 2 and 4 are satisfied, and investors know average prior beliefs and share a common degree of risk aversion (see appendix C).

In fully revealing REE, investors' information sets coincide by (6) and (7). Consequently, the expected relative excess return $\xi^i = \xi$ (5) coincides. It becomes

$$\xi = \frac{\mathbb{E}[\theta] - RP}{RP} = \frac{A\bar{x}}{\beta} = \frac{A\bar{x}}{\bar{\beta} + \sum_{k=1}^{I}N^k} \in (0, \bar{\xi}] \quad \text{where} \quad \bar{\xi} \equiv \frac{A\bar{x}}{\bar{\beta}}. \tag{9}$$

The upper bound $\bar{\xi}$ is the *elementary excess return*: the maximal expected excess return absent information acquisition.

The expected relative excess return over opportunity cost $\mathbb{E}^i[\theta - RP]/RP$ is crucial for individual incentives to acquire information. Information acquisition diminishes the expected relative excess return. Equilibrium price P will reveal signal realizations. So, private information will become publicly known to investors through informative price, and risk averse investors will value the risky asset more thus bidding up price. Therefore, investors expect higher opportunity cost of the risky asset $\mathbb{E}_{ante}[RP]$ in the face of reduced uncertainty. The diminishing effect of public information on the expected relative excess return also occurs in additive signal-return models for any distribution with a moment-generating function (Muendler forthcoming) and when price is partially revealing (Admati 1985, Easley and O'Hara 2004).

Lemma 2 (Diminishing excess return). *Under assumptions 1 through 4, the expected relative excess return ξ in asset market REE strictly falls in the number of signals, while the expected opportunity cost of the risky asset $\mathbb{E}_{ante}[RP]$ strictly increases in the number of signals* ante notitiam.

Proof. Note that $\xi = \mathbb{E}_{ante}[\xi]$ by (9). The number of signals $\bar{N} = \sum_{k=1}^{I} N^k$ strictly diminishes ξ by (9). The number of signals strictly raises $\mathbb{E}_{ante}[RP] = (\bar{\alpha} + \bar{\alpha}\bar{N}/\bar{\beta})/(A\bar{x} + \beta)$ since $\partial\mathbb{E}_{ante}[RP]/\partial\bar{N} = \bar{\xi}/\beta^2(1+\xi)^2 > 0$. ∎

3 Information Market Equilibrium

Given the expected financial market equilibrium, how much information do investors acquire in RICE? Investors dislike the diminishing effect of information on the expected relative excess return ξ but anticipate a more educated portfolio choice if they can receive signal realizations. In their *ante notitias* choice of the optimal number signals, risk averse investors weigh the diminishing excess return and the marginal cost of a signal against the benefit of a more informed intertemporal consumption allocation.

Signals raise asset price $\mathbb{E}_{ante}^i[P]$ *ante notitias* by Lemma 2. So, investors who are endowed with the risky asset x_0^i experience a positive endowment revaluation effect of signal acquisition *ante notitias*. In other words, investors who hold the risky asset have an incentive to acquire information and raise the value of their endowment $W_0^i = b_0^i + Px_0^i$ by buying signals. To distinguish between the diminishing effect of information on the expected relative excess return ξ and the positive endowment revaluation effect of information, it is instructive to define the *relative risky asset endowment* of investor i as

$$\omega^i \equiv \frac{x_0^i}{\bar{x}} \in [0, I].$$

If one investor initially owns all risky assets, then $\omega^i = I$. If an investor owns the market average amount of assets, then $\omega^i = 1$.

Investors evaluate *ante notitias* expected utility for their signal choice, taking the potential revaluation of their endowments into account. *Ante notitias* expected utility has a closed form if R is constant, which assumption 5 guarantees.

For heterogeneous investors with arbitrary endowments $x_0^i = \omega^i \bar{x}$ and Poisson-gamma signal-return distributions, *ante notitias* expected utility becomes

$$
\mathbb{E}_{ante}^i\left[U^i\right] = -\delta^i \exp\left\{-A\tfrac{R}{1+R}(W_0^i - cN^i)\right\} \tag{10}
$$
$$
\times\left[1 + \left(\left[(1+\xi)\exp\left\{\tfrac{\xi}{1+\xi}(\omega^i-1)\right\}\right]^{\frac{1}{1+R}} - 1\right)\tfrac{\bar{\xi}}{\xi}\right]^{-\bar{\alpha}}
$$

(see appendix E).

Although the number of signals is discrete, one can take the derivative of *ante notitias* utility with respect to N^i to describe the optimal signal choice. Strict monotonicity of the first-order condition in the relevant range will prove this to be admissible. Differentiating (10) with respect to the number of signals yields the incentive to purchase information. As long as $\partial\mathbb{E}_{ante}^i[U^{i*}]/\partial N^i > 0$, investor i will generically purchase more signals. If $\partial\mathbb{E}_{ante}^i[U^{i*}]/\partial N^i \leq 0$ for all N^i, she purchases no information at all.

Differentiate (10) with respect to N^i, and divide by $-\mathbb{E}_{ante}^i[U^{i*}] > 0$ for clarity, to find

$$
-\frac{1}{\mathbb{E}_{ante}^i[U^{i*}]}\frac{\partial\mathbb{E}_{ante}^i[U^{i*}]}{\partial N^i} = -A\frac{R}{1+R}\,c \tag{11}
$$
$$
+\frac{\bar{\alpha}}{\bar{\beta}}\frac{\left[(1+\xi)\exp\left\{\tfrac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}}\left(1-\tfrac{1}{1+R}\tfrac{\xi(\xi+\omega^i)}{(1+\xi)^2}\right)-1}{1+\left(\left[(1+\xi)\exp\left\{\tfrac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}}-1\right)\tfrac{\bar{\xi}}{\xi}}.
$$

The first term on the right hand side of (11) is negative and represents the *marginal cost* of a signal, *MC*. The second term expresses the *potential marginal signal benefit* $MB^i(\xi,\omega^i)$, which can be positive or negative.[9] Note that the incentive for information acquisition does not depend on an investor's patience.

---

[9]In Muendler (2005), where investors are homogeneous, I call the marginal utility benefit the action value of information. Contrary to the action value of information, the potential marginal signal benefit is not homogeneous at any given information level (any given ξ). The potential benefit can be a strict utility loss for some investors at a given ξ while other investors still gain from more information.
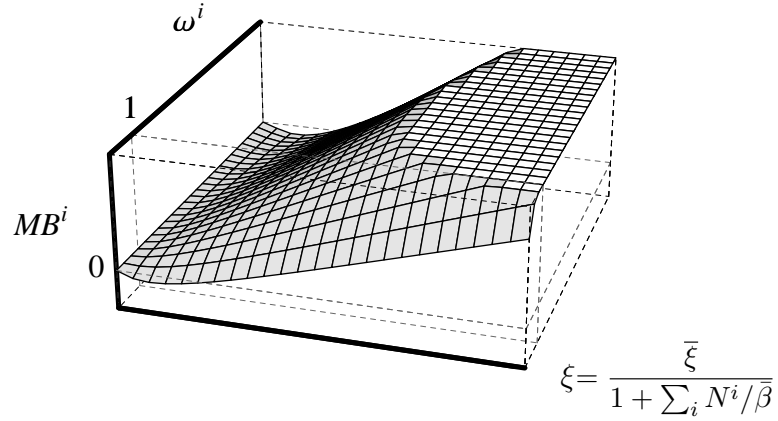
The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$.

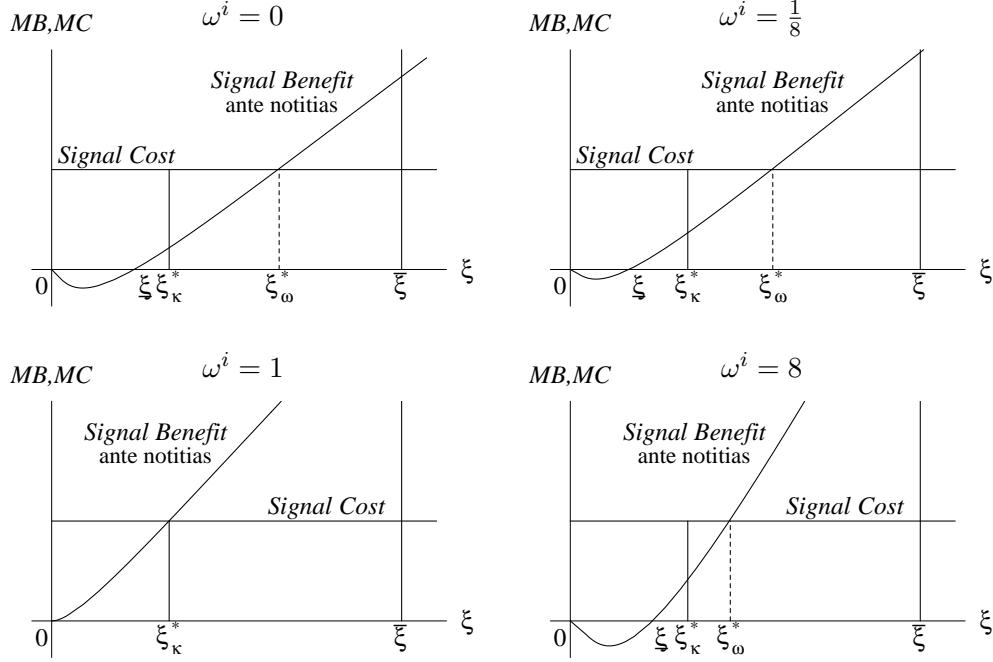Figure 2: **Potential marginal signal benefits**

3.1 The potential marginal signal benefit

The potential marginal signal benefit $MB^i(\xi, \omega^i)$ of a signal varies with ω^i and is therefore investor specific. Figure 2 depicts the range of individual marginal benefit schedules $MB^i(\xi, \omega^i)$ by relative risky asset endowment $\omega^i \in [0, I]$ (along the axis running from front to back) and $\xi \in (0, \bar{\xi}]$ (ξ increases along the axis running from west to east).

Rational investors view the choice of the total number of signals $\sum_{k=1}^{I} N^k$ as the converse of a choice of the expected relative excess return ξ because more signals $\sum_{k=1}^{I} N^k$ imply a lower expected relative excess return ξ and vice versa (Lemma 2). Figure 2 therefore also depicts how the marginal signal benefit $MB^i(\xi, \omega^i)$ varies as the expected relative excess return ξ changes (ξ falls along the axis running from east to west). When there is no information ($\sum_{k=1}^{I} N^k = 0$), the expected relative excess return ξ is at its maximum elementary level $\bar{\xi}$ (at the upper bound to the east). When there is perfect information ($\sum_{k=1}^{I} N^k \to \infty$), the risky asset becomes identical to a safe bond so that there is no excess return ($\xi = 0$ at the lower bound to the west).

3.2 An investor's natural transparency limit

There is a fundamental tradeoff behind the potential marginal signal benefit MB. An additional signal can diminish the expected relative excess return ξ so strongly

The expected relative excess return $\xi = \overline{\xi}/(1 + \sum_i N^i/\overline{\beta})$ strictly decreases in $\sum_i N^i$.
Parameters: $A = 2$, $\overline{\alpha} = 1.3$, $\overline{\beta} = 1$, $R = 1.1$, $\overline{x} = 7$, $c = .1$.

Figure 3: **Potential marginal signal benefits by endowment**

that this negative effect more than outweighs the benefits of information.[10]

Figure 3 depicts sections of the graph in Figure 2 for four relative risky asset endowments ω^i and shows the individual marginal benefit schedules $MB^i(\xi, \omega^i)$. These sections could represent an economy with ten investors, for instance, where eight investors hold $\omega^i = 1/8$ and one investor each holds $\omega^i = 1$ and $\omega^i = 8$.

Starting from zero information (at the eastern most excess return $\xi = \overline{\xi}$), the individual marginal benefit schedule $MB^i(\xi, \omega^i)$ decreases monotonically in the number of signals for every investor (as we move westwards). Ultimately, the potential marginal benefit MB of an additional signal reaches zero. Most important, the potential marginal signal benefit MB turns strictly negative when the expected relative excess return ξ drops very far. Call the threshold where

---

[10]It can be shown for the case of a normal-normal pair of signal-return distributions that the diminishing effect of information on the expected excess return is so strong that the MB never turns positive for $\omega^i = 0$ (an implication of Theorem 4, Muendler forthcoming). For a Poisson-gamma pair of distributions, however, the MB term in (11) can take a negative or a positive sign for $\omega^i = 0$.

the potential marginal signal benefit turns weakly negative investor i's *natural transparency limit* $\underline{\xi}^i$.

At the natural transparency limit $\underline{\xi}^i$, investor i pays to inhibit further information disclosure. The potential benefit MB does not constitute a benefit but a cost once ξ falls below an investor's natural transparency limit. The negative MB for low ξ reflects that, given a relatively large number of available signals, the negative effect of an additional signal on the expected relative excess return ξ outweighs the benefit from a more informed expected portfolio choice *ante notitias*. A low ξ means that investors currently hold relatively many signals given market size and the mean-variance ratio of the asset.

The transparency limit is investor specific because information revalues the risky asset, and the revaluation affects investors with heterogeneous endowments in different ways. The diminishing effect of an additional signal on the excess return ξ weighs heavily for investors with large initial risky asset positions because *ante notitias* they are strongly affected by expected asset price volatility, anticipating that information will move price *post notitias*. The diminishing effect of an additional signal on the excess return is also particularly strong for investors with no endowment of the risky asset ($x_0^i = 0$) since the increase in the opportunity cost RP is not mitigated by any positive wealth effect of asset price on their endowments.

Facing the marginal utility cost of information MC, every investor comes up with an individually optimal choice of $\xi_{\omega^i}^*$ given her relative stock endowment ω^i. When the elementary excess return $\bar{\xi} \equiv A\bar{x}/\bar{\beta}$ (the maximal expected excess return absent information acquisition) is high, then a given choice of excess return $\xi_{\omega^i}^*$ requires a large amount of information acquisition, and vice versa. So, information demand is strong if many risky assets are in the market, or if investors are highly risk averse, or if prior expectations of the mean-variance ratio of the asset return are relatively low so that uncertainty matters strongly compared to expected returns.

Proposition 1 (Potential marginal signal benefit). *The following is true for the potential marginal signal benefit* $\mathrm{MB}(\xi, \omega^i)$ *under assumptions 1 to 5.*

1. *Investor i's natural transparency limit $\underline{\xi}^i \in (0, \infty)$ uniquely solves the zero potential marginal signal benefit condition* $\mathrm{MB}(\underline{\xi}^i, \omega^i) = 0$ *given $R \in (0, \infty)$ and ω^i. The potential marginal signal benefit depends on ω^i but not on $\bar{\xi}$.*

2. *The potential marginal signal benefit* $\mathrm{MB}(\xi, \omega^i)$ *takes strictly positive values if and only if the prevailing expected relative excess return exceeds investor i's natural transparency limit $\xi > \underline{\xi}^i$.*

3. *If investor i's natural transparency limit is less than the elementary excess return $\underline{\xi}^i < \overline{\xi}$ then, in the range $\xi \in [\underline{\xi}^i, \overline{\xi}]$, the potential marginal signal benefit $\overline{\mathrm{MB}}(\xi, \omega^i)$ strictly monotonically increases in ξ and is unbounded for arbitrarily large ξ.*

4. *If investor j has a risky asset endowment around the market average with $\omega^j \in [\sqrt{R}/(\sqrt{1+R} + \sqrt{R}), \sqrt{1+R}/(\sqrt{1+R} - \sqrt{R})]$, then the potential marginal signal benefit $\mathrm{MB}(\xi, \omega^j)$ attains strictly positive values for any $\xi \in (0, \overline{\xi}]$ so that investor j's natural transparency limit is infinite information.*

Proof. See appendix F. ∎

Proposition 1 establishes that there is an investor specific natural transparency limit below which the investor pays to inhibit further disclosure. The natural transparency limit is unique for each investor i and is determined by the investor's risky asset endowment ω^i (statements 1 and 2). So, the incentive for information acquisition varies with the risky asset endowment.

Proposition 1 also shows, however, that there is always a market size $\bar{x}$, or a degree of risk aversion A, or a level of the prior mean-variance ratio of the risky asset $\bar{\beta}$ behind $\overline{\xi}$ so that, for any investor i with endowment ω^i, at least one costly signal becomes worthwhile to acquire in equilibrium as $\bar{x}$ is raised (statement 3).

Finally, Proposition 1 identifies a range of risky asset endowments $\omega^i \in [\sqrt{R}/(\sqrt{1+R} + \sqrt{R}), \sqrt{1+R}/(\sqrt{1+R} - \sqrt{R})]$ where the potential marginal signal benefit never turns negative (statement 4), not even as ξ approaches zero. This range contains the market average endowment $\omega^i = 1$ as depicted in Figure 3. An investor with initial stock holdings in this range will hire unboundedly many spy robots when their fee goes to zero.

These properties of the potential marginal signal benefit MB characterize the information market equilibrium in RICE. Signals are public goods and therefore perfect strategic substitutes under fully revealing price because any fellow investor's signal is as useful (or detrimental) as an own signal. Consequently, the information market equilibrium does not pin down how many signals a single investor holds. In RICE, one investor may acquire all $\sum_i N^i$ signals while nobody else buys any signal, or all investors may hold the same number of signals. Monotonicity of the individual potential marginal signal benefit schedules implies, however, that the equilibrium information level $\sum_{k=1}^{I} N^{k,*}$ is unique.

Proposition 2 (Information market equilibrium). *A RICE results in the following informational outcomes for any $R \in (0, \infty)$ under assumptions 1 to 5.*

1. *If the cost of a signal is strictly positive, then the equilibrium information level $\sum_{k=1}^{I} N^{k,*}$ is unique.*

2. *If the cost of a signal is strictly positive, any permutation of the signal allocation that maintains the information level $\sum_{k=1}^{I} N^{k,*}$ is an equilibrium.*

3. *If the cost of a signal is nil but $R > 0$, and if there is at least one investor j with a risky asset endowment $\omega^j \in [\sqrt{R}/(\sqrt{1+R}+\sqrt{R}), \sqrt{1+R}/(\sqrt{1+R}-\sqrt{R})]$, then the unique signal market equilibrium involves an infinite amount of freely received signals.*

It remains to determine how investors arrive at $\sum_{k=1}^{I} N^{k,*}$ in the equilibrium for spy robots.

3.3 Dominant investor valuation of signals

Information demand is intricately tied to investors' risky asset endowments in RICE. There is, in fact, a single dominant investor with an above-average endowment of the risky asset. This dominant investor's marginal signal valuation dominates everyone else's valuation so that she single-handedly picks the information market outcome. In the sample economy of Figure 3, the average investor κ with $\omega^\kappa = 1$ has the strongest incentive for information acquisition among the ten investors and continues to acquire signals until the expected relative excess return ξ is diminished into a neighborhood around ξ_κ^*. All other investors would stop acquiring signals earlier: at some expected relative excess return $\xi_\omega^* > \xi_\kappa^*$. In Figure 3, the dominant investor κ's endowment revaluation effect is so strong that the individual marginal signal benefit MB^κ never turns negative for any level of the expected relative excess return ξ. Proposition 3 shows that the dominant investor is to be found in an open set around investors with endowments of $\omega^\kappa = 1$ and above.

The individual marginal signal benefit in equation (11) involves the expected relative excess return ξ and investor i's relative risky asset endowment ω^i in non-algebraic ways. Proposition 3 states properties of the information market equilibrium for intervals of endowments.

Proposition 3 (Dominant Investor Valuation of Signals). *A RICE has the following allocation properties for any $R \in (0, \infty)$ under assumptions 1 to 5.*

1. *Given any ξ^*, the individual marginal signal benefit $\mathrm{MB}(\xi^*, \omega^\kappa)$ is maximal in equilibrium for a dominant investor κ. The dominant investor's unique relative risky asset endowment falls into the interval $\omega_{\max MB}^\kappa \in (1, 1 + R(1+\xi^*))$. This investor determines the total number of signals $\sum_{k=1}^{I} N^{k,*}$ in equilibrium and diminishes expected relative excess return to ξ^*.*
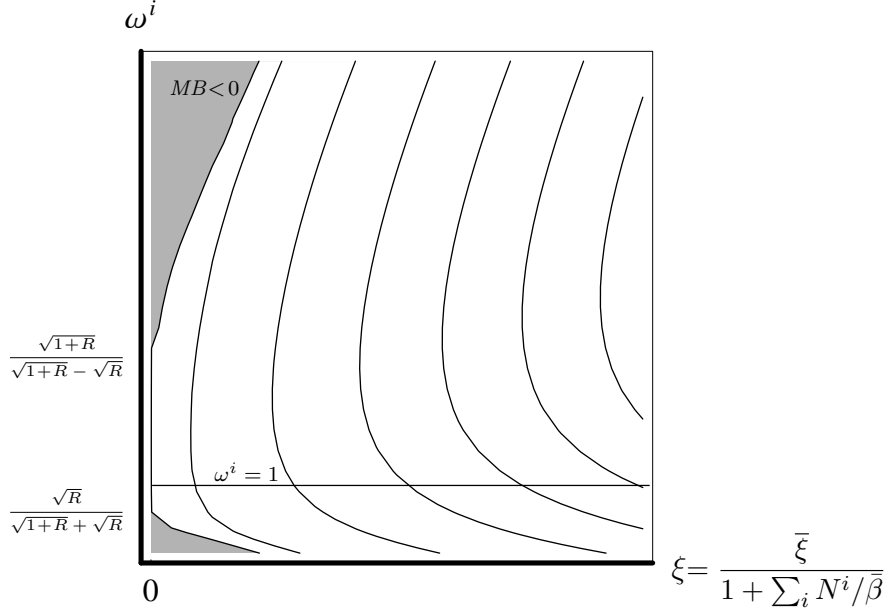
18

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$.

Figure 4: **Contours of potential marginal signal benefits**

2. *The individual marginal signal benefit* $\text{MB}(\xi^*, \omega^i)$ *at expected relative excess return* ξ^* *is strictly positive for investors with endowments* ω^i *in an open interval* Ω^+ *that includes* $[1, \omega^\kappa] \subset \Omega^+$.

Proof. See appendix G. ∎

Figure 4 shows contours of the potential marginal signal benefit $MB(\xi, \omega^i)$ and serves to illustrate Proposition 3. Proposition 3 establishes that, for a given equilibrium excess return ξ^* (or any ξ), there is a unique $\omega^\kappa_{\max MB}$ that maximizes the potential marginal signal benefit (statement 1). Equivalently, since $MB(\xi, \omega^i)$ is monotonic in ξ when positive (Proposition 1), every marginal signal benefit $MB(\xi, \omega^i)$ contour must have one unique $\omega^\kappa_{\max MB}$, for which ξ is minimal. Figure 4 depicts these $\omega^\kappa_{\max MB}$ as the western most points of the MB contours.

Fix the marginal cost of a signal in utility terms, $cAR/(1 + R)$. This cost is the same for all investors by the first order condition (11). So, in equilibrium, the decisive investor must sit on the marginal signal benefit contour $MB(\xi, \omega^i) = cAR/(1 + R)$. Pick any relative risky asset endowment ω^j; the $MB(\xi, \omega^i)$ contour returns the expected relative excess return ξ that investor

19

j finds optimal. Investor j will acquire signals until the expected relative excess return in equilibrium is pushed down to her desired level. Move along the $MB(\xi, \omega^i)$ to its western most point. This is the relative risky endowment level of the unique dominant investor, $\omega^\kappa_{\max MB} \in (1, 1 + R(1 + \bar{\xi}))$, for whom the incentives to acquire information strictly exceed those of any other investor.

For investors with relative risky asset endowments below or above $\omega^\kappa_{\max MB}$, the diminishing effect of signals on the expected excess return ξ weighs more heavily. So, the investor with relative risky asset endowment $\omega^\kappa_{\max MB}$ single-handedly determines the information market outcome. This investor κ will continue acquiring signals and diminish the expected relative excess return ξ until the total number of signals $\sum_{k=1}^{I} N^{k,*}$ satisfies her first-order condition (11) for signal demand. Proposition 3 also shows, however, that investors with relative risky asset endowments in a range from below unity to above $\omega^\kappa_{\max MB}$ do not suffer a utility loss from the dominant investor's signal choice (statement 2). What is the overall welfare impact of transparency?

4 Informational Efficiency

The rational Bayesian framework permits the application of a Pareto criterion to judge transparency in financial markets.
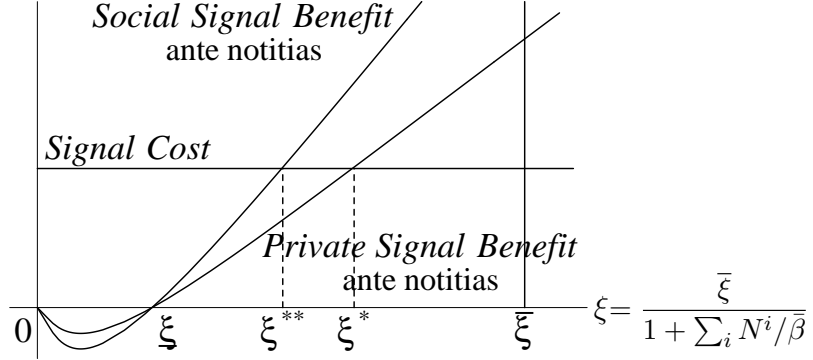
Definition 2 (Informational Pareto efficiency) *An allocation of x^{i**} risky assets, b^{i**} safe bonds, and N^{i**} signals to investors $i = 1, ..., I$ is called informationally Pareto efficient in a given market environment $(\bar{\xi}, R)$ if there is no other allocation such that all investors are at least as well off and at least one investor is strictly better off.*

To investigate whether RICE is Pareto efficient, imagine a benevolent social planner who can instruct every consumer j to buy exactly N^{j**} signals. This social planner maximizes $\sum_{j=1}^{I} \mathbb{E}^i_{ante}[U^j]$ with respect to $\{N^1, ..., N^I\}$. Thus, similar to Samuelson's (1954) condition for public good provision, a benevolent social planner's first-order conditions for information allocation are not (11) but instead

$$-\frac{1}{\mathbb{E}^j_{ante}[U^j]} \frac{\partial \sum_{k=1}^{I} \mathbb{E}^k_{ante}[U^{k**}]}{\partial N^k} = -A\frac{R}{1+R} c \tag{12}$$

$$+\frac{\bar{\alpha}}{\bar{\beta}} \frac{\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} \left(1 - \frac{1}{1+R}\frac{\xi^2}{(1+\xi)^2}\right) - 1}{1 + \left(\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\frac{\bar{\xi}}{\xi}} \left(1 + \sum_{k \neq j}^{I} \frac{\mathbb{E}^k_{ante}[U^{k**}]}{\mathbb{E}^j_{ante}[U^{j**}]}\right)$$

SB, MB, MC

Social Signal Benefit
ante notitias

Signal Cost

Private Signal Benefit
ante notitias

0 $\underline{\xi}$ ξ^{**} ξ^{*} $\overline{\xi}$ $\xi = \dfrac{\overline{\xi}}{1 + \sum_i N^i / \overline{\beta}}$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 5: **Socially desirable information choice for homogeneous investors**

for any $j \in 1, ..., I$, written in terms of that investor j's utility. Thus, compared to the privately perceived benefits, the potential social benefits SB that a social planner considers scale up the private benefits MB by a factor of $1 + (1/\mathbb{E}^j_{ante}[U^{j**}]) \cdot \sum_{k \neq j}^{I} \mathbb{E}^k_{ante}[U^{k**}] > 1$.

4.1 Welfare effects for homogeneous investors

To simplify the welfare analysis, consider the case of $I - 1$ homogeneous investors with $\omega^i = 0$ and one single owner j of the risky asset with $\omega^j = I$. The upper left and lower right graphs in figure 3 exemplify this case for a sample economy with $I = 8$ investors, seven of whom hold $\omega^i = 0$ while one owns $\omega^j = I = 8$. Neither the sole owner nor investors with safe endowments may value information much. In fact, the marginal signal benefit approaches negative infinity for the sole owner of a risky project as her relative risky asset endowment ω^j (the project size $I\bar{x}$) increases for a given average endowment $\bar{x}$ (claim 4 in appendix G).

Not even for homogeneous investors with no initial risky asset holdings need information be desirable. Proposition 1 implies that signals can turn from a public good into a public bad as market conditions change. These market conditions are captured by $\overline{\xi}$. If $\overline{\xi}$ drops below the homogeneous investors' joint transparency limit, information becomes a public bad, and a benevolent social planner wants to implement an even smaller amount of information than the private market. For no information is acquired in private markets in that case

21

anyway, the market equilibrium is informationally efficient when information is a public bad.

On the other hand, if information is a public good under given market conditions, a social planner wants (weakly) more information to be allocated than markets provide. Individual investors do not take into account that their signal acquisition also benefits other investors through fully revealing price. In this case, markets allocate (weakly) less information than desirable. Signals are not divisible, however, and we cannot infer from condition (12) that a social planner wants to implement strictly more information. In Figure 5, a social planner wants to allocate information so that the relative excess return is brought down from around ξ^* to around ξ^{**}. If a single additional signal makes the implementable level of ξ drop far below ξ^{**}, however, investors could be better off if relative excess return ξ remains at the market equilibrium level around ξ^*.

Even if signals are free with $c = 0$, only the market outcome with finite information is efficient but not the one with infinite information. As long as the bond is valuable ($R > 0$), neither markets nor the social planner want to remove uncertainty. In incomplete markets, investors with no initial risky asset holdings prefer to have a second asset around that is not a perfect substitute to the bond. Risk-averse investors want to hold risky assets that yield a positive excess return ξ over opportunity cost. Only if the bond becomes useless and $R \to 0$, does unbounded information Pareto become efficient.

4.2   Welfare effects for heterogeneous investors

The welfare analysis changes considerably when endowments with the risky asset differ across investors. Now, one dominant investor κ with close-to-market-average holdings of the risky asset determines the equilibrium number of signal in RICE (Proposition 3). Investors with endowments in an open interval around $[1, \omega^\kappa]$ strictly benefit from investor κ's additional information choice since their marginal utility benefit of signals is strictly positive and they do not have to pay for the public good.

For investors outside the open set of endowments of $\omega^\kappa = 1$ and above, however, the individual marginal signal benefit $MB^i(\xi, \omega^i)$ can turn strictly negative even in the presence of the endowment revaluation effect. Reduce the signal cost in Figure 3, for instance. Then the individual marginal benefit schedules $MB^i(\xi, \omega^i)$ can dip into the strictly negative range before investor κ reaches her natural transparency limit. Similarly, consider the western-most (strictly positive) potential marginal benefit contour in Figure 4. At the equilibrium outcome ξ^* for the dominant investor on this contour, the potential marginal signal benefit has turned strictly negative for investors at the extremes of the ω^i distribution.

Formally, at any ξ, the marginal signal benefit approaches negative infinity as ω^i increases for a given average endowment $\bar{x}$ (claim 4 in appendix G). So, when the distribution of risky asset endowments is unequal and many investors hold risky asset endowments far from average, the dominant investor's information choice inflicts a strict negative externality on numerous investors. If, on the other hand, investors' risky asset endowments are distributed closely around the market average, the endowment revaluation effect makes signals similarly valuable to all investors. Proposition 4 summarizes these insights.

Proposition 4 (Informational Efficiency). *A RICE has the following informational efficiency properties for any $R \in (0, \infty)$ under assumptions 1 to 5.*

1. *If every investor i's endowment ω^i falls into the interval $\omega^i \in [1, \omega^\kappa]$, where ω^κ is the dominant investor κ's endowment (Proposition 3), then RICE is not informationally Pareto efficient. Up to discrete tolerance, an increase in the number of signals raises every investor's* ante notitias *utility.*

2. *If there is one investor j with an endowment outside the range where infinite information is beneficial, $\omega^j \notin [\sqrt{R}/(\sqrt{1+R}+\sqrt{R}), \sqrt{1+R}/(\sqrt{1+R}-\sqrt{R})]$, then there is an information cost c low enough so that RICE is informationally Pareto efficient. An increase in the number of signals strictly reduces investor j's* ante notitias *utility while, up to discrete tolerance, the increase in the number of signals strictly raises the dominant investor's* ante notitias *utility.*

Information acquisition creates a two-group society of investors. The endowment revaluation effect of more signals strictly outweighs the diminishing effect on the expected excess return ξ for a first group of investors in an open set Ω^+ of relative risky asset endowments around $[1, \omega^\kappa] \subset \Omega^+$ (Proposition 3). In fact, given the choice of free signals, investors k with endowments inside the range $\omega^k \in [\sqrt{R}/(\sqrt{1+R} + \sqrt{R}), \sqrt{1+R}/(\sqrt{1+R} - \sqrt{R})]$ (Proposition 1), would remove all uncertainty from the market—just to enjoy the endowment revaluation.

For the second group of investors, endowments are either too small so that the diminishing effect on the expected excess return starts to outweigh the endowment revaluation effect at some small enough ξ, or endowments are too large so that the expected variance of price outweighs the endowment revaluation effect at some small enough ξ. Outside the endowment range where infinite costless information is beneficial (outside $[\sqrt{R}/(\sqrt{1+R} + \sqrt{R}), \sqrt{1+R}/(\sqrt{1+R} - \sqrt{R})]$), the marginal *ante notitias* utility of an additional signal is strictly negative for low ξ. The second group of investors suffers a strict negative externality in *ante notitias* utility terms from the rush to information of the first group of investors.

The Pareto criterion applied to a social planner's transparency choice loses its bite: investors in the second group are worse off with every additional signal.

Overall, transparency leads to an unambiguous Pareto improvement only in markets with sufficiently similar investors—after the risky asset has been issued and all investors hold it in their initial portfolios to some degree. If an asset has not been issued yet so that only one agent holds the asset in his or her initial endowment, public disclosure can violate everyone's transparency limit.

4.3 Asset price precision

Most commonly, the informational efficiency of financial markets is judged with criteria that do not relate to welfare but to the degree of information transmission through asset price. Fama (1970) discerns three degrees of market efficiency in this welfare-independent sense: Strong, semi-strong, and weak. Prices are fully revealing in RICE under the assumptions of this paper (Lemma 3 in appendix C). So, $\mathbb{E}^i[RP - \mathbb{E}^i[RP]] = 0$ and RICE satisfies strong-form efficiency. An alternative statistically well defined measure of the informativeness of a signal is its precision, the inverse of the *ante notitias* variance. Informational efficiency in this non-welfare sense relates to price as a source of information.

The precision of price, a Poisson variable by (4) and fact 3 (appendix B), is

$$\frac{1}{\mathbb{E}^i_{ante}[\mathbb{V}^i(P|\theta)]} = \frac{\bar{\beta}^2}{\bar{\alpha}} \frac{\left((1+\bar{\xi}) + \sum_{i=1}^I N^{i*}/\bar{\beta}\right)^2}{\sum_{i=1}^I N^{i*}/\bar{\beta}}.$$

So, the precision of price can fall with the number of signals purchased. For

$$\frac{\partial}{\partial N^i}\left(\frac{1}{\mathbb{E}^i_{ante}[\mathbb{V}^i(P|\theta)]}\right) = \frac{\bar{\beta}}{\bar{\alpha}} \frac{(\sum_{i=1}^I N^{i*}/\bar{\beta})^2 - (1+\bar{\xi})^2}{(\sum_{i=1}^I N^{i*}/\bar{\beta})^2},$$

each additional signal reduces the precision of the market clearing price if the amount of pre-existing information $\sum_{k=1}^I N^{k,*}$ is small.

Proposition 5 (Precision loss of the price system). *In asset market REE under assumptions 1 through 4, the* ante notitias *precision of the price system decreases with every additional signal if and only if $\sum_{k=1}^I N^{k,*}/\bar{\beta} < 1 + \bar{\xi}$.*

Precision of price can fall with the number of signals purchased.[11] Each investor anticipates that she and all others will respond to signals in their portfolio

---

[11]Bushee and Noe (2000) provide empirical evidence that more information may worsen price volatility.

choice. From an *ante notitias* perspective, asset demand (4) can become more volatile with the anticipated arrival of information. The expected variance of asset demand is

$$\mathbb{E}^i_{ante}\left[\mathbb{V}^i\left(x^{i*}|\theta\right)|RP\right] = \frac{\bar{\alpha}}{\bar{\beta}A^2(RP)^2}\left(\sum_{k=1}^{I}N^k\right)$$

by fact 3. Financial markets need to clear and every investor ends up holding $\bar{x}$ risky assets in equilibrium by Proposition 3, irrespective of information. Hence, market price has to fully absorb any demand moves that stem from information revelation. As a consequence, the variance of price can increase with more information acquisition. When there is relatively little pre-existing information $\sum_{k=1}^{I}N^{k,*}$, an additional signal will affect individual demands strongly and thus add to the price's variance. If, on the other hand, a lot of information is available already, an additional signal that gets fully revealed through price will move investors' demands little. If investors receive many signals, an additional piece of information is likely to confirm previous observations and tends to stabilize demand. So, equilibrium price is expected to become less volatile with an additional signal if the pre-existing information level $\sum_{k=1}^{I}N^{k,*}$ is high.

Rational investors completely internalize this change in price volatility when they maximize *ante notitias* utility. In that sense, the precision of price is irrelevant for the Pareto efficiency of RICE.

5   Conclusion

By adding a stage of signal acquisition to a canonical model of portfolio choice, this paper has shown that investors with heterogeneous initial holdings of the risky asset find financial transparency, even if free, only desirable to a degree. In reducing uncertainty, transparency raises expected asset price and thus benefits holders of the risky asset. At their natural transparency limits, however, investors pay to inhibit further disclosure in order to prevent the information-induced increase in expected asset price and to avert an erosion of the asset's expected excess return. The natural transparency limit varies with an investor's initial portfolio position. The natural transparency limit shifts to perfect information for investors around the market average risky asset endowment because these investors benefit most from rising expected asset price. There is, in fact, a dominant investor with a risky asset endowment modestly above market average who keeps acquiring signals after all other investors' demand for transparency is exhausted. The dominant investor's transparency choice strictly improves welfare for investors with similar (modestly smaller or larger) risky asset endowments.

Investors with strongly different risky asset endowments, however, suffer a strict welfare loss if the dominant investor keeps acquiring signals beyond their natural transparency limits.

Financial transparency not only changes its utility benefit with market conditions. Transparency raises expected asset price and thus also affects investors' portfolio positions *ante notitias*. So, the value of public signals is intricately linked to the distribution of risky asset endowments across heterogeneous investors. This poses an intriguing problem for the analysis of informational efficiency in the presence of heterogeneous investors. A common distinction in economics between pure allocative efficiency in the Pareto sense and distributive judgements is hard to uphold for financial information in a rational Bayesian framework.

Extensions of the framework in this paper remain for future work: an analysis of information values in complete markets, an investigation of partially revealing equilibrium (such as for investors with different degrees of risk aversion), and the consideration of investors who engage in strategic demand decisions to partly conceal their information. The driving force behind the effects of transparency, however, is the diminishing effect of information on an asset's excess return. Information diminishes the asset's excess return because a sufficient statistic of private signals is publicly inferrable from price. Neither complete markets nor partially revealing equilibrium nor strategic investors can make asset price entirely uninformative, or else price would lose its allocation function. So information continues to diminish excess returns in those settings albeit in a mitigated manner (Admati 1985, Easley and O'Hara 2004). It thus appears plausible that main results of this paper will carry over to generalizations and extensions.

Appendix

A Optimality conditions and portfolio value

Define $t \equiv -Ax^i \in (-\infty, 0)$ for the moment generating function (MGF) $M_{\theta|\mathcal{F}^i}(t)$. Maximizing (3) over x^i and b^i for CARA (assumption 2 and 3) yields the first-order conditions

$$\frac{P}{\rho^i} = H^i M'_{\theta|\mathcal{F}^i}(t) \quad \text{and} \quad \frac{1}{\rho^i R} = H^i M_{\theta|\mathcal{F}^i}(t), \tag{A.1}$$

where $H^i \equiv \exp\{-A[(1+R)b^i + Px^i - W_0^i - cN^i]\}$. Note that H^i, W_0^i, C_1^i and C_0^i are functions of $\mathcal{F}^i$ since RP depends on $\mathcal{F}^i$.

With the definition of H^i, the optimal portfolio value can be written

$$
\begin{aligned}
b^i + Px^i &= \tfrac{1}{1+R}\left(W_0^i - cN^i + RP\, x^i - \tfrac{1}{A}\ln H^i\right) \\
&= \tfrac{1}{1+R}\left[b_0^i + RP(x_0^i/R + x^i) + \tfrac{1}{A}\ln \rho^i R M_{\theta|\mathcal{F}^i}(-Ax^i) - cN^i\right],
\end{aligned}
\tag{A.2}
$$

where the second line follows from the bond first-order condition in (A.1).

The matrix of cross-derivatives for the two assets b^i and x^i reflects the second-order conditions:

$$\mathbf{B} = -A^2 \rho^i \exp\{-ARb^i\} \begin{vmatrix} R(1+R)M_{\theta|\mathcal{F}^i}(t) & \cdot \\ (1+R)M'_{\theta|\mathcal{F}^i}(t) & PM'_{\theta|\mathcal{F}^i}(t) + M''_{\theta|\mathcal{F}^i}(t) \end{vmatrix} \tag{A.3}$$

by (A.1). If $\mathbf{B}$ is negative definite, a unique global utility maximum results. Equivalently, require $-\mathbf{B}$ to be positive definite and all upper-left sub-matrices must have positive determinants. Since the upper-left entry in $\mathbf{B}$ is strictly positive, negative definiteness of $\mathbf{B}$ is equivalent to

$$\det(-\mathbf{B}) = A^4 (\rho^i)^2 \exp\{-2ARb^i\} R(1+R) \left[M''_{\theta|\mathcal{F}^i}(t)M_{\theta|\mathcal{F}^i}(t) - M'_{\theta|\mathcal{F}^i}(t)^2\right] > 0,$$

which in turn is equivalent to

$$\frac{M''_{\theta|\mathcal{F}^i}(t)}{M_{\theta|\mathcal{F}^i}(t)} - \left(\frac{M'_{\theta|\mathcal{F}^i}(t)}{M_{\theta|\mathcal{F}^i}(t)}\right)^2 > 0 \tag{A.4}$$

since $M_{\theta|\mathcal{F}^i}(t) > 0$. This condition implies that $M'_{\theta|\mathcal{F}^i}(t)/M_{\theta|\mathcal{F}^i}(t)$ strictly monotonically increases in t, or strictly monotonically decreases in x^i for $t \equiv -Ax^i$. The condition is satisfied for a gamma distribution.

B Poisson and gamma distributions

Fact 1 in the text states how Poisson signals update beliefs about gamma distributed returns. This appendix lists further useful properties of Poisson and gamma distributions

B.1 Poisson signals

Poisson distributed signals $S_n^i|\theta \overset{i.i.d.}{\sim} \mathcal{P}(\theta)$ have a density

$$f\left(s_n^i \,|\theta\right) = \begin{cases} \exp\{-\theta\}\,\theta^{s_n^i}/s_n^i! & \text{for } s_n^i > 0 \\ 0 & \text{for } s_n^i \le 0 \end{cases}$$

Fact 2 (Poisson MGF). *The MGF of a Poisson signal is*

$$M_{S|\theta}(t) = \exp\{\theta(\exp\{t\} - 1)\}.$$

Proof. Casella and Berger (1990). ∎

Fact 3 (Sum of Poisson signals). *The sum of N independently Poisson distributed signals with a common mean and variance θ, $S_1 + ... + S_N$, has a Poisson distribution with parameter $N\theta$.*

Proof. The distribution of the sum of N independent Poisson variables is the product $\Pi_{n=1}^N f\left(s_n^i \,|\theta\right) = \exp\{-N\theta\}\,\theta^{\sum_{n=1}^N s_n^i}/\sum_{n=1}^N s_n^i!$, a Poisson distribution with parameter $N\theta$. ∎

B.2 Gamma payoffs

Given an individual investor i's information set $\{\alpha^i, \beta^i\}$, the risky asset payoff is distributed $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$ so that its density is

$$\pi\left(\theta \,|\alpha^i, \beta^i\right) = \begin{cases} (\beta^i)^{\alpha^i}\,\theta^{\alpha^i - 1}\,\exp\{-\beta^i\theta\}/\Gamma(\alpha^i) & \text{for } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

where the gamma function is given by $\Gamma(\alpha^i) \equiv \int_0^\infty z^{\alpha^i - 1} e^{-z}\,\mathrm{d}z$. The two parameters α^i and β^i must be positive.

Fact 4 (Gamma MGF). *The MGF of a gamma distributed return is*

$$M_{\theta|\alpha^i, \beta^i}(t) = \left(\frac{\beta^i}{\beta^i - t}\right)^{\alpha^i}.$$

Proof. Casella and Berger (1990). ∎

C Conditions for fully revealing price

Lemma 3 *Suppose signals are Poisson, the asset return is gamma distributed (assumption 1), and expected utility is CARA (assumption 2). Then equilibrium price P fully reveals all investors' information $\sum_{i=1}^{I} \sum_{n=1}^{N^i} s_n^i$ in RICE if and only if*

1. *investors know average prior beliefs and share a common degree of risk aversion (a weaker version of assumption 3),*

2. *investors know market size (assumption 4),*

3. *investors know the total number of all other investors' signals $\sum_{k=1}^{I} N^k$ at the time of portfolio choice, and*

4. *investors are not borrowing constrained.*

Proof. Lemma 1 establishes sufficiency. Necessity of assumptions 3 and 4 follows by inspection of the general solution for market price given individual beliefs $\alpha^i = \bar{\alpha}^i + \sum_{n=1}^{N^i} s_n^i$ and $\beta^i = \bar{\beta}^i + N^i$, based on heterogeneous priors $\bar{\alpha}^i$ and $\bar{\beta}^i$, and arbitrary degrees of risk aversion A^i:

$$RP = \frac{\frac{1}{I}\sum_{i=1}^{I}\frac{\alpha^i}{A^i}}{\bar{x} + \sum_{i=1}^{I}\frac{\beta^i}{A^i}} = \frac{\left(\frac{1}{I}\sum_{i=1}^{I}\frac{\bar{\alpha}^i}{A^i}\right) + \frac{1}{I}\sum_{i=1}^{I}\frac{1}{A^i}\sum_{n=1}^{N^i}s_n^i}{\bar{x} + \left(\frac{1}{I}\sum_{i=1}^{I}\frac{\bar{\beta}^i}{A^i}\right) + \frac{1}{I}\sum_{i=1}^{I}\frac{1}{A^i}N^i}.$$

If investors have a common degree of risk aversion $A^i = A$, only knowledge of the average prior beliefs $\frac{1}{I}\sum_{i=1}^{I}\bar{\alpha}^i$ and $\frac{1}{I}\sum_{i=1}^{I}\bar{\beta}^i$ is necessary to make price fully revealing.

Conditional independence of signals is necessary since investor i would not know the correlation between RP and her signals if perfect copies or correlated signals had been sent to other investors. If $\sum_{k=1}^{I} N^k$ were unknown to investor i, she would not be able to extract the sufficient statistic $\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$ from price.

For necessity of unconstrained borrowing, consider the case in which some investors cannot go short in the risky asset due to a borrowing constraint. Then another investor will not know whether the equilibrium price is low because many relatively poor investors received bad signals and hit their borrowing constraint or whether only a few relatively wealthy investors received extremely bad signals. As a consequence, price uncertainty remains. ∎

D Bond return response to stock information

Taking logs of both sides of the bond first-order condition in (A.1) yields

$$A(1+R)b^i - Ab_0^i + AP(x^i - x_0^i) = \ln[\rho^i RM_{\theta|\mathcal{F}^i}(-A\,x^i)] + AcN^i,$$

a permissible operation since $\rho^i, R, M_{\theta|\mathcal{F}^i}(\cdot) > 0$ by their definitions. Summing up both sides over investors i and dividing by their total number yields

$$AR\bar{b} - \ln \rho^i R - \ln M_{\theta|\mathcal{F}^i}(t) - Ac\sum_{k=1}^{I} N^k/I = 0 \qquad (D.1)$$

where $\bar{b} \equiv \sum_{i=1}^{I} b_0^i/I$ is the average initial bond endowment per investor and $t \equiv -A\overline{x}$. Equation (D.1) implicitly determines the gross bond return R. *Post notitias*, $M_{\theta|\mathcal{F}^i}(t)$ and R respond to the signal realization. Define $\bar{s} \equiv \sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$.

Applying the implicit function theorem to (D.1) for the MGF of the gamma distribution $M_{\theta|\alpha,\beta}(t) = [\beta/(\beta - t)]^\alpha$ yields

$$\frac{dR}{d\bar{s}} = -\frac{\ln(1+\xi)}{A\bar{b} - 1/R}$$

for $\alpha = \bar{\alpha} + \bar{s}, \beta = \bar{\beta} + \sum_{k=1}^{I} N^k$ by (6) and $\xi = A\bar{x}/\beta$ given $\sum_{\underline{k}=1}^{I} N^k$. The bond return falls in response to a favorable signal realization $\bar{s}$ iff $\bar{b} > 1/(AR)$. So, in principle, R too is a function of the signal realization $\bar{s}$. For large bond endowments $\bar{b}$, however,

$$\lim_{\bar{b}\to\infty} dR/d\bar{s} = 0.$$

Similarly, $dR/d\bar{s} = 0$ for $\xi = \bar{x} = 0$.

E *Ante notitias* expected indirect utility

The following property of the Poisson-gamma signal-return distributions proves useful for the derivation of *ante notitias* expected indirect utility.

Fact 5 (Expected signal effect on utility). *For two arbitrary constants B and ξ, $\bar{N}$ Poisson distributed signals $S_1, ..., S_{\bar{N}}$ and a conjugate prior gamma distribution of their common mean θ, the following is true:*

$$\mathbb{E}_{ante}\left[(1+\xi)^{-B\cdot\sum\limits_{n=1}^{\bar{N}} s_n} \cdot \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\cdot\sum_{n=1}^{\bar{N}} s_n\right\}\right]$$

$$= (1+\xi)^{\bar{\alpha}B}\exp\left\{\bar{\alpha}\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\left(1 + \left[(1+\xi)^B\exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}B\right\} - 1\right]\frac{\beta}{\bar{\beta}}\right)^{-\bar{\alpha}},$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the parameters of the prior gamma distribution of θ, and $\beta = \bar{\beta} + \bar{N}$ is the according parameter of the post notitias *distribution.*

Proof. By iterated expectations $\mathbb{E}_{ante}\left[\cdot\right] = \mathbb{E}_\theta\left[\mathbb{E}\left[\cdot|\theta\right]\right]$. The 'inner' expectation $\mathbb{E}\left[\cdot|\theta\right]$ is equal to

$$
\begin{aligned}
\mathbb{E}\left[\cdot|\theta\right] &= \sum_{(\sum_{n=1}^{\bar{N}} s_n)=0}^{\infty} (1+\xi)^{-B\sum_{n=1}^{\bar{N}} s_n} \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi} B \sum_{n=1}^{\bar{N}} s_n\right\} f\left(\sum_{n=1}^{\bar{N}} s_n\right) \\
&= \exp\left\{-\bar{N}\theta\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)\right\},
\end{aligned}
$$

because the sum $\sum_{n=1}^{\bar{N}} s_n$ is Poisson distributed with mean $\bar{N}\theta$ (fact 3). Thus, by the MGF of a gamma distribution (fact 4),

$$
\begin{aligned}
\mathbb{E}_{ante}\left[\cdot\right] &= \mathbb{E}_\theta\left[\exp\left\{-\theta\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta-\bar{\beta})\right\}\right] \\
&= (\bar{\beta})^{\bar{\alpha}}\left(\bar{\beta}+\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta-\bar{\beta})\right)^{-\bar{\alpha}}.
\end{aligned}
$$

since $\bar{N} = \beta - \bar{\beta}$ (fact 1). Simplifying the last term and factoring out $(1+\xi)^B \exp\{\frac{\xi(\omega^i-1)}{1+\xi}B\}$ proves fact 5. ∎

For a gamma distributed asset return, *post notitias* expected indirect utility becomes

$$
\mathbb{E}^i\left[U^{i*}\right] = -\delta^i \exp\left\{-A\frac{R}{1+R}(W_0^i - cN^i)\right\} \exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}\right\}^{-\frac{\alpha^i}{1+R}} (1+\xi)^{-\frac{\alpha^i}{1+R}} \quad \text{(E.1)}
$$

where $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$ is the relative endowment of investors with the risky asset, and $\xi \equiv A\bar{x}/\beta$. With fact 5 at hand, one can set $B \equiv 1/(1+R)$ (by assumption 5) and obtains *ante notitias* expected utility (10) for $\omega^i = 0$ and (10) for arbitrary $\omega^i \in [0, I]$.

F   Monotone marginal signal benefit schedule (proof of Proposition 1)

Define the relative endowment of investors with the risky asset as $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$. The expected relative excess return ξ is bounded by $\xi \in (0, \bar{\xi}]$. Under assumptions 1 through 5, the potential marginal signal benefit $MB(\xi, \omega^i)$ is $MB(\xi, \omega^i) = g(\xi, \omega^i)/h(\xi, \omega^i)$ by (11) with

$$
m(\xi, \omega^i) \equiv \left[(1+\xi)\exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}}, \tag{F.1}
$$

$$
h(\xi, \omega^i) \equiv 1 + \left[m(\xi, \omega^i) - 1\right]\frac{\bar{\xi}}{\xi}, \tag{F.2}
$$

$$
g(\xi, \omega^i) \equiv -\frac{\xi^2}{\bar{\xi}}\frac{\partial h(\xi, \omega^i)}{\partial \xi} = m(\xi, \omega^i)\left(1 - \frac{1}{1+R}\frac{\xi(\xi+\omega^i)}{(1+\xi)^2}\right) - 1. \tag{F.3}
$$

31

The proof of Proposition 1 proceeds in four steps.

First, claim 1 sates useful properties of $m(\xi, \omega^i)$ for the discussion of $g(\xi, \omega^i)$ and $h(\xi, \omega^i)$. Second, claim 2 establishes that the numerator $g(\xi, \omega^i)$ strictly increases in ξ iff $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that it is not bounded above. So, the numerator boosts the marginal benefit $MB(\xi, \omega^i)$ higher and higher as ξ rises. Third, claim 3 establishes that the denominator $h(\xi, \omega^i)$ is bounded below and above in the positive range, and that it strictly decreases in ξ iff the numerator is strictly positive. So, the denominator cannot explode and boosts the marginal benefit $MB(\xi, \omega^i)$ higher where the potential benefit $MB(\xi, \omega^i)$ is positive. The latter two claims imply that $MB(\xi, \omega^i)$ strictly increases in ξ iff $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that $MB(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, fourth and last, $MB(\xi, \omega^i)$ ultimately attains strictly positive values and continues to strictly increase in that positive range.

Claim 1 $m(\xi, \omega^i)$ *strictly increases in* ω^i; $m(0, \omega^i) = 1$; *and* $m(\xi, \omega^i) > 1$ *for any* $\xi > 0$, $\omega^i \geq 0$ *and* $R \in (0, \infty)$.

Proof. By (F.1), $\partial m(\xi, \omega^i)/\partial \xi = m(\xi, \omega^i)\xi/(1 + \xi) > 0$, which establishes the first part of the claim.

Taking natural logs of both sides of (F.1) is permissible since $m(\xi, \omega^i) > 0$ and shows that $m(\xi, \omega^i) \geq 1$ iff $\ln(1 + \xi) \geq -\xi(\omega^i - 1)/(1 + \xi)$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, consider $\omega^i = 0$. So, $m(\xi, 0) \geq 1$ iff $\ln(1 + \xi) \geq \xi/(1 + \xi)$. Note that equality holds at $\xi = 0$ but $\ln(1 + \xi)$ increases strictly faster in ξ than $\xi/(1 + \xi)$ increases in ξ for any $\xi > 0$. So, $m(\xi, 0) > 1$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, $m(\xi, \omega^i) > 1$. ∎

Claim 2 $g(\xi, \omega^i)$ *strictly increases in* ξ *iff* $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$. *In addition,* $\lim_{\xi \to 0} g(\xi, \omega^i) = 0$ *and* $\lim_{\xi \to \infty} g(\xi, \omega^i) = +\infty$.

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ξ is

$$\frac{\partial g(\xi, \omega^i)}{\partial \xi} = \frac{\xi}{(1 + R)^2 (1 + \xi)^4} m(\xi, \omega^i) \left[R(\xi + \omega^i)^2 - (1 + R)(\omega^i - 1)^2 \right].$$

So, $\partial g(\xi, \omega^i)/\partial \xi = 0$ at $\xi = 0$ and at $\xi = |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ (the negative root is ruled out by $\xi \geq 0$). Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around the zero points shows that $g(\xi, \omega^i)$ strictly decreases in ξ if $\xi \in (0, |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i)$ and strictly increases if $\xi \in (|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i, \infty)$.

$\lim_{\xi \to 0} g(\xi, \omega^i) = m(0, \omega^i) - 1 = 0$ by claim 1. $\lim_{\xi \to \infty} g(\xi, \omega^i) = -1 + \lim_{\xi \to \infty} \exp\{\xi/(1 + R)\} = +\infty$ since $R \in (0, \infty)$. ∎

Claim 2 implies that, if $|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i > 0$, then there must be a strictly positive $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ that solves $g(\underline{\xi}^i, \omega^i) = 0$ because $g(\xi, \omega^i)$ strictly decreases as long as $\xi < |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ but subsequently strictly increases in ξ. If $|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i \leq 0$, however, then $g(\underline{\xi}^i, \omega^i) = 0$ only at $\underline{\xi}^i = 0$.

Claim 3 $h(\xi, \omega^i)$ *strictly decreases in ξ iff $g(\xi, \omega^i) > 0$. $h(\xi, \omega^i)$ is bounded in $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ for any $\xi \in (0, \overline{\xi}]$ and $R \in (0, \infty)$, where $\overline{\xi}$ is given by (9) and $\underline{\xi}^i$ solves $g(\underline{\xi}^i, \omega^i) = 0$. Moreover, $h(\underline{\xi}^i, \omega^i) > 1$.*

Proof. By (F.3), $\partial h(\xi, \omega^i)/\partial \xi < 0$ iff $g(\xi, \omega^i) > 0$. So, $h(\xi, \omega^i)$ attains its global maximum at $\underline{\xi}^i$, which solves $g(\underline{\xi}^i, \omega^i) = 0$, and $h(\xi, \omega^i)$ attains its minimum either if $\xi \to 0$ or if $\xi \to \infty$. By L'Hôpital's rule, $\lim_{\xi \to 0} m(\xi, \omega^i)/\xi - 1/\xi = 0$ so $\lim_{\xi \to 0} h(\xi, \omega^i) = 1$. Similarly, for any $R \in (0, \infty)$, $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1$ (but, as $R \to 0$, $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1 + \overline{\xi} \exp\{\omega^i - 1\}$). This establishes that $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ for any $\xi \in (0, \overline{\xi}]$. ∎

Claims 2 and 3 imply that, if $|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i > 0$, then $MB(\xi, \omega^i)$ strictly increases in ξ iff $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and $MB(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, $MB(\xi, \omega^i)$ attains strictly positive values if and only if $\xi > \underline{\xi}^i$, where $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i > 0$ solves $g(\underline{\xi}^i, \omega^i) = 0$, and $\underline{\xi}^i \in (0, \infty)$ is independent of $\overline{\xi}$ and unique given $R \in (0, \infty)$.

If $|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i \leq 0$, however, then $MB(\xi, \omega^i)$ attains strictly positive and increasing values for any $\xi \in (0, \overline{\xi}]$. $|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i \leq 0$ is satisfied if $\omega^i \in [\sqrt{R}/(\sqrt{1 + R} + \sqrt{R}), \sqrt{1 + R}/(\sqrt{1 + R} - \sqrt{R})]$.

G   Dominant investor valuation of signals (proof of Proposition 3)

Define the relative endowment of investors with the risky asset as $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$. The expected relative excess return ξ is bounded by $\xi \in (0, \overline{\xi}]$. Under assumptions 1 through 5, the potential marginal signal benefit $MB(\xi, \omega^i)$ is $MB(\xi, \omega^i) = g(\xi, \omega^i)/h(\xi, \omega^i)$ by (11) with $h(\xi, \omega^i)$ and $g(\xi, \omega^i)$ given by (F.3) and (F.2).

The proof of the remainder of Proposition 3 draws on properties of $g(\xi, \omega^i)$ and $h(\xi, \omega^i)$, which claims 4 and 5 establish. Claim 6 evaluates the potential marginal signal benefit $MB(\xi, \omega^i)$ at $\omega^i = 1$.

Claim 4 $g(\xi, \omega^i)$ *strictly decreases in ω^i iff $\omega^i > 1 + R(1 + \xi)$. In addition, $\lim_{\omega^i \to \infty} g(\xi, \omega^i) = -\infty$.*

33

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ω^i is

$$\frac{\partial g(\xi, \omega^i)}{\partial \omega^i} = \frac{1}{1+R} \frac{\xi^2}{(1+\xi)^2} m(\xi, \omega^i) \left[R(1+\xi) - (\omega^i - 1) \right],$$

where $m(\xi, \omega^i)$ is given by (F.1). So, $\partial g(\xi, \omega^i)/\partial \omega^i = 0$ at $\omega^i = 1 + R(1+\xi)$. Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around this unique zero point shows that $g(\xi, \omega^i)$ strictly increases in ω^i if $\omega^i \in [0, 1 + R(1+\xi))$ and strictly increases if $\omega^i \in (1 + R(1+\xi), I]$. So, $\lim_{\omega^i \to \infty} g(\xi, \omega^i) = -\infty$ for $R \in (0, \infty)$ and $\xi \in (0, \bar{\xi}]$. ∎

Claim 5 $h(\xi, \omega^i)$ *strictly increases in* ω^i *and is strictly convex in* ω^i *at any* $\xi > 0$.

Proof. The first and second derivatives of $h(\xi, \omega^i)$ with respect to ω^i are

$$\frac{\partial h(\xi, \omega^i)}{\partial \omega^i} = \frac{1}{1+R} \frac{\bar{\xi}}{1+\xi} m(\xi, \omega^i) > 0$$

and

$$\frac{\partial^2 h(\xi, \omega^i)}{\partial (\omega^i)^2} = \frac{1}{1+R} \frac{\xi}{1+\xi} \frac{\partial h(\xi, \omega^i)}{\partial \omega^i} > 0.$$

∎

Claim 6 *The potential marginal signal benefit* $\mathrm{MB}(\xi, 1)$ *is strictly positive at* $\omega^i = 1$ *for* $\xi > 0, R > 0$. *At* $\omega^i = 1$, *the potential marginal signal benefit* $\mathrm{MB}(\xi, 1)$ *strictly increases in* ω^i.

Proof. At $\omega^i = 1$, $MB(\xi, 1) > 0$ iff

$$\tfrac{1}{1+R} \ln(1 + \xi) > -\ln \left(1 - \tfrac{1}{1+R} \tfrac{\xi}{1+\xi} \right).$$

Note that equality holds at $\xi = 0$ but the left-hand side increases strictly faster in ξ (it increases by $1/(1+R)(1+\xi)$) than the right-hand side increases (which increases in ξ by $1/(1+\xi)^2[1+R-\xi/(1+\xi)]$) for any $\xi R > 0$. So, $MB(\xi, 1) > 0$.

The first derivative of $MB(\xi, \omega^i)$ with respect to ω^i is

$$\frac{\partial MB(\xi, \omega^i)}{\partial \omega^i} = \left(\frac{\partial g(\xi, \omega^i)/\partial \omega^i}{g(\xi, \omega^i)} - \frac{\partial h(\xi, \omega^i)/\partial \omega^i}{h(\xi, \omega^i)} \right) MB(\xi, \omega^i).$$

So, $\partial MB(\xi, \omega^i)/\partial \omega^i > 0$ at $\omega^i = 1$ iff

$$\left. \left(\frac{\partial g(\xi, \omega^i)/\partial \omega^i}{g(\xi, \omega^i)} \middle/ \frac{\partial h(\xi, \omega^i)/\partial \omega^i}{h(\xi, \omega^i)} \right) \right|_{\omega^i = 1} > 1 \qquad (G.1)$$

since $h(\xi, \omega^i) > 1$ by claim 3 and $\partial h(\xi, \omega^i)/\partial \omega^i > 1$ by claim 5 for $\xi > 0$. A round of simplifications shows that inequality (G.1) is equivalent to

$$\xi(\overline{\xi} + \xi R) > \overline{\xi}(1+R)\left[(1+\xi)^{\frac{1}{1+R}} - 1\right].$$

Note that this condition holds with equality at $\xi = 0$ but the left-hand side increases strictly faster in ξ (it increases by $\overline{\xi}(1+2R\xi/\overline{\xi}) > \overline{\xi}$) than the right-hand side increases (which increases in ξ by $\overline{\xi}(1+\xi)^{-\frac{1}{1+R}} < \overline{\xi}$). So, $\partial MB(\xi, 1)/\partial \omega^i > 0$. ∎

These claims help establish Proposition 3. $g(\xi, \omega^i)$ attains its unique maximum in ω^i at $\omega^i = 1 + R(1 + \xi)$ by claim 4 while $h(\xi, \omega^i)$ strictly increases in ω^i but is convex. So, given any ξ, $MB(\xi, \omega^i)$ must attain its global maximum at some $\omega^\kappa < 1 + R(1 + \xi)$. At $\omega^i = 1$, $MB(\xi, 1)$ strictly increases. This proves the first statement of Proposition 3 that, given any ξ, $MB(\xi, \omega)$ must attain its unique global maximum for some $\omega^\kappa \in (1, 1 + R(1 + \overline{\xi}))$. The second statement that $MB(\xi, \omega) > 0$ for ω in an open interval Ω^+ that includes $[1, \omega^\kappa] \subset \Omega^+$ follows because $MB(\xi, \omega^i)$ is strictly positive and strictly increases at $\omega = 1$ for any $\xi > 0$. So, $MB(\xi, \omega) > 0$ in an open interval around $\omega = 1$. $MB(\xi, \omega)$ is maximal at ω^κ so that the open interval Ω^+ must in fact extend to $[1, \omega^\kappa] \subset \Omega^+$.

References

Admati, Anat R., "A Noisy Rational Expectations Equilibrium for Multi-asset Securities Markets," *Econometrica*, May 1985, *53* (3), 629–57.

Angeletos, George Marios and Alessandro Pavan, "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review*, May 2004, *94* (2), 91–98.

Bolton, Patrick and Christopher Harris, "Strategic Experimentation," *Econometrica*, March 1999, *67* (2), 349–74.

Bushee, Brian J. and Christopher F. Noe, "Corporate Disclosure Practices, Institutional Investors, and Stock Return Volatility," *Journal of Accounting Research*, Supplement 2000, *38*, 171–202.

Casella, George and Roger L. Berger, *Statistical inference*, 6th ed., New York: Duxbury, June 1990.

Cripps, Martin William, Godfrey Keller, and Sven Rady, "Strategic Experimentation with Exponential Bandits," *Econometrica*, January 2005, *73* (1), 39–68.

Davis, Ronald E., "Certainty-Equivalent Models for Portfolio Optimization Using Exponential Utility and Gamma-Distributed Returns," in Kenneth D. Lawrence, John B. Jr. Guerard, and Gary R. Reeves, eds., *Advances in mathematical programming and financial planning*, Vol. 3, Greenwich, CT and London: JAI Press, 1993, pp. 69–108.

Diamond, Douglas W., "Optimal Release of Information by Firms," *Journal of Finance*, September 1985, *40* (4), 1071–94.

Easley, David and Maureen O'Hara, "Information and the Cost of Capital," *Journal of Finance*, August 2004, *59* (4), 1553–83.

_____ , **Soeren Hvidkjaer, and Maureen O'Hara**, "Is Information Risk a Determinant of Asset Returns?," *Journal of Finance*, October 2002, *57* (5), 2185–2221.

Fama, Eugene F., "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, May 1970, *25* (2), 383–417.

Grossman, Sanford J. and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, June 1980, *70* (3), 393–408.

Heston, Steven L., "Invisible Parameters in Option Prices," *Journal of Finance*, July 1993, *48* (3), 933–47.

Kim, Oliver and Robert E. Verrecchia, "Market Reaction to Anticipated Announcements," *Journal of Financial Economics*, December 1991, *30* (2), 273–309.

Krebs, Tom, "Fundamentals, Information, and International Capital Flows: A Welfare Analysis," *European Economic Review*, April 2005, *49* (3), 579–98.

Morris, Stephen and Hyun Song Shin, "Social Value of Public Information," *American Economic Review*, December 2002, *92* (5), 1521–34.

Moscarini, Giuseppe and Lones Smith, "The Optimal Level of Experimentation," *Econometrica*, November 2001, *69* (6), 1629–44.

Muendler, Marc-Andreas, "The Action Value of Information and the Natural Transparency Limit," *University of California, San Diego, Department of Economics Discussion Paper*, September 2005, *2005-09*.

——, "The Possibility of Informationally Efficient Markets," *Journal of Economic Theory*, forthcoming.

Nelson, Daniel B., "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, March 1991, *59* (2), 347–70.

Raiffa, Howard and Robert Schlaifer, *Applied Statistical Decision Theory* Studies in managerial economics, Boston, Massachusetts: Graduate School of Business Adminitration, Harvard University, 1961.

Robert, Christian P., *The Bayesian Choice: From Decision-Theoretic Foundations to Computational Implementation* Springer Texts in Statistics, first ed., New York: Springer, 1994.

Samuelson, Paul A., "The Pure Theory of Public Expenditure," *Review of Economics and Statistics*, November 1954, *36* (4), 387–89.

Svensson, Lars E. O., "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro Transparency, Not Con," *American Economic Review*, 2005. forthcoming.