Risk Response in Agriculture

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Abstract: Crop production is subject to supply shocks, and both expected and realized outputs as well as output prices are unknown when inputs are chosen. The process by which producers form expectations is difficult to model, especially when working with aggregate data. We present a necessary and sufficient condition on cost and technology to allow variable input demand equations to be specified as functions of input prices, quasi-fixed inputs, and total variable cost. These all are observable when inputs are committed to production, so that *ex ante* demands can be estimated with observable data. A flexible, exactly aggregable, and economically regular model of variable input demands is derived and applied to aggregate U.S. agricultural data for the period 1960-1999. We use the empirical results of this model to aid in the specification of a dynamic life-cycle model for agricultural producers facing output and output price risk, with investment in an off-farm, conditionally risk free asset, risky financial assets, savings, consumption, and agricultural production opportunities. This framework admits a coherent, structural, econometric model of input use, output production, savings, investment, and consumption for agriculture. We apply this model to U.S. data for the period 1960-1999. Ongoing work focuses on updating the data set to the 21st century and applying both components of the model at the state-level.

Key Words: Aggregation, ex ante variable cost, expected utility, functional form, life cycle consumption, rank, risk

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1. Introduction

Farm and food policies affect crop acres, asset management, intensive and extensive margin decisions, and risk management choices in agricultural production. For example, in 1991, less than 25% of cropland (82 million acres) was covered by a Federally subsidized crop insurance contract, with $11.2 billion in total liability, $740 million in insurance premiums, premium subsidies of 25% ($190 million) of gross farm premiums, and total indemnity payments of $955 million. Relative to premiums paid by farmers ($550 million), for each $1.00 in premiums paid by the typical insured farmer, $1.75 in indemnity payments were received.

Even with this relatively profitable insurance program, farmer participation rates remained quite low. This outcome is likely due to the race to the bottom problem in a pooling equilibrum (LaFrance, Shimshack, and Wu 2000, 2001, 2002, 2004). However, Congress responded to the *appearance* of an incomplete insurance market with increased subsidies and many new forms of insurance.

The 1996 Federal Agricultural Improvement and Reform Act and the amendments to the 1938 Federal Crop Insurance Act that are commonly known as the Agricultural Risk Protection Act of 2000 mandated higher subsidy rates, the development and marketing of new insurance products for virtually every crop and livestock product produced in the U.S., and substantial subsidies for crop insurance marketing firms and large private reinsurance companies.

This change in farm policy greatly expanded the Federal crop insurance program. In 2003, the Federal Crop Insurance Corporation (FCIC) provided insurance products for

more than 100 crops on 217 million acres (2/3 of all cropland). The total insurance liability was $40.6 billion, with $3.4 billion in insurance premiums, subsidies of almost 60% of gross premiums ($2.0 billion), and total indemnity payments of $3.2 billion. The current program includes subsidy payments to private companies marketing Federal crop insurance equal to 24.5% of gross premiums for administration and oversight (A&O), and to private reinsurance companies equal to 13.6% of gross premiums. Reinsurance companies also have the right to sell up to 50% of their contracts back to the FCIC (that is, to the taxpayer) at cost. The FCIC's Risk Management Agency's (RMA) book of business shows that 20% of the insured farms account for nearly 80% of indemnity payments. This suggests substantial adverse selection, as well as moral hazard, since the majority of the Federally subsidized crop insurance products calculate premiums based on deviations from county-level yield trends. That is to say, FCIC insurance products are based on a pooling equilibrium established at the county level, and in some cases larger areas known as risk regions.

The net effect is that for each $1.00 in premiums actually paid by farmers they receive an average of $2.40 in indemnity payments, insurance marketing firms receive $0.40 in A&O subsidies, and reinsurance companies make in the neighborhood of $0.45 in profit due to the combined direct subsidies on premiums and their reinsurance rights with the FCIC, which allow them to cream, or high grade, the insurance pool.

In 2004 the RMA issued an RFP to develop subsidized pasture and range insurance for 440 million acres of private, public, and Native American pasture and rangeland in the country. Many agricultural economists at land grant universities across the country

actively consult with the RMA and private insurance companies to develop new and expand existing Federally subsidized crop insurance products.

Although this is only one example of the ubiquitous nature of Federal intervention in U.S. agriculture, there is a large literature on the impacts of subsidized crop insurance on variable input use and the intensive margin (Nelson and Loehman 1987, Chambers 1989, and Quiggin 1992, Horowitz and Lichtenberg 1994, Smith and Goodwin 1996, and Babcock and Hennessy 1996). The effects of subsidized crop insurance programs on the extensive margin also has been the subject of considerable analysis (Gardner and Kramer 1986, Goodwin, Smith and Hammond 1999, Keeton, Skees and Long 1999, and Young, Schnepf, Skees, and Lin 1999, and LaFrance, Shimshack, and Wu 2000, 2001, 2002, 2004), all of which conclude that subsidized crop insurance results in additional of marginal crop acres. Williams (1988), Turvey (1992), Wu (1999), and Soule, Nimon, and Mullarkey (2000) examine the impacts of subsidized crop insurance on choices of crop mixes and acreage decisions. Empirical results in this component of the literature suggest that economically marginal land also is environmentally marginal. These results all suggest that subsidized crop insurance tends to increase environmental degradation. Even so, very little of the previous work in this area uses structural models, or takes into account the dynamic nature of agricultural decision making under risk.

To better understand these and many other longstanding issues in U.S. agricultural policy, this paper develops a comprehensive structural econometric model of variable input use, crop mix and acreage choices, investment and asset management decisions, and consumption, savings and wealth accumulation in a stochastic dynamic programming

model of farm-level decision making over time. This model develops and establishes clear and intuitively appealing relationships between dynamic life-cycle consumption theory, the theory of the competitive firm subject to risk, and modern finance theory.

We present, discuss, and apply a new class of variable input demand systems in a multi-product production setting. All of the models in this class can be estimated with observable data, are exactly aggregable, are consistent with economic theory for any von Neumann-Morgenstern expected utility function, and can be used to nest and test exact aggregation, economic regularity, functional form, and flexibility. Implications of monotonicity, concavity in prices, and convexity in outputs and quasi-fixed inputs are developed for a specific subset of this class of models. We then apply this to 13 variable inputs in U.S. agriculture over the sample period 1960-1999.

The results obtained from this empirical variable cost model are used to help develop a structural model of the dynamic decision problems faced by a generic agricultural producer. In this life-cycle model of agricultural decisions under risk, farmers create income and wealth through savings, investment in risky financial assets, own-labor choices both on- and off-farm, and agricultural production and investment activities. This disciplines the economic theory of agricultural production over time and under risk, and helps to better identify risk preferences and other model parameters.

2. The Production Model and Two Results

Four longstanding questions in economics, econometrics, and agricultural economics are the choice of functional form, the degree of flexibility, the conditions required for and regions of economic regularity, consistency with aggregation from micro- to macro-level

data, and how best to handle simultaneous equations bias, errors in variables, and latent variables in a structural econometric models. In this paper, we attempt to deal with all of these issues in a coherent framework for the analysis of a life-cycle model of agricultural production, investment, consumption, and savings decisions.

Analysis of multi-product behavior of firms is common in economics (Färe and Primont 1995; Just, Zilberman, and Hochman 1988; Shumway 1983, Lopez 1983; Akridge and Hertel 1986). A large literature on functional structure and duality guides empirical formulations and testing based on concepts of non-jointness and separability (Lau 1972, 1978; Blackorby, Primont and Russell 1977, 1978; Chambers 1984). Non-joint production processes reduce to additivity in costs (Hall 1973; Kohli 1983). Separability in a partition of inputs or outputs often results in separability in a similar partition of prices (Blackorby, Primont and Russell 1977; Lau 1978).

The neoclassical model of conditional demands for variable inputs with joint production, quasi-fixed inputs, and production and output price risk is

$$\boldsymbol{x}(\boldsymbol{w}, \bar{\boldsymbol{y}}, \boldsymbol{z}) = \arg\min \left\{ \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x} : F(\boldsymbol{x}, \bar{\boldsymbol{y}}, \boldsymbol{z}) \leq 0 \right\}, \tag{1}$$

where $\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}_+^{n_x}$ is an n_x–vector of variable inputs, $\boldsymbol{w} \in \mathcal{W} \subseteq \mathbb{R}_+^{n_x}$ is an n_x–vector of variable input prices, $\bar{\boldsymbol{y}} \in \mathcal{Y} \subseteq \mathbb{R}_+^{n_y}$ is an n_y–vector of planned outputs, $\boldsymbol{z} \in \mathcal{Z} \subseteq \mathbb{R}_+^{n_k}$ is an n_z–vector of quasi-fixed inputs.[1] $F : \mathcal{X} \times \mathcal{Y} \times \mathcal{Z} \to \mathbb{R}$ is the joint production transforma-

[1] In this section, we use $\bar{\boldsymbol{y}} \in \mathbb{R}_+^{n_y}$ to denote the n_y–vector of planned/expected outputs to simplify notation. In later sections, we modify this notation to $\bar{\boldsymbol{Y}} = \boldsymbol{a} \bullet \bar{\boldsymbol{y}}$, where $\boldsymbol{a}$ is the n_y–vector of acres planted to crops, $\bar{\boldsymbol{y}}$ now is the n_y–vector of expected yields, and $\bullet$ is the Hadamard product. We also define $\boldsymbol{z}$ explicitly below.

tion function, which is the boundary of a closed and convex production possibilities set that is characterized by free disposal in inputs and outputs. Let the variable cost function be denoted by $c(\boldsymbol{w}, \bar{\boldsymbol{y}}, z) \equiv \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}(\boldsymbol{w}, \bar{\boldsymbol{y}}, z)$. We assume throughout that the production process is subject to supply shocks of the general form

$$\boldsymbol{y} = \bar{\boldsymbol{y}} + \boldsymbol{h}(\bar{\boldsymbol{y}}, z, \boldsymbol{\varepsilon}),\ E\left[\boldsymbol{h}(\bar{\boldsymbol{y}}, z, \boldsymbol{\varepsilon}) \mid \boldsymbol{x}, \bar{\boldsymbol{y}}, z\right] = \mathbf{0}. \tag{2}$$

In either a static or a dynamic setting, it is a simple matter to show that (1) is implied by (2) and the expected utility hypothesis for *all* von Newman-Morgenstern preferences (Pope and Chavas 1994; Ball, et al., 2010).

Planned output is a vector of latent, unobservable variables in production with supply risk. Hence, to estimate the demand system in (1) directly, one must either identify and estimate the expectations formation process or address the errors in variables problem associated with using $\boldsymbol{y}$ in place of $\bar{\boldsymbol{y}}$ in the demand equations (Pope and Chavas 1994). One branch of the literature advocates specifying an *ex ante* cost function where planned output is replaced by cost, which is observable when the variable inputs are committed to the production process (Pope and Chavas 1994; Pope and Just 1998; Chambers and Quiggin 2000; Chavas 2008; Ball, et al. 2010; LaFrance and Pope 2010). In a joint production process, this requires making assumptions such that the input demands are functions of input prices, the levels of quasi-fixed inputs, and the variable cost of production,

$$\boldsymbol{x}(\boldsymbol{w}, \bar{\boldsymbol{y}}, z) = \tilde{\boldsymbol{x}}(\boldsymbol{w}, z, c(\boldsymbol{w}, \bar{\boldsymbol{y}}, z)). \tag{3}$$

This approach makes particular sense in agriculture where outputs and output prices are observed *ex post*. The main result of LaFrance and Pope (2010) on this question is as follows (a proof of this result is presented in Appendix A of this paper).

Proposition 1*: The following functional structures are equivalent:*

$$\boldsymbol{x}(\boldsymbol{w}, \overline{\boldsymbol{y}}, \boldsymbol{z}) \equiv \tilde{\boldsymbol{x}}(\boldsymbol{w}, c(\boldsymbol{w}, \overline{\boldsymbol{y}}, \boldsymbol{z}), \boldsymbol{z}); \tag{4}$$

$$c(\boldsymbol{w}, \overline{\boldsymbol{y}}, \boldsymbol{z}) \equiv \tilde{c}(\boldsymbol{w}, \boldsymbol{z}, \theta(\overline{\boldsymbol{y}}, \boldsymbol{z})); \tag{5}$$

$$F(\boldsymbol{x}, \overline{\boldsymbol{y}}, \boldsymbol{z}) \equiv \tilde{F}(\boldsymbol{x}, \boldsymbol{z}, \theta(\overline{\boldsymbol{y}}, \boldsymbol{z})). \tag{6}$$

In other words, outputs must be weakly separable from the variable input prices in the variable cost function. This, in turn, is equivalent to outputs being weakly separable from the variable inputs in the joint production transformation function.

This is a tight result – separability is both necessary and sufficient for the variable inputs to be estimable in *ex ante* form. Hereafter, we will call any such demand model an *ex ante joint production system.*

A second common issue in the empirical analysis of agricultural supply decisions is that some level of aggregation is virtually unavoidable. Micro-level data needed to study input use, acreage allocations, and asset management choices at the farm level does not exist. Aggregation from micro-level decision makers to macro-level data has been studied extensively in consumer theory.[2] This has received less attention in production economics (Chambers and Pope 1991, 1994; Ball et, al., 2010; LaFrance and Pope 2008, 2010).

Recently, LaFrance and Pope (2009) obtained the indirect preferences for all exactly

[2] An important subset of the literature on this topic includes: Gorman (1953, 1961, 1981); Muellbauer (1975, 1976); Howe, Pollak and Wales (1979); Deaton and Muellbauer (1980); Jorgenson, Lau and Stoker (1980, 1982); Russell (1983, 1996); Jorgenson and Slesnick (1984, 1987); Lewbel (1987, 1988; 1989, 1990, 1991, 2003); Jorgenson (1990); Diewert and Wales (1987, 1988); Blundell (1988);; van Daal and Merkies (1989); Jerison (1993); Russell and Farris (1993, 1998); and Banks, Blundell, and Lewbel (1997), LaFrance, Beatty, Pope and Agnew (2002), LaFrance (2004), LaFrance, Beatty and Pope (2006), and LaFrance and Pope (2009). The focus in the literature has been interior solutions and smooth demand equations. We remain faithful to this approach throughout the present paper.

aggregable, full rank systems of consumer demand equations. Their result extends directly to production in the following way. Let $K \in \{1,2,3,4\}$ and define the smooth real-valued function, $\omega : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, by

$$\omega(\eta(\boldsymbol{w}),\theta) = \begin{cases} \theta, & \text{if } K = 1,2, \text{ or } K = 3 \text{ and } \lambda'(s) = 0, \\ \theta + \int_0^{\eta(\boldsymbol{w})} \left[\lambda(s) + \omega(s,\theta)^2 \right] ds, & \text{if } K = 3,4, \text{ and } \lambda'(s) \neq 0, \end{cases} \tag{7}$$

subject to $\omega(0,\theta) = \theta$ and $\partial\omega(0,\theta)/\partial s = \lambda(0) + \theta^2$, where $\eta : \boldsymbol{\mathcal{W}} \to \mathbb{R}$ and $\lambda : \mathbb{R} \to \mathbb{R}$ are smooth, real-valued functions, and η is 0° homogeneous. A class of full rank and exactly aggregable *ex ante* production systems can be characterized as follows.[3]

Proposition 2: *Let* $\pi : \boldsymbol{\mathcal{W}} \to \mathbb{R}_{++}$, $\pi \in \boldsymbol{\mathcal{C}}^{\infty}$, *be strictly positive valued, increasing, concave, and* 1° *homogeneous; let* $\eta : \boldsymbol{\mathcal{W}} \to \mathbb{R}_{+}$, $\eta \in \boldsymbol{\mathcal{C}}^{\infty}$, *be positive valued and* 0° *homogeneous; let* $\alpha,\beta,\gamma,\delta : \boldsymbol{\mathcal{W}} \to \mathbb{C} = \{a + \imath b,\ a,b \in \mathbb{R}\}$, $\alpha,\beta,\gamma,\delta \in \boldsymbol{\mathcal{C}}^{\infty}$, *be* 0° *homogeneous and satisfy* $\alpha\delta - \beta\gamma \equiv 1$, $\imath = \sqrt{-1}$; *and let* $f : \mathbb{R}_{++} \to \mathbb{C}$, $f \in \boldsymbol{\mathcal{C}}^{\infty}$, *and* $f' \neq 0$. *Then the variable cost function for any full rank, exactly aggregable, ex ante joint production system is a special case of*

$$f\left(\frac{c(\boldsymbol{w},\bar{\boldsymbol{y}},z)}{\pi(\boldsymbol{w})}\right) = \frac{\alpha(\boldsymbol{w})\omega(\eta(\boldsymbol{w}),\theta(\bar{\boldsymbol{y}},z)) + \beta(\boldsymbol{w})}{\gamma(\boldsymbol{w})\omega(\eta(\boldsymbol{w}),\theta(\bar{\boldsymbol{y}},z)) + \delta(\boldsymbol{w})}. \tag{8}$$

LaFrance and Pope (2009) present a complete proof of necessity in the case of con-

[3] This result is consistent with exact aggregation as defined by Gorman (1981). One part of our ongoing work is to extend this class to Lau's (1982) definition of exact aggregation, generalizing the left-hand-side of (8) to $f\left(c(\boldsymbol{w},\bar{\boldsymbol{y}},z)/\pi,z\right)$, wherein cost and quasi-fixed inputs vary across individual economic agents.

sumer choice theory. Their proof applies to the current problem with only minor changes in notation. Sufficiency is shown here by considering the structure of the input demands generated by (8). This is accomplished simply enough by differentiating with respect to $\boldsymbol{w}$ and applying Shephard's lemma. To make the notation as compact as possible, let a bold subscript $\boldsymbol{w}$ denote a vector of partial derivatives with respect to the variable input prices and suppress the arguments of the functions $\{\alpha,\beta,\gamma,\delta,\eta,\pi\}$ to yield (after a large amount of straightforward but tedious algebra, which is presented in Appendix B):

$$\begin{aligned}\boldsymbol{x} = \frac{\pi_{\boldsymbol{w}}}{\pi}c + \pi\Bigg\{&\Big[\alpha\beta_{\boldsymbol{w}} - \beta\alpha_{\boldsymbol{w}} + (\alpha^2\lambda + \beta^2)\eta_{\boldsymbol{w}}\Big]\frac{1}{f'} \\ &-\Big[\alpha\delta_{\boldsymbol{w}} - \delta\alpha_{\boldsymbol{w}} + \gamma\beta_{\boldsymbol{w}} - \beta\gamma_{\boldsymbol{w}} + 2(\alpha\gamma\lambda + \beta\delta)\eta_{\boldsymbol{w}}\Big]\frac{f}{f'} \\ &+\Big[\gamma\delta_{\boldsymbol{w}} - \delta\gamma_{\boldsymbol{w}} + (\gamma^2\lambda + \delta^2)\eta_{\boldsymbol{w}}\Big]\frac{f^2}{f'}\Bigg\}.\end{aligned} \tag{9}$$

Thus, (8) generates input demands that have the finitely additive and multiplicatively separable structure of any full rank, exactly aggregable system (Gorman 1981; Lau 1982; Lewbel 1989). Note that there are potentially up to four linearly independent variable cost terms on the right with four associated linearly independent vectors of input price functions. Hence, any system generated by (8) will have rank up to, but no greater than four, the highest possible rank (Lewbel 1987, 1990, 1991; LaFrance and Pope 2009).

A third issue when estimating a system of variable input demand equations such as (9) is the fact that quasi-fixed inputs, planned outputs, variable input prices, and total variable cost all are jointly determined with the input demands. Consistent estimation under these conditions is addressed in the empirical application below.

3. The Econometric Cost Model, Data, and Estimates

Previous work at both state and national levels of aggregation with our data set strongly suggests that full rank 3 seriously over-parameterizes the structural model for this data. As a result, we restrict attention here to a rank two model. In this part of the paper, we analyze the conditional demands for 13 variable inputs in U.S. agriculture: pesticides and herbicides; fertilizer; fuel and natural gas; electricity; purchased feed; purchased seed; purchased livestock; machinery repairs; building repairs; custom machinery services; veterinary services; other materials; and labor. The specification of the variable cost function normalized by the farm wage rate is,

$$\begin{aligned} c_t(\tilde{\boldsymbol{w}}_t, A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t) &= \left[\, \alpha_{10} + \boldsymbol{\alpha}_1^{\mathrm{T}} \tilde{\boldsymbol{w}}_t \,\right] A_t + \left[\, \alpha_{20} + \boldsymbol{\alpha}_2^{\mathrm{T}} \tilde{\boldsymbol{w}}_t \,\right] K_t \\ &\quad + \sqrt{\tilde{\boldsymbol{w}}_t^{\mathrm{T}} \boldsymbol{B} \tilde{\boldsymbol{w}}_t + 2\boldsymbol{\gamma}^{\mathrm{T}} \tilde{\boldsymbol{w}}_t + 1} \times \theta(A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t), \end{aligned} \tag{10}$$

where $\boldsymbol{z}_t = [A_t\, K_t\, \boldsymbol{a}_t]^{\mathrm{T}}$, A_t is farmland, K_t is the value of farm capital, $\boldsymbol{a}_t = [a_{1t}\, a_{2t} \cdots a_{n_y t}]^{\mathrm{T}}$ is the n_y–vector of acres planted to crops, $A_t = a_{0t} + \boldsymbol{\iota}^{\mathrm{T}} \boldsymbol{a}_t$, with a_{0t} denoting farmland that is not devoted to crop production, $\bar{\boldsymbol{Y}}_t = [a_{1t}\bar{y}_{1t} \cdots a_{n_y t}\bar{y}_{n_y t}]^{\mathrm{T}}$ is the n_y–vector of planned crop production, with each element defined as the product of acres planted to the crop times the expected yield per acre, and $\tilde{\boldsymbol{w}}_t = [w_{1t}/w_{n_x t}, \cdots, w_{n_x - 1t}/w_{n_x t}]^{\mathrm{T}}$ is the $(n_x - 1)$–vector of variable input prices except the farm wage normalized by $w_{n_x t}$.

We treat the n_x^{th} input, labor, asymmetrically with respect to the other inputs both in the structural and stochastic parts of the econometric model. To conserve and simplify notation from this point forward, we drop the ~ over the first n_x–1 input prices, absorb the

normalization by w_{n_x} into the notation for variable cost and the n_x–1 first input prices, and define $N = n_x - 1$.

We assume constant returns to scale, so that $\theta(A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t)$ is 1° homogeneous. Define $\alpha_1(\boldsymbol{w}_t) = \alpha_{10} + \boldsymbol{\alpha}_1^{\mathsf{T}}\boldsymbol{w}_t$, $\alpha_2(\boldsymbol{w}_t) = \alpha_{20} + \boldsymbol{\alpha}_2^{\mathsf{T}}\boldsymbol{w}_t$, and $\beta(\boldsymbol{w}_t) = \sqrt{\boldsymbol{w}_t^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}_t + 2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}_t + 1}$. The necessary and sufficient conditions for the variable cost function to be increasing and concave in the variable input prices throughout an open set containing the data points are as follows (see Appendix C for a complete derivation of the cost function and θ):

Monotonicity in $\boldsymbol{w}$:

$$\begin{aligned}\frac{\partial c(\boldsymbol{w}_t, A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t)}{\partial \boldsymbol{w}} &= \boldsymbol{\alpha}_1 A_t + \boldsymbol{\alpha}_2 K_t + \frac{\theta}{\beta(\boldsymbol{w}_t)}(\boldsymbol{B}\boldsymbol{w}_t + \boldsymbol{\gamma}) \\ &= \boldsymbol{\alpha}_1 A_t + \boldsymbol{\alpha}_2 K_t + \left[\frac{c_t - \alpha_1(\boldsymbol{w}_t)A_t - \alpha_2(\boldsymbol{w}_t)K_t}{\boldsymbol{w}_t^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}_t + 2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}_t + 1}\right](\boldsymbol{B}\boldsymbol{w}_t + \boldsymbol{\gamma}) \geq \mathbf{0};\end{aligned} \tag{11}$$

Concavity in $\boldsymbol{w}$:

$$\begin{aligned}\frac{\partial^2 c(\boldsymbol{w}_t, A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t)}{\partial \boldsymbol{w}\partial \boldsymbol{w}^{\mathsf{T}}} &= \frac{\theta}{\beta(\boldsymbol{w}_t)}\boldsymbol{B} - \frac{\theta}{\beta(\boldsymbol{w}_t)^2}(\boldsymbol{B}\boldsymbol{w} + \boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w} + \boldsymbol{\gamma})^{\mathsf{T}} \\ &= \left[\frac{c(\boldsymbol{w}_t, A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t) - \alpha_1(\boldsymbol{w}_t)A_t - \alpha_2(\boldsymbol{w}_t)K_t}{\boldsymbol{w}_t^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}_t + 2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}_t + 1}\right] \times \\ &\qquad \left[\boldsymbol{B} - \frac{(\boldsymbol{B}\boldsymbol{w}_t + \boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w}_t + \boldsymbol{\gamma})^{\mathsf{T}}}{(\boldsymbol{w}_t^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}_t + 2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}_t + 1)}\right],\end{aligned} \tag{12}$$

symmetric, negative semi-definite. Setting $\boldsymbol{B} = \boldsymbol{L}\boldsymbol{L}^{\mathsf{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathsf{T}}$, where $\boldsymbol{L}$ is a (lower or upper) triangular matrix with nonzero main diagonal elements implies

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L} & \boldsymbol{\gamma} \\ \mathbf{0}^{\mathsf{T}} & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{L}^{\mathsf{T}} & \mathbf{0}^{\mathsf{T}} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}\boldsymbol{L}^{\mathsf{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix} \tag{13}$$

is positive definite. It follows that $\left[\boldsymbol{B}-\frac{(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})^{\top}}{(\boldsymbol{w}_t^{\top}\boldsymbol{B}\boldsymbol{w}_t+2\boldsymbol{\gamma}^{\top}\boldsymbol{w}_t+1)}\right]$ is positive semri-definite and that

$$\begin{bmatrix}\boldsymbol{w}_t^{\top} & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{B} & \boldsymbol{\gamma}\\ \boldsymbol{\gamma}^{\top} & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_t\\ 1\end{bmatrix}=\boldsymbol{w}_t^{\top}\boldsymbol{B}\boldsymbol{w}_t+2\boldsymbol{\gamma}^{\top}\boldsymbol{w}_t+1>0\ \forall\ \boldsymbol{w}_t\in\mathbb{R}_{+}^{n_x-1}. \tag{14}$$

Given this, the variable cost function is concave in $\boldsymbol{w}$ if and only if

$$c_t(\tilde{\boldsymbol{w}}_t,A_t,K_t,\boldsymbol{a}_t,\bar{\boldsymbol{Y}}_t)<\left[\alpha_0+\boldsymbol{\alpha}_1^{\top}\tilde{\boldsymbol{w}}_t\right]A_t+\left[(\alpha_2+\boldsymbol{\alpha}_2^{\top}\tilde{\boldsymbol{w}}_t\right]K_t, \tag{15}$$

(LaFrance, Beatty, and Pope 2006). Hence, we impose $\boldsymbol{B}=\boldsymbol{L}\boldsymbol{L}^{\top}+\boldsymbol{\gamma}\boldsymbol{\gamma}^{\top}$ during estimation and check the monotonicity conditions (11) at all data points once the model is estimated, and find that they are satisfied. We develop the specification for $\theta(A_t,K_t,\boldsymbol{a}_t,\bar{\boldsymbol{Y}}_t)$ in the section on life-cycle consumption and investment decisions and Appendix C.

Applying Shephard's Lemma to (10) and rearranging terms then gives the empirical variable input demand equations in normalized expenditures per dollar of capital as

$$\boldsymbol{e}_t=\boldsymbol{W}_t\left[\boldsymbol{\alpha}_1\frac{A_t}{K_t}+\boldsymbol{\alpha}_2+\left(\frac{(c_t/K_t)-\alpha_1(\boldsymbol{w}_t)(A_t/K_t)-\alpha_2(\boldsymbol{w}_t)}{\boldsymbol{w}_t^{\top}\boldsymbol{B}\boldsymbol{w}_t+2\boldsymbol{\gamma}^{\top}\boldsymbol{w}_t+1}\right)(\boldsymbol{B}\boldsymbol{w}_t+\boldsymbol{\gamma})\right]+\boldsymbol{u}_t, \tag{16}$$

where $\boldsymbol{W}_t=\mathbf{diag}[w_{it}]$ is the diagonal matrix with $w_{i,t}$ as the ith main diagonal element and $\boldsymbol{e}_t=[w_{1,t}x_{1,t}\ \cdots\ w_{n_x-1,t}x_{n_x-1,t}]^{\top}$ is the (n_x-1)-vector of normalized expenditures per dollar of capital on all inputs except labor, and we follow standard practice in the empirical analysis of demand systems and add a vector of random errors to the right-hand-side to obtain the empirical model. We assume that the errors terms for the 12 equations estimated follow to an unrestricted AR(1) process,

$$\boldsymbol{u}_t = \boldsymbol{R}\boldsymbol{u}_{t-1} + \boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t\ i.i.d.\ (\mathbf{0}, \boldsymbol{\Sigma}),\ t = 1, \cdots, T. \tag{17}$$

As noted above, we apply this model to annual aggregate data on 13 variable inputs in U.S. agriculture (pesticides and herbicides, fertilizer, fuel and natural gas, electricity, purchased feed, purchased seed, purchased livestock, machinery repairs, building repairs, custom machinery services, veterinary services, other materials, and farm labor). The sample period is 1960-1999. This data was compiled by the United States Department of Agriculture's (USDA), Economic Research Service (ERS) and is described in detail in Ball, Halahan, and Nehring (2004). Farmland, equipment, buildings, and structures are treated as quasi-fixed inputs. Hereafter, this data set is called the Ball data.

Due to the way that several variables are constructed in the Ball data, it is necessary to modify and augment this data for empirical implementation. First, we define the replacement cost of owner-operator labor by the farm wage rate. This implies that the return to owner-operator labor in the Ball data due to management skill is treated as a part of the residual claimant's quasi-rent. Second, we use a direct measure of the value of capital obtained from the ERS rather than the measures constructed in the Ball data. Third, estimates of the price of farmland are taken from state-level surveys conducted by the National Agricultural Statistics Service (NASS), rather than the constructed measures in the Ball data. Finally, we adjust the measure of agricultural land. The Census of Agriculture has reported land in farms in four- to five-year intervals for 1954, 1959, 1964, 1969, 1974, 1978, 1982, 1987, 1992, 1997, 2002, and 2002. These are the total farmland numbers used in the sample years that match the Census years. ERS reports the harvested acres for all major crops by state and year since 1947. This data is used to adjust the

farmland measures in the Ball data as follows. First, the difference between total farmland in the Ball data and harvested acres is calculated for each non-census year by state. Second, in each period between adjacent censuses, the average of this difference is calculated. This mean difference is treated as fixed in each of the three- or four-year intervals between census years and added to harvested acres to obtain the measure of farmland used in this study in those years of our sample period. We normalize costs, expenditures, and acres by capital rather than total land because we are more confident in the capital measure and Pope, LaFrance and Just (2007) have shown that deflating by a variable that is subject to measurement error leads to difficult econometric issues.

Estimation is by nonlinear generalized method of moments (GMM), which assumes a parametric 12×12 AR(1) process for the time series component and White/Huber robust covariance matrix estimator that is consistent under heteroskedasticity of an unknown form. The instruments are variable cost per unit of capital, land per unit of capital, and variable input prices all lagged two periods, plus the following general economy variables lagged one period: real per capita disposable personal income; unemployment rate; the real rate of return on AAA corporate 30-year bonds; real manufacturing wage rate; real index of prices paid by manufacturers for materials and components; and real index of prices paid by manufacturers for fuel, energy and power. Per capita disposable personal income is deflated by the consumer price index for all items. The aggregate wholesale price variables are deflated by the implicit price deflator for gross domestic product. The real rate of return on corporate bonds is calculated as the nominal rate of return minus the midyear annual inflation rate.

Table 1 presents the estimated 12×12 AR(1) matrix. The Eigen values of the implied autocovariance structure are well within the stability region, with two real roots and five complex conjugate pairs:

$$\begin{aligned}
\lambda_1 &= 0.6772; \\
\lambda_2 &= 0.1294; \\
\lambda_{3,4} &= 0.7594 \pm 0.2998\,\imath, \quad \text{modulus} = 0.8165; \\
\lambda_{5,6} &= 0.4104 \pm 0.5273\,\imath, \quad \text{modulus} = 0.6682; \\
\lambda_{7,8} &= 0.3056 \pm 0.3371\,\imath, \quad \text{modulus} = 0.4550; \\
\lambda_{9,10} &= -0.4222 \pm 0.0832\,\imath, \ \text{modulus} = 0.4304; \\
\lambda_{11,12} &= -0.0863 \pm 0.2638\,\imath, \ \text{modulus} = 0.2776.
\end{aligned} \tag{18}$$

A system of 12 linear first-order difference equations has the same dynamic structure as a single 12th-order linear difference equation. This implies that the time series properties of this model are quite complex. No evidence is found for any additional serial correlation in the data.

The single equation and system-wide 1st - and 2nd-order Brownian bridge tests for specification error and parameter instability developed in LaFrance (2008) provide no evidence of misspecification or parameter instability. (Appendix D presents and discusses this set of within-sample residual test statistics.)

Table 2 presents the parameter estimates for the structural part of the model. To obtain a positive definite $\boldsymbol{B}$ matrix, the lower 4 main diagonal elements of the Choleski factor $\boldsymbol{L}$ were restricted to 0.01 and the off-diagonal elements in the last four columns were restricted at 0.0. In other words, the estimated symmetric but not curvature restricted $\boldsymbol{B}$ matrix has four negative Eigen values. As a consequence, the standard errors in table 2 are conditional on these inequality restrictions. The estimated structural parameters re-

ported in table 2 generate a variable cost function that is increasing and weakly concave in all variable input prices throughout the data set. We conclude that this is a coherent and reasonable model of the short-run cost of production in U.S. agriculture.

4. Crop Acres, Capital, Savings and Investment, and Consumption in Agriculture

Although the organizational form of farms can vary widely, a recent report by Hoppe and Banker (2006) finds that 98% of U.S. farms remained family farms as of 2003. In a family farm, the entrepreneur controls the means of production and makes investment, consumption, and production decisions. In this section, we develop and analyze a model of the intertemporal nature of these decisions. The starting point is a model similar in spirit to Hansen and Singleton's (1983), but generalized to include consumption decisions and farm investments as well as financial investments and production decisions. The additional variable definitions required for this are as follows:

W_t = beginning-of-period total wealth,

b_t = current holding of bonds with a risk free rate of return r_t,

f_t = current holding of a risky financial asset,

$p_{F,t}$ = beginning-of-period market price of the financial asset,

$\rho_{F,t+1}$ = dividend plus capital gains rate on the financial asset,

$a_{i,t}$ = current allocation of land to the ith crop, $i = 1,\ldots,n_Y$,

A_t = total quantity of farm land,

$p_{L,t}$ = beginning-of-period market price of land,

$\rho_{L,t+1} = (p_{L,t+1} - p_{L,t}) / p_{L,t}$ = capital gain rate on land,

$\bar{y}_{i,t}$ = expected yield per acre for the ith crop, $i = 1,\ldots,n_Y$,

$y_{i,t+1}$ = realized yield of the ith crop,

$p_{Y_i,t+1}$ = end-of-period realized market price for the ith farm product,

$\boldsymbol{q}_t$ = vector of quantities of consumption goods,

$\boldsymbol{p}_{Q,t}$ = vector of market prices for consumer goods,

m_t = total consumption expenditures,

$u(\boldsymbol{q}_t)$ = periodic utility from consumption.

As with all discrete time models, timing can be represented in multiple ways. In the model used here, all financial returns and farm asset gains are assumed to be realized at the end of each time period (where depreciation is represented by a negative asset gain). Variable inputs are assumed to be committed to farm production activities at the beginning of each decision period and the current period market prices for the variable inputs are known when these use decisions are made. Agricultural production per acre is realized stochastically at the end of the period such that

$$y_{i,t+1} = \bar{y}_{i,t}(1+\varepsilon_{i,t+1}),\ i = 1,\ldots,n_Y\ , \tag{19}$$

where $\varepsilon_{i,t+1}$ is a random output shock with $E(\varepsilon_{i,t+1}) = 0$. Consumption decisions are made at the beginning of the decision period and the current market prices of consumption good are known when these purchases are made. Utility is assumed to be strictly increasing and concave in $\boldsymbol{q}_t$. The total beginning-of-period quantity of land is $A_t = \boldsymbol{\iota}^{\mathrm{T}}\boldsymbol{a}_t$, with $\boldsymbol{\iota}$ denoting an n_Y–vector of ones. Homogeneous land is assumed with a scalar price, $p_{L,t}$.

To simplify our derivations, we require an uncommon piece of matrix notation. The Hadamard/Schur product of two $n\times m$ matrices **A** and **B** is the matrix whose elements are

element-by-element products of the elements of **A** and $\boldsymbol{B}$, $\boldsymbol{A}\bullet\boldsymbol{B} = \boldsymbol{C} \Leftrightarrow c_{ij} = a_{ij}b_{ij}\ \forall i, j$. This definition assists the derivation of the arbitrage conditions present in what follows.

Revenue at $t + 1$ is the random price times production

$$R_{t+1} = \sum_{i=1}^{n_Y}(p_{Y_i,t+1}\bar{y}_{i,t}a_{i,t}(1+\varepsilon_{i,t+1})) \equiv (\boldsymbol{p}_{Y,t+1}\bullet\boldsymbol{a}_t\bullet\bar{\boldsymbol{y}}_t)^{\mathsf{T}}(\boldsymbol{\iota}+\boldsymbol{\varepsilon}_{t+1}). \tag{20}$$

Wealth is allocated at the beginning of period t to investments, the variable cost of production, and consumption,

$$W_t = b_t + f_t + p_{L,t}A_t + K_t + c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, K_t, \bar{\boldsymbol{Y}}_t) + m_t. \tag{21}$$

Although some costs occur at or near harvest (near $t + 1$), we include all costs in (16.26) at time t because they are incurred before revenues are received. Consumer utility maximization yields the indirect utility function conditioned on consumer good prices and consumption expenditure,

$$\upsilon(\boldsymbol{p}_{Q,t}, m_t) \equiv \max_{\boldsymbol{q}\in R_+^{n_Q}}\left\{u(\boldsymbol{q}) : \boldsymbol{p}_{Q,t}^{\mathsf{T}}\boldsymbol{q} = m_t\right\}. \tag{22}$$

Realized end of period wealth is

$$\begin{aligned} W_{t+1} &= (1+r_t)b_t + (1+\rho_{F,t+1})f_t + (1+\rho_{L,t+1})p_{L,t}A_t \\ &\quad + (1+\rho_{K,t+1})K_t + (\boldsymbol{p}_{Y,t+1}\bullet\boldsymbol{a}_t\bullet\bar{\boldsymbol{y}}_t)^{\mathsf{T}}(\boldsymbol{\iota}+\boldsymbol{\varepsilon}_{t+1}), \end{aligned} \tag{23}$$

where $\rho_{K,t+1}$ is the proportional change in the value of capital held at the beginning of the production period. Thus, the decision maker's wealth is increased by net returns on assets and farm revenue. The owner/operator decision maker's intertemporal utility function is assumed to be

$$U_T(\boldsymbol{q}_1,...,\boldsymbol{q}_T) = \sum_{t=0}^{T}(1+\rho)^{-t}u(\boldsymbol{q}_t). \tag{24}$$

The producer is assumed to maximize von Neumann-Morgenstern expected utility of the discounted present value of the periodic utility flows from goods consumption.

By Euler's theorem, constant returns to scale implies linear homogeneity of the variable cost function in capital, land, and output. For the variable cost function derived and estimated in this paper, this implies

$$\begin{aligned} c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{Y}}_t) \equiv & \frac{\partial c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{Y}}_t)}{\partial \boldsymbol{a}_t^{\top}} \boldsymbol{a}_t + \frac{\partial c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{Y}}_t)}{\partial A_t} A_t \\ & + \frac{\partial c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{Y}}_t)}{\partial K_t} K_t + \frac{\partial c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{Y}}_t)}{\partial \bar{\boldsymbol{Y}}_t^{\top}} \bar{\boldsymbol{Y}}_t. \end{aligned} \tag{25}$$

The vector of expected crop outputs satisfies

$$\bar{\boldsymbol{Y}}_t = \bar{\boldsymbol{y}}_t \bullet \boldsymbol{a}_t, \tag{26}$$

where $\bar{y}_{j,t}$ is the expected yield per acre and $a_{j,t}$ is the number of acres planted for the jth crop. The variable cost function might depend on time due to technological change or other dynamic forces, and the subscript t indicates this possibility. To distinguish quasi-fixed from variable inputs and to account for the possibility of hysteresis in agricultural investments, we allow for adjustment costs for total farmland and capital,

$$C_{Adj}(A_t - A_{t-1}, K_t - K_{t-1}) = \tfrac{1}{2}\gamma_A (A_t - A_{t-1})^2 + \tfrac{1}{2}\gamma_K (K_t - K_{t-1})^2, \tag{27}$$

with $\gamma_A, \gamma_K \geq 0$.

This problem is solved by stochastic dynamic programming working backwards recursively from the last period in the planning horizon to the first. In the last period, the optimal decision is to invest or produce nothing and consume all remaining wealth, i.e., $m_T = W_T$. Denote the last period's optimal value function by $v_T(W_T, A_{T-1}, K_{T-1})$. Then

$v_T(W_T, A_{T-1}, K_{T-1}) = \upsilon(\boldsymbol{p}_{Q,T}, W_T)$ is the optimal utility for the terminal period. For all other time periods, stochastic dynamic programming yields the Bellman backward recursion (Bellman and Dreyfus 1962). For an arbitrary $t < T$, the Lagrangean for the problem at time t is

$$\begin{aligned}\ell_t &= \upsilon(\boldsymbol{p}_{Q,t}, m_t) + (1+r)^{-1} E_t \Big\{ V_{t+1} \Big[(1+r) b_t + (1+\rho_{F,t+1}) f_t \\ &+ p_{L,t+1} A_t + (1+\rho_{K,t+1}) K_t + (\boldsymbol{p}_{Y,t+1} \bullet \bar{\boldsymbol{y}}_t \bullet \boldsymbol{a}_t)^{\mathsf{T}} (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1}), A_t, K_t \Big] \Big\} \\ &+ \lambda_t \Big\{ W_t - m_t - b_t - f_t - p_{L,t} A_t - K_t - c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, A_t, K_t, \bar{\boldsymbol{y}}_t \bullet \boldsymbol{a}_t) \\ &\quad - \tfrac{1}{2}\gamma_A (A_t - A_{t-1})^2 - \tfrac{1}{2}\gamma_K (K_t - K_{t-1})^2 \Big\} + \mu_t (A_t - \boldsymbol{\iota}^{\mathsf{T}} \boldsymbol{a}_t),\end{aligned} \tag{28}$$

where $E_t(\bullet)$ is the conditional expectation at the beginning of period t given information available at that point in time, λ_t is the shadow price for the beginning-of-period wealth allocation constraint, and μ_t is the shadow price for the land allocation constraint. The first-order, necessary and sufficient Kuhn-Tucker conditions are the two constraints and the following:

$$\frac{\partial \ell_t}{\partial m_t} = \frac{\partial \upsilon_t}{\partial m_t} - \lambda_t \le 0,\ m_t \ge 0,\ m_t \frac{\partial \ell_t}{\partial m_t} = 0; \tag{29}$$

$$\frac{\partial \ell_t}{\partial b_t} = E_t \left(\frac{\partial V_{t+1}}{\partial W_{t+1}} \right) - \lambda_t \le 0,\ b_t \ge,\ b_t \frac{\partial \ell_t}{\partial b_t} = 0; \tag{30}$$

$$\frac{\partial \ell_t}{\partial f_t} = (1+r)^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W_{t+1}} (1+\rho_{F,t+1}) \right] - \lambda_t \le 0,\ f_t \ge 0,\ f_t \frac{\partial \ell_t}{\partial f_t} = 0. \tag{31}$$

$$\begin{aligned}\frac{\partial \ell_t}{\partial A_t} &= (1+r)^{-1} E_t \left(\frac{\partial V_{t+1}}{\partial W_{t+1}} p_{L,t+1} + \frac{\partial V_{t+1}}{\partial A_t} \right) \\ -\lambda_t &\left[p_{L,t} + \frac{\partial c_t}{\partial A_t} + \gamma_A (A_t - A_{t-1}) \right] + \mu_t \le 0,\ A_t \ge 0,\ A_t \frac{\partial \ell_t}{\partial A_t} = 0;\end{aligned} \tag{32}$$

$$\frac{\partial \ell_t}{\partial K_t} = (1+r)^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W_{t+1}} (1+\rho_{K,t+1}) + \frac{\partial V_{t+1}}{\partial K_{t+1}} \right]$$
$$-\lambda_t \left[1 + \frac{\partial c_t}{\partial K_t} + \gamma_K (K_t - K_{t-1}) \right] \leq 0, K_t \geq 0, K_t \frac{\partial \ell_t}{\partial K_t} = 0; \tag{33}$$

$$\frac{\partial \ell_t}{\partial \boldsymbol{a}_t} = (1+r)^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W_{t+1}} (\boldsymbol{p}_{Y,t+1} \bullet \overline{\boldsymbol{y}}_t) \bullet (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1}) \right] - \lambda_t \left(\frac{\partial c_t}{\partial \boldsymbol{a}_t} + \frac{\partial c_t}{\partial \overline{\boldsymbol{Y}}_t} \bullet \overline{\boldsymbol{y}}_t \right) - \mu_t \boldsymbol{\iota} \leq 0,$$
$$\boldsymbol{a}_t \geq \mathbf{0},\ \boldsymbol{a}_t^{\top} \frac{\partial \ell_t}{\partial \boldsymbol{a}_t} = 0; \tag{34}$$

$$\frac{\partial \ell_t}{\partial \overline{\boldsymbol{y}}_t} = (1+r)^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W} \boldsymbol{p}_{Y,t+1} \bullet \boldsymbol{a}_t \bullet (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1}) \right] - \lambda_t \frac{\partial c_t}{\partial \overline{\boldsymbol{Y}}_t} \bullet \boldsymbol{a}_t \leq \mathbf{0},$$
$$\overline{\boldsymbol{y}}_t \geq \mathbf{0},\ \overline{\boldsymbol{y}}_t^{\top} \frac{\partial \ell_t}{\partial \overline{\boldsymbol{y}}_t} = 0. \tag{35}$$

We also have the following implications of the envelope theorem:

$$\frac{\partial V_t}{\partial W_t} = \lambda_t;$$
$$\frac{\partial V_t}{\partial A_{t-1}} = \lambda_t \gamma_A (A_t - A_{t-1}); \tag{36}$$
$$\frac{\partial V_t}{\partial K_{t-1}} = \lambda_t \gamma_K (K_t - K_{t-1});$$

where the variables $\{\lambda_t, A_t, K_t\}$ are all evaluated at their optimal choices.

Combining the Kuhn-Tucker conditions with the results of the envelope theorem and assuming an interior solution for consumption, bonds, and risky financial assets, we obtain the standard Euler equations for smoothing the marginal utility of consumption and wealth,

$$\frac{\partial \upsilon_t}{\partial m_t} = E_t \left(\frac{\partial \upsilon_{t+1}}{\partial m_{t+1}} \right) = \frac{\partial V_t}{\partial W_t} = E_t \left(\frac{\partial V_{t+1}}{\partial W_{t+1}} \right) = \lambda_t = E_t(\lambda_{t+1}), \tag{37}$$

and the standard arbitrage condition for excess returns to risky financial assets,

$$E_t\left[\,(\rho_{F,t+1}-r)\frac{\partial V_{t+1}}{\partial W_{t+1}}\right]=0 \tag{38}$$

The complementary slackness of the Kuhn-Tucker condition (35), implies that for each crop we have the supply condition under risk,

$$E_t\left[\frac{\partial V_{t+1}}{\partial W_{t+1}}\left(p_{Y_i,t+1}-(1+r)\frac{\partial c_t}{\partial \overline{Y}_{i,t}}\right)\right]\overline{Y}_{i,t}=0,\ i=1,\cdots,n_y. \tag{39}$$

For each crop produced in positive quantity, this reduces to the well-known result that the conditional covariance between the marginal utility of future wealth and the difference between the ex post realized market price the marginal cost of production must vanish. The multiplicative factor $1+r$ is multiplied by ex ante marginal cost so that these two economic values are measured at a common point in time – in the present case at the end of the production period.

To obtain the arbitrage condition for the level of investment in agriculture, we combine the linear homogeneity property of the variable cost function in $(\boldsymbol{a}_t, A_t, K_t, \overline{\boldsymbol{Y}}_t)$ from equation (25) with complementary slackness in Kuhn-Tucker conditions (33)–(37),

$$0=\frac{\partial \ell_t}{\partial \boldsymbol{a}_t^{\mathrm{T}}}\boldsymbol{a}_t+\frac{\partial \ell_t}{\partial A_t}A_t+\frac{\partial \ell_t}{\partial K_t}K_t, \tag{40}$$

which, after considerable rearranging and combining of terms, gives

$$\begin{aligned}E_t\{\partial V_{t+1}/\partial W_{t+1}[\,&s_{K,t}(\rho_{K,t+1}-r)+s_{L,t}(\rho_{L,t+1}-r)+\pi_{t+1}\\&+s_{K,t}\gamma_K\,(\,K_{t+1}-(2+r)K_t+(1+r)K_{t-1}\,)\\&+s_{A,t}\gamma_A\,(\,A_{t+1}-(2+r)A_t+(1+r)A_{t-1}\,)\,]\}=0,\end{aligned} \tag{41}$$

where $s_{K,t}=K_t/(p_{L,t}A_t+K_t)$ is capital's share of the value of the investment in agriculture in period t, $s_{L,t}=p_{L,t}A_t/(p_{L,t}A_t+K_t)$ is land's share of the value of the investment

in agriculture in period t, $s_{A,t} = A_t/(p_{L,t}A_t + K_t)$ is the ratio of the quantity of land to the value of the investment in agriculture at the beginning of the production period, and

$$\pi_{t+1} = \frac{R_{t+1} - (1+r)c_t}{p_{L,t}A_t + K_t} \tag{42}$$

is the *ex post* net return to crop production over the variable cost of production relative to the *ex ante* value of agricultural investment, so that it is measured as a rate of return to agricultural production. The first 3 terms inside of the square brackets of equation (41) represent the total sum of the excess returns to agriculture, including the rate of net return to crop production over variable costs. The last two terms in square brackets capture the effects of adjustment costs for farm capital and farmland. This has the standard one-period ahead and one period behind 2nd–order difference structure common to quadratic adjustment cost models in dynamic optimization problems.

To implement this system of Euler equations, we assume that the indirect utility function for consumption goods is a member of the certainty equivalent class,

$$\upsilon(\boldsymbol{p}_{Qt}, m_t) = \frac{m_t}{\pi_C(\boldsymbol{p}_{Qt})} - ½\beta\left(\frac{m_t}{\pi_C(\boldsymbol{p}_{Qt})}\right)^2, \tag{43}$$

where $0 \le \beta < \pi_C(\boldsymbol{p}_{Qt})/m_t \ \forall\ t$ and $\pi_C(\boldsymbol{p}_{Qt})$ is the consumer price index (CPI) for all items. Then the marginal utility of money in each period is

$$\lambda_t = \frac{1 - \beta\left[m_t/\pi_C(\boldsymbol{p}_{Qt})\right]}{\pi_C(\boldsymbol{p}_{Qt})}. \tag{44}$$

This allows us to identify the effects of risk aversion separately from those of adjustment costs and hysteresis in agricultural investment decisions. We assume that the preferences

of agricultural producers are of the same class as all other individuals in the economy. This allows use of the observable variable per capita personal consumption expenditure, rather than the latent variable wealth, to model the empirical arbitrage equations.

Empirical Arbitrage Equations and Data

Let $n \le n_y$ be the number of crops included in the empirical model. The specification that we choose for $\partial c_t / \partial \overline{Y}_{i,t}$ is (see Appendix C for a complete derivation),

$$\begin{aligned} \frac{\partial c_t}{\partial \overline{Y}_{i,t}} &= \widehat{\beta(\boldsymbol{w}_t)}\left(\theta_i + \sum_{j=1}^{n_x} \theta_{ij} \frac{\overline{Y}_{j,t}}{K_t} \right), \text{ with} \\ \widehat{\beta(\boldsymbol{w}_t)} &= \sqrt{\tilde{\boldsymbol{w}}_t^{\mathsf{T}} \hat{\boldsymbol{B}} \tilde{\boldsymbol{w}}_t + 2\hat{\boldsymbol{\gamma}}^{\mathsf{T}} \tilde{\boldsymbol{w}}_t w_{n_x,t} + w_{n_x,t}^2}. \end{aligned} \tag{45}$$

We use the estimated $\widehat{\beta(\boldsymbol{w}_t)}$ obtained from the *ex ante* variable input demand system, and $\tilde{\boldsymbol{w}}_t = [w_{1,t} \cdots w_{n_x,t-1}]^{\mathsf{T}}$ is the vector of variable input prices other than the farm wage.[4] The $n+3$ empirical arbitrage/Euler equations therefore are

$$\begin{aligned} &\textbf{Consumption / Bonds}: && \beta(m_{t+1} - m_t) = u_{1,t+1}, \\ &\textbf{Risky Assets}: && (1 - \beta m_{t+1})(\rho_{F,t+1} - r) = u_{2,t+1}, \\ &\textbf{Crops}: && (1 - \beta m_{t+1})\left[p_{Y_i,t+1} - (1+r)\widehat{\beta(\boldsymbol{w}_t)}\left(\theta_i + \sum_{j=1}^{n} \frac{\theta_{ij} Y_{j,t+1}}{K_t} \right) \right] = u_{i,t+1}, \\ & && i = 3, \cdots, n+2, \\ &\textbf{Agriculture}: && (1 - \beta m_{t+1})\big[s_{K,t}(\rho_{K,t+1} - r) + s_{L,t}(\rho_{L,t+1} - r) + \pi_{t+1} \\ & && + s_{A,t}\gamma_A (A_{t+1} - (2+r)A_t + (1+r)A_{t-1}) \\ & && + s_{K,t}\gamma_K (K_{t+1} - (2+r)K_t + (1+r)K_{t-1}) \big] = u_{n+3,t+1}. \end{aligned} \tag{46}$$

[4]All prices, costs, and revenues, including the value of farm capital, are deflated by the consumer price index in this part of the empirical analysis.

The estimation method for this part of the modeling exercise again is NL3SLS/GMM with a parameteric AR(1) correction for autocorrelation and White/Huber heteroskedasticity consistent estimated covariance matrix. We restrict the parameter matrix $\boldsymbol{\Theta} = [\theta_{ij}]$ to be positive semi-definite by estimating it in Choleski factored form, $\boldsymbol{\Theta} = \boldsymbol{QQ}^{\mathrm{T}}$, where $\boldsymbol{Q}$ is a lower triangular matrix.

Empirical Results

We analyze acreage and supply decisions under risk for 10 crops with the greatest value in the U.S. in 2006: soybeans; corn; cotton; hay; potatoes; rice; sugar beets; sugarcane; tobacco; and wheat. Crop revenues includes the value of government payments that is been imputed in the Ball data, to at least partially capture the effects of farm-level price, income, and other subsidy and stabilization programs on the distribution of realized farm revenues. The 10 crops analyzed in this study account for 94–95% of total farm revenue from crop production and an even larger share of crop acreage. In addition to the 10 crop production decisions under risk, we estimate Euler equations for the excess return to investing in agriculture, personal consumption expenditures, and the rate of return to stocks as measured by the S& Poor 500 index.

To ensure a consistent definition of real values in this component of the model, we deflate all nominal prices, revenues, costs, and other values by that year's consumer price index for all items. We scale all aggregate economic data – e.g., the total value of agricultural investment in U.S. agriculture – by the U.S. population to measure these variables all in per capita units. As noted above, real per capita personal consumption expenditures represents the Euler equation for the marginal utility of money over time.

Table 3 presents the unrestricted 13×13 AR(1) coefficient matrix. Similar to the variable cost function model, Eigen values of the implied autocovariance structure are well within the stability region, with five real roots and four complex conjugate pairs:

$$
\begin{aligned}
\lambda_1 &= \quad 0.8960; \\
\lambda_2 &= -0.5510; \\
\lambda_3 &= \quad 0.2829; \\
\lambda_4 &= \quad 0.1800; \\
\lambda_5 &= -0.0471; \\
\lambda_{6,7} &= \quad 0.2966 \pm 0.6058\iota, \text{ modulus} = 0.6745; \\
\lambda_{8,9} &= \quad 0.5789 \pm 0.0674\iota, \text{ modulus} = 0.5828; \\
\lambda_{10.11} &= -0.0048 \pm 0.5157\iota, \text{ modulus} = 0.5157; \\
\lambda_{12,13} &= -0.4497 \pm 0.1827\iota, \text{ modulus} = 0.4854.
\end{aligned}
$$

Also similar to the properties of the cost function estimates, there is no evidence of any additional serial correlation in the error terms, and all of the system-wide and single equation Brownian bridge tests fail to reject the null hypothesis of no model specification errors or parameter instability at all standard levels of significance.

Table 4 presents the parameter estimates for the conditional mean components of the arbitrage model. To obtain a positive definite $\boldsymbol{\Theta}$ matrix, the lower 6 main diagonal elements of the Choleski factor $\boldsymbol{Q}$ were restricted to 0.01 and the off-diagonal elements in the last four columns were restricted at 0.0. In other words, the estimated symmetric but not curvature restricted $\boldsymbol{\Theta}$ matrix has four negative Eigen values. As a consequence, the standard errors in table 2 are conditional on these inequality restrictions. The estimated structural parameters reported in table 4 generate a system of 10 linear marginal cost functions that are increasing in planned output levels throughout the sample period. We conclude that this is a coherent and reasonable model of U.S. agricultural production.

The point estimate for the curvature parameter in the quadratic indirect utility function is $\hat{\beta} = 6.571 \times 10^{-5}$, with an estimated classical Gaussian asymptotic standard error of 2.793×10^{-7} and an estimated White/Huber robust standard error of 5.914×10^{-7}, both implying a highly significant risk aversion parameter. On the other hand, the point estimates for the quadratic adjustment cost parameters are mixed. The point estimate for adjustment costs on farmland is $\hat{\gamma}_A = -4.455 \times 10^{-6}$, with an estimated classical Gaussian asymptotic standard error of 1.985×10^{-6} and an estimated White/Huber robust standard error of 1.477×10^{-6}. In both cases, this is statistically different from significant at the 5% significance level, although economically, the sign is not what we would expect a priori. The point estimate for adjustment costs in farm capital is $\hat{\gamma}_K = 4.012 \times 10^{-11}$, with an estimated classical Gaussian asymptotic standard error of 2.738×10^{-11} and an estimated White/Huber robust standard error of 2.383×10^{-11}. While this has the expected sign, the classical standard error implies this is not statistically different from zero at the 10% level of significance, while the robust standard error implies that it marginally is significant at the same level. We suspect that either this level of aggregation across agents cannot capture these effects or else there is at most only a small level of adjustment cost in the farm sector. On the other hand, if there is no adjustment cost mechanism in U.S. agriculture, and if the quadratic indirect utility model is correctly specificed, then the Euler equations estimated here are theoretically and empirically correct even with national aggregate data.

Conclusions

This paper has developed and analyzed a new structural model of variable input use, production, acreage allocations, capital investment, and consumption choices in the U.S. farm sector. The theoretical framework identifies and incorporates the restrictions that are necessary and sufficient to estimate variable input use using only observable data, and to aggregate from micro units of behavior to county-, state-, region-, or country-levels of data and analyses. We defined, specified and estimated a dynamic life-cycle model of decision making under risk. We disciplined the model and associated parameter estimates for risk aversion in agricultural production and investment decisions with the interactions that naturally occur among the available alternative investment and savings opportunities in the economy. Our current work is focused on applying this analysis to state-level data, which should to mitigate somewhat the issues related to aggregating across different production regions, climates, and reasonable output choice sets.

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Table 1. First-Order Autocorrelation Parameters for the Variable Cost Model.

	Pesticide	**Fertilizer**	**Fuel & Nat. Gas**	**Electricity**	**Purchased Feed**	**Purchased Seed**	**Purchased Livestock**	**Machinery Repairs**	**Building Repairs**	**Hired Machinery**	**Veterinary Services**	**Other Materials**
Pesticide	-.235 (.323) [.230]	.280 (.184) [.139**]	-.556 (.303*) [.242**]	-.199 (.235) [.141]	-.188 (.126) [.070***]	-.291 (.394) [.281]	-.029 (.096) [.057]	-.029 (.283) [.223]	-.341 (.482) [.290]	-.283 (.320) [.206]	.690 (.464) [.254***]	.040 (.084) [.068]
Fertilizer	.333 (.524) [.259]	.699 (.298**) [.161***]	-.265 (.491) [.256]	-.664 (.382*) [.191***]	.062 (.204) [.121]	-2.01 (.639***) [.459***]	.154 (.155) [.101]	1.22 (.459***) [.324***]	-1.50 (.782*) [.399***]	.704 (.519) [.273***]	-.339 (.753) [.412]	.046 (.136) [.075]
Fuel & Nat. Gas	.256 (.229) [.072***]	.193 (.130) [.047***]	.503 (.215**) [.085***]	-.263 (.167) [.071***]	-.040 (.089) [.039]	-1.11 (.279***) [.132***]	.034 (.068) [.029]	.577 (.200***) [.107***]	-.159 (.342) [.117]	.321 (.227) [.093***]	.131 (.329) [.135]	-.048 (.060) [.025**]
Electricity	.066 (.256) [.178]	-.147 (.146) [.087*]	-.168 (.240) [.155]	.455 (.187**) [.107***]	-.052 (.100) [.055]	.049 (.312) [.227]	-.061 (.076) [.046]	.254 (.224) [.088***]	.120 (.382) [.209]	-.268 (.254) [.141*]	.067 (.368) [.259]	-.064 (.067) [.035*]
Purchased Feed	-.463 (.626) [.376]	.252 (.356) [.257]	.199 (.587) [.433]	-.404 (.456) [.253]	.066 (.244) [.181]	-.018 (.763) [.483]	.308 (.186*) [.157**]	-.770 (.548) [.351**]	-.016 (.934) [.625]	.345 (.620) [.282]	.025 (.900) [.625]	.220 (.163) [.098**]
Purchased Seed	-.120 (.276) [.217]	.185 (.157) [.121]	-.282 (.258) [.234]	-.303 (.201) [.155*]	-.043 (.107) [.070]	-.447 (.336) [.236*]	.012 (.082) [.068]	.607 (.241**) [.245**]	-.296 (.411) [.328]	-.185 (.273) [.190]	-.139 (.396) [.273]	-.019 (.072) [.055]
Purchased Livestock	.0450 (.649) [.399]	-.244 (.369) [.226]	-.577 (.608) [.391]	-.332 (.472) [.312]	.073 (.252) [.176]	-.222 (.790) [.600]	.362 (.192*) [.158**]	.615 (.568) [.402]	-.769 (.967) [.603]	-.951 (.643) [.328***]	-.975 (.932) [.562*]	-.296 (.169*) [.118**]
Machinery Repairs	.030 (.245) [.162]	.135 (.140) [.090]	-.359 (.230) [.187*]	-.008 (.178) [.139]	-.045 (.095) [.059]	-.616 (.299**) [.246**]	.064 (.073) [.059]	.657 (.215***) [.127***]	.0763 (.366) [.206]	-.003 (.243) [.136]	-.024 (.352) [.195]	-.048 (.064) [.049]
Building Repairs	.084 (.160) [.112]	.064 (.091) [.065]	-.079 (.150) [.144]	-.007 (.116) [.083]	.011 (.062) [.047]	-.105 (.195) [.132]	.009 (.047) [.041]	.376 (.140***) [.119***]	-.071 (.238) [.162]	-.041 (.158) [.094]	-.337 (.230) [.175*]	-.021 (.042) [.031]
Hired Machinery	.113 (.253) [.252]	-.097 (.144) [.124]	.011 (.237) [.200]	.032 (.184) [.150]	-.043 (.098) [.083]	-.119 (.308) [.304]	-.123 (.075) [.079]	.239 (.221) [.197]	-.060 (.377) [.324]	-.060 (.250) [.219]	-.209 (.363) [.416]	-.043 (.066) [.047]
Veterinary Services	.041 (.149) [.076]	-.089 (.085) [.059]	-.006 (.140) [.110]	.135 (.108) [.049***]	.004 (.058) [.035]	.096 (.181) [.122]	.021 (.044) [.020]	-.129 (.130) [.079]	.329 (.222) [.132**]	-.356 (.148**) [.084***]	.602 (.214***) [.152***]	.023 (.039) [.019]
Other Materials	-1.45 (.997) [.622**]	-.062 (.567) [.355]	-.085 (.934) [.568]	.272 (.726) [.473]	-.188 (.388) [.272]	2.46 (1.21**) [1.09**]	.178 (.295) [.232]	-2.37 (.873***) [.881***]	-.409 (1.49) [1.01]	.477 (.987) [.633]	.986 (1.43) [.841]	.211 (.259) [.140]

Numbers in parentheses (·) and in square brackets [·] are standard Gaussian and White heteroskedasticity consistent asymptotic standard errors, respectively. *, **, and *** indicate significantly different from zero at the 10%, 5%, and 1% level, respectively, for the corresponding asymptotic standard error.

Table 2. Estimates of α and B Parameters for the Variable Cost Function.

Variable	Pesticide	Fertilizer	Fuel & Nat. Gas	Electric	Purchased Feed	Purchased Seed	Purchased Livestock	Machinery Repairs	Building Repairs	Hired Machinery	Veterinary Services	Other Materials	Labor
Land	-55.21 (74.23)	-64.95 (81.50)	-21.99 (29.73)	-15.54 (23.17)	-67.00 (101.7)	-44.30 (58.38)	-51.87 (77.94)	-20.32 (31.02)	-9.286 (11.61)	-19.31 (28.14)	-8.374 (12.76)	-114.7 (171.2)	36.84 (30.43)
Capital	3.554 (3.854)	4.140 (4.233)	1.515 (1.572)	1.001 (1.223)	5.044 (5.314)	2.837 (3.040)	3.808 (4.102)	1.615 (1.637)	.6197 (.6152)	1.362 (1.479)	.5819 (.6678)	8.634 (8.920)	-1.564 (1.596)
Pesticide	3.695*** (1.288)	2.995* (1.824)	.9374 (.8018)	.7107 (.6566)	5.042*** (1.800)	2.558** (1.179)	2.832 (1.851)	1.020 (.7793)	.3742 (.3850)	1.201* (.6345)	.4549 (.3043)	7.442** (3.324)	-1.390*** (.5430)
Fertilizer	2.995* (1.824)	5.119*** (1.390)	1.469** (.6455)	1.388*** (.4419)	4.709** (2.284)	2.480* (1.364)	3.806** (1.671)	.5221 (1.116)	1.332*** (.2998)	1.425** (.5894)	.5213 (.3355)	6.914* (4.174)	-1.501*** (.5742)
Fuel & Nat. Gas	.9374 (.8018)	1.469** (.6455)	1.027*** (.2538)	.3949* (.2106)	1.303 (1.068)	.8504 (.6167)	1.764*** (.5496)	.7740*** (.2860)	.5493** (.2480)	.7398*** (.2268)	.0858 (.1824)	2.189 (1.736)	-.6180*** (.1851)
Electric	.7107 (.6566)	1.388*** (.4419)	.3949* (.2106)	1.372*** (.4332)	1.559** (.6354)	-.4480 (1.038)	1.129** (.5554)	-.2855 (.7581)	.7367* (.3984)	.0407 (.4945)	-.0248 (.2198)	1.322 (1.547)	-.4507** (.2276)
Purchased Feed	5.042*** (1.800)	4.709** (2.284)	1.303 (1.068)	1.559** (.6354)	7.302*** (2.414)	3.229* (1.736)	4.087* (2.426)	1.382 (1.111)	.4051 (.4920)	1.857** (.7602)	.6293 (.4126)	9.850** (4.878)	-2.067*** (.6721)
Purchased Seed	2.558** (1.179)	2.480* (1.364)	.8504 (.6167)	-.4480 (1.038)	3.229* (1.736)	5.547*** (1.135)	2.694** (1.305)	1.608 (1.032)	-.2849 (1.027)	2.494*** (.8282)	.9234*** (.2894)	4.455 (3.182)	-1.408*** (.2930)
Purchased Livestock	2.832 (1.851)	3.806** (1.671)	1.764*** (.5496)	1.129** (.5554)	4.087* (2.426)	2.694** (1.305)	4.303*** (1.638)	1.161 (.9572)	.3629 (.4089)	1.424** (.5827)	.6567** (.3026)	7.340* (3.950)	-1.585*** (.5396)
Machinery Repairs	1.020 (.7793)	.5221 (1.116)	.7740*** (.2860)	-.2855 (.7581)	1.382 (1.111)	1.608 (1.032)	1.161 (.9572)	6.508* (3.475)	-2.860 (1.825)	2.635*** (.8530)	.2365 (.3453)	3.098** (1.528)	-.8805*** (.2250)
Building Repairs	.3742 (.3850)	1.332*** (.2998)	.5493** (.2480)	.7367* (.3984)	.4051 (.4920)	-.2849 (1.027)	.3629 (.4089)	-2.860 (1.825)	7.456** (3.378)	-1.735* (1.032)	-.6479 (.4479)	.4392 (.9521)	-.4085*** (.1467)
Hired Machinery	1.201* (.6345)	1.425** (.5894)	.7398*** (.2268)	.0407 (.4945)	1.857** (.7602)	2.494*** (.8282)	1.424** (.5827)	2.635*** (.8530)	-1.735* (1.032)	2.493** (1.055)	.3542* (.2006)	.8858 (2.280)	-.8651*** (.1362)
Veterinary Services	.4549 (.3043)	.5213 (.3355)	.0858 (.1824)	-.0248 (.2198)	.6293 (.4126)	.9234*** (.2894)	.6567** (.3026)	.2365 (.3453)	-.6479 (.4479)	.3542* (.2006)	.2980*** (.1159)	1.410** (.6818)	-.2534** (.1150)
Other Materials	7.442** (3.324)	6.914* (4.174)	2.189 (1.736)	1.322 (1.547)	9.850** (4.878)	4.455 (3.182)	7.340* (3.950)	3.098** (1.528)	.4392 (.9521)	.8858 (2.280)	1.410** (.6818)	21.99*** (6.588)	-2.845** (1.374)
Labor	-1.390*** (.5430)	-1.501*** (.5742)	-.6180*** (.1851)	-.4507** (.2276)	-2.067*** (.6721)	-1.408*** (.2930)	-1.585*** (.5396)	-.8805*** (.2250)	-.4085*** (.1467)	-.8651*** (.1362)	-.2534** (.1150)	-2.845** (1.374)	1.000 (——)

White's heteroskedasticity consistent asymptotic standard error is in parentheses below each point estimate. *, **, and *** indicate statistically different from zero at the 10%, 5%, and 1% significance level, respectively.

Table 3. First-Order Autocorrelation Parameters for the Arbitrage/Euler Equations.

	Soybeans	**Corn**	**Cotton**	**Hay**	**Potatoes**	**Rice**	**Sugar Beets**	**Sugarcane**	**Tobacco**	**Wheat**	**Agriculture**	**Financial Asset**	**Consumption**
Soybeans	-.418 (.334) [.202**]	.696 (.321**) [.203***]	-.813 (1.23) [.587]	.027 (.064) [.049]	.162 (.207) [.130]	.065 (.193) [.121]	-.062 (.094) [.057]	.036 (.115) [.065]	-.670 (1.00) [.536]	.396 (.382) [.190**]	-1.25 (5.83) [3.30]	-.037 (1.54) [.940]	-.078 (2.46) [1.54]
Corn	.341 (.264) [.140**]	.277 (.253) [.122**]	1.46 (.974) [.470***]	-.025 (.050) [.034]	-.0044 (.163) [.089]	.074 (.153) [.080]	-.085 (.074) [.033**]	.045 (.090) [.054]	-.383 (.795) [.298]	.0085 (.301) [.129]	1.90 (4.60) [2.45]	-1.36 (1.21) [.629**]	-1.38 (1.94) [.986]
Cotton	-.054 (.066) [.055]	.022 (.063) [.051]	-.261 (.243) [.206]	.014 (.012) [.014]	-.045 (.040) [.027*]	-.064 (.038*) [.038]	-.00012 (.018) [.013]	.013 (.022) [.018]	.070 (.198) [.131]	.011 (.075) [.055]	-.558 (1.15) [1.10]	.070 (.304) [.210]	.436 (.486) [.361]
Hay	-1.71 (1.48) [.788**]	2.62 (1.42*) [1.08**]	-3.11 (5.49) [2.54]	.244 (.285) [.177]	.302 (.920) [.459]	-.824 (.862) [.559]	.167 (.418) [.359]	.103 (.512) [.318]	-1.40 (4.48) [2.37]	1.67 (1.69) [.983*]	27.6 (25.9) [21.2]	2.55 (6.85) [3.57]	-12.2 (10.9) [7.16*]
Potatoes	-.081 (.358) [.230]	.146 (.344) [.182]	1.06 (1.32) [.712]	.159 (.068**) [.041***]	.238 (.222) [.162]	.033 (.207) [.125]	.036 (.100) [.051]	.139 (.123) [.079*]	-.904 (1.08) [.484*]	-.191 (.409) [.233]	1.61 (6.26) [3.45]	2.99 (1.65*) [1.17**]	.696 (2.64) [.799]
Rice	.092 (.474) [.295]	-.428 (.455) [.312]	1.72 (1.75) [1.02*]	-.104 (.091) [.054*]	.230 (.293) [.167]	-.021 (.274) [.186]	-.148 (.133) [.092]	-.067 (.163) [.083]	-.188 (1.42) [.800]	.216 (.541) [.326]	-1.19 (8.28) [6.26]	.396 (2.18) [1.42]	-2.34 (3.49) [2.07]
Sugar Beets	-.044 (1.00) [.707]	-.693 (.966) [.749]	6.03 (3.71) [3.63*]	-.065 (.193) [.156]	-.793 (.622) [.462*]	.492 (.583) [.529]	.072 (.282) [.277]	.0049 (.346) [.283]	1.15 (3.03) [2.44]	-.172 (1.14) [.976]	13.9 (17.5) [10.33]	2.15 (4.63) [4.02]	-1.80 (7.42) [7.27]
Sugarcane	-.404 (.821) [.413]	-.658 (.788) [.518]	4.33 (3.03) [2.21**]	-.083 (.157) [.067]	-1.17 (.508**) [.342***]	.367 (.476) [.339]	.413 (.230*) [.191**]	-.482 (.282*) [.196**]	.135 (2.47) [1.69]	.148 (.938) [.562]	-4.40 (14.3) [7.58]	-6.93 (3.78*) [2.25***]	-6.35 (6.05) [4.90]
Tobacco	-.0075 (.130) [.050]	-.096 (.124) [.045]	.866 (.480*) [.183***]	-.0052 (.024) [.0078]	.018 (.080) [.028]	-.028 (.075) [.024]	.00048 (.036) [.011]	-.0051 (.044) [.018]	.103 (.392) [.135]	-.175 (.148) [.047***]	.240 (2.27) [.912]	.467 (.600) [.175***]	-.881 (.960) [.311***]
Wheat	-.377 (.342) [.200*]	.095 (.328) [.173]	-1.59 (1.26) [.682**]	.020 (.065) [.044]	-.099 (.212) [.095]	.138 (.198) [.095]	-.090 (.096) [.051*]	.018 (.118) [.068]	.085 (1.03) [.520]	.572 (.391) [.181***]	-3.57 (5.98) [2.00*]	-.909 (1.57) [.973]	3.15 (2.52) [1.22***]
Agriculture	-.0015 (.012) [.0067]	.0092 (.012) [.0091]	.047 (.046) [.025*]	-.0014 (.0024) [.0014]	.010 (.0078) [.0044**]	-.00046 (.0073) [.0045]	-.0035 (.0035) [.0023]	.0040 (.0043) [.0030]	.048 (.038) [.026*]	.018 (.014) [.0089**]	.505 (.220**) [.151***]	.032 (.058) [.040]	.0077 (.093) [.040]
Financial Asset	-.036 (.047) [.031]	-.082 (.045) [.036**]	.114 (.175) [.099]	.0011 (.0091) [.0057]	.0021 (.029) [.016]	.048 (.027) [.021]	.0016 (.013) [.012]	-.0057 (.016) [.011]	-.235 (.143) [.102]	-.010 (.054) [.039]	.761 (.831) [.787]	.383 (.219) [.143]	-.191 (.351) [.222]
Consumption	.016 (.036) [.017]	.027 (.034) [.012]	-.054 (.133) [.048]	.0017 (.0069) [.0035]	.015 (.022) [.0067]	-.010 (.020) [.010]	-.0012 (.010) [.0038]	.010 (.012) [.0048]	.022 (.108) [.045]	.017 (.041) [.013]	.119 (.629) [.227]	-.033 (.166) [.081]	.392 (.265) [.134]

Numbers in parentheses (·) and in square brackets [·] are standard Gaussian and White heteroskedasticity consistent asymptotic standard errors, respectively. *, **, and *** indicate significantly different from zero at the 10%, 5%, and 1% level, respectively, for the corresponding asymptotic standard error.

Table 4. Estimates of the θ Parameters for the Arbitrage/Euler Equations.

Variable	Soybeans	Corn	Cotton	Hay	Potatoes	Rice	Sugar Beets	Sugarcane	Tobacco	Wheat
Constant	0.448166***	0.216785***	0.082527***	3.612791***	0.4599***	1.048429***	2.33634***	1.584347***	0.145884***	0.409247***
	(0.036632)	(0.024595)	(0.011218)	(0.197076)	(0.038374)	(0.064534)	(0.172455)	(0.161094)	(0.013593)	(0.033842)
Soybeans	78.34646***	-21.4571***	-1.80305	387.3322***	-32.5804*	41.71316**	-248.589***	-207.111***	3.799219	-0.08132
	(10.91386)	(3.992182)	(4.006355)	(88.54985)	(17.18041)	(20.3605)	(61.53115)	(52.65531)	(2.398324)	(7.195695)
Corn	-21.4571***	10.15897***	-2.01571	-83.6499***	0.028554	6.208851	79.67549***	65.14134***	1.87953**	-4.10214
	(3.992182)	(2.021877)	(1.236323)	(31.09819)	(4.755907)	(6.176972)	(17.03617)	(17.90792)	(0.767446)	(3.139472)
Cotton	-1.80305	-2.01571	35.56266***	-15.4651	115.6149***	-77.2543***	-215.395***	-369.447***	-3.23171*	0.82913
	(4.006355)	(1.236323)	(10.44578)	(59.45403)	(19.98758)	(29.06395)	(70.0208)	(65.29259)	(1.670253)	(2.366206)
Hay	387.3322***	-83.6499***	-15.4651	3920.242***	13.57345	-532.877**	523.7243	876.7413	-106.728***	101.2035*
	(88.54985)	(31.09819)	(59.45403)	(839.7853)	(205.9008)	(265.5106)	(593.9006)	(765.733)	(30.21382)	(55.11529)
Potatoes	-32.5804*	0.028554	115.6149***	13.57345	406.3396***	-353.176***	-427.114*	-916.150***	-27.1575***	16.57514*
	(17.18041)	(4.755907)	(19.98758)	(205.9008)	(110.575)	(80.82956)	(239.1877)	(247.4911)	(7.104441)	(8.869283)
Rice	41.71316**	6.208851	-77.2543***	-532.877**	-353.176***	577.8639***	-421.694*	-196.946	77.80513***	-67.5467***
	(20.3605)	(6.176972)	(29.06395)	(265.5106)	(80.82956)	(155.478)	(253.3207)	(271.0725)	(12.01163)	(11.23272)
Sugar Beets	-248.589***	79.67549***	-215.395***	523.7243	-427.114*	-421.694*	3761.957***	4775.894***	-124.254***	112.5914***
	(61.53115)	(17.03617)	(70.0208)	(593.9006)	(239.1877)	(253.3207)	(1065.617)	(1221.091)	(21.47398)	(30.83312)
Sugarcane	-207.111***	65.14134***	-369.447***	876.7413	-916.150***	-196.946	4775.894***	6539.396***	-132.38***	135.5798***
	(52.65531)	(17.90792)	(65.29259)	(765.733)	(247.4911)	(271.0725)	(1221.091)	(1500.074)	(16.05683)	(31.36628)
Tobacco	3.799219	1.87953**	-3.23171*	-106.728***	-27.1575***	77.80513***	-124.254***	-132.38***	12.70347***	-11.9499***
	(2.398324)	(0.767446)	(1.670253)	(30.21382)	(7.104441)	(12.01163)	(21.47398)	(16.05683)	(1.405087)	(2.30313)
Wheat	-0.08132	-4.10214	0.82913	101.2035*	16.57514*	-67.5467***	112.5914***	135.5798***	-11.9499***	12.13339**
	(7.195695)	(3.139472)	(2.366206)	(55.11529)	(8.869283)	(11.23272)	(30.83312)	(31.36628)	(2.30313)	(5.291329)

White's heteroskedasticity consistent asymptotic standard erroris in parentheses below each point estimate. *, **, and *** indicate statistically different from zero at the 10%, 5%, and 1% significance level, respectively.

Appendix A

Let $\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}_{++}^{n_x}$ be an n_x–vector of variable inputs, let $\boldsymbol{w} \in \mathcal{W} \subseteq \mathbb{R}_{++}^{n_x}$ be an n_x–vector of variable input prices, let $\boldsymbol{y} \in \mathcal{Y} \subseteq \mathbb{R}_{++}^{n_y}$ be an n_y–vector of outputs, let $\boldsymbol{z} \in \mathcal{Z} \subseteq \mathbb{R}_{++}^{n_z}$ be an n_z–vector of quasi-fixed inputs, let $F : \mathcal{X} \times \mathcal{Y} \times \mathcal{Z} \to \mathbb{R}$ be a transformation function that defines the boundary of a closed, convex production possibilities set with free disposal in inputs and outputs, let $\boldsymbol{X} : \mathcal{W} \times \mathcal{Y} \times \mathcal{Z} \to \mathcal{X}$, be an n_x–vector of variable input demand functions, and let $C : \mathcal{W} \times \mathcal{Y} \times \mathcal{Z} \to \mathbb{R}_{++}$ be a variable cost function,

$$c = C(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}) \equiv \min_{\boldsymbol{x}} \left\{ \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x} : F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \le 0, \boldsymbol{x} \ge \boldsymbol{0} \right\} \equiv \boldsymbol{w}^{\mathsf{T}} \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}), \tag{A.1}$$

where the symbol ^{T} denotes vector and matrix transposition. The purpose of this appendix is to prove that short-run cost-minimizing variable input demands, $\boldsymbol{x} = \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z})$, can be written in the form $\boldsymbol{x} = \tilde{\boldsymbol{X}}(\boldsymbol{w}, c, \boldsymbol{z})$ if and only if $c = C(\boldsymbol{w}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})) \Leftrightarrow F(\boldsymbol{x}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z}))$.

The neoclassical model of conditional demands for variable inputs with joint production, quasi-fixed inputs, and production uncertainty is

$$\boldsymbol{X}(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}) = \arg\min \left\{ \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x} : F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \le 0,\ \boldsymbol{x} \ge \boldsymbol{0} \right\}, \tag{A.2}$$

where $\boldsymbol{x}$ is an n_x–vector of positive variable inputs with corresponding positive prices, $\boldsymbol{w}$, $\boldsymbol{y}$ is an n_y–vector of planned outputs, $\boldsymbol{z}$ is an n_z–vector of quasi-fixed inputs, F is the real valued transformation function that defines the boundary of a closed, convex production possibilities set with free disposal in the inputs and the outputs, $\boldsymbol{X}$ maps variable input prices, planned outputs, and quasi-fixed inputs into variable input demand functions, and $C(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}) \equiv \boldsymbol{w}^{\mathsf{T}} \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z})$, is the positive-valued variable cost function. By

Shephard's Lemma, we have

$$\boldsymbol{X}(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z}) = \nabla_{\boldsymbol{w}} C(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z}) \equiv (\partial C/\partial w_1, \cdots, \partial C/\partial w_{n_x})^{\mathsf{T}}. \tag{A.3}$$

$\boldsymbol{X}$ is homogeneous of degree zero in $\boldsymbol{w}$ by the derivative property of homogeneous functions. Integrating with respect to $\boldsymbol{w}$ to recover the variable cost function, we obtain

$$c = C(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z}) \equiv \tilde{C}(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z})), \tag{A.4}$$

where $\theta : \mathcal{Y} \times \mathcal{Z} \to \mathbb{R}$ is the *constant of integration*. In the present case, this means that θ is constant with respect to $\boldsymbol{w}$. In general, θ is a function of $\boldsymbol{y}$ and $\boldsymbol{z}$ and its structure cannot be identified from the variable input demands because it captures that part of the joint production process relating to quasi-fixed inputs and outputs that is separable from the variable inputs.

Under standard conditions, the variable cost function is strictly decreasing in $\boldsymbol{z}$, strictly increasing in $\boldsymbol{y}$, jointly convex in $(\boldsymbol{y},\boldsymbol{z})$, increasing, concave and homogeneous of degree one in $\boldsymbol{w}$. We are free to choose the *sign* of θ so that, with no loss of generality, $\partial\tilde{C}/\partial\theta > 0$.

Since $\tilde{C}$ is strictly increasing in θ, a unique inverse exists such that $\theta = \gamma(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},c)$, where $\gamma : \mathcal{W} \times \mathcal{Y} \times \mathcal{Z} \times \mathbb{R}_+ \to \mathbb{R}$, is the inverse of $\tilde{C}$ with respect to θ. $\gamma(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},c)$ is called the *quasi-indirect production transformation function,* analogous to the quasi-indirect utility function of consumer theory (Hausman 1981; Epstein 1982; LaFrance 1985, 1986, 1990, 2004; and LaFrance and Hanemann 1989). For all interior and feasible $(\boldsymbol{y},\boldsymbol{z})$, the function γ is strictly increasing in c, strictly decreasing and quasi-convex in $\boldsymbol{w}$, and positively homogeneous of degree zero in $(\boldsymbol{w}, c)$.

The following two identities are simple implications of the inverse function theorem:

$$c \equiv \tilde{C}(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},\gamma(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},c)); \tag{A.5}$$

and

$$\theta \equiv \gamma(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},\tilde{C}(\boldsymbol{w},\boldsymbol{y},\boldsymbol{z},\theta)). \tag{A.6}$$

This lets one write the conditional demands for the variable inputs as

$$\boldsymbol{x} = \nabla_{\boldsymbol{w}} \tilde{C} \equiv \boldsymbol{G}(\boldsymbol{w}, \boldsymbol{y}, z, c). \tag{A.7}$$

Equation (A.7) gives the rationale for writing the factor demands as a function of c as well as $(\boldsymbol{w}, \boldsymbol{y}, z)$. Thus, given the above regularity conditions for *F* and *C*, one can always write the system of factor demands as functions of cost.

Now define the quasi-production transformation function by

$$\upsilon(\boldsymbol{x}, \boldsymbol{y}, z) \equiv \min_{\boldsymbol{w} \geq \boldsymbol{0}} \left\{ \gamma(\boldsymbol{w}, \boldsymbol{y}, z, \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x}) \right\}. \tag{A.8}$$

The terminology *quasi-production transformation function* indicates that $\upsilon(\boldsymbol{x}, \boldsymbol{y}, z)$ only reveals part of the structure of the joint production process. It cannot, and does not, reveal $\theta(\boldsymbol{y}, z)$. This is analogous to the situation where one only recovers part of a direct utility function when analyzing the market demands for a subset of consumption goods.

The identity $\theta(\boldsymbol{y}, z) \equiv \gamma(\boldsymbol{w}, \boldsymbol{y}, z, \tilde{C}(\boldsymbol{w}, \boldsymbol{y}, z, \theta(\boldsymbol{y}, z)))$ implies

$$\theta(\boldsymbol{y}, z) \equiv \gamma(\boldsymbol{w}, \boldsymbol{y}, z, \tilde{C}(\boldsymbol{w}, \boldsymbol{y}, z, \theta(\boldsymbol{y}, z))) \geq \min_{\boldsymbol{w} \geq \boldsymbol{0}} \left\{ \gamma(\boldsymbol{w}, \boldsymbol{y}, z, \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x}) \right\} \equiv \upsilon(\boldsymbol{x}, \boldsymbol{y}, z), \tag{A.9}$$

for all interior and feasible $(\boldsymbol{x}, \boldsymbol{y}, z)$. This inequality follows from the fact that $\theta(\boldsymbol{y}, z)$ is feasible but not necessarily optimal in the minimization problem. The part of $F(\boldsymbol{x}, \boldsymbol{y}, z)$ not contained in $\upsilon(\boldsymbol{x}, \boldsymbol{y}, z)$ is given by (Diewert 1975; Epstein 1975; Hausman 1981; and LaFrance and Hanemman 1989),

$$F(\boldsymbol{x}, \boldsymbol{y}, z) \equiv \tilde{F}(\boldsymbol{x}, \boldsymbol{y}, z, \theta(\boldsymbol{y}, z)). \tag{A.10}$$

The quasi-production transformation function is the unique solution, $\theta = \upsilon(\boldsymbol{x}, \boldsymbol{y}, z)$, to the implicit function, $\tilde{F}(\boldsymbol{x}, \boldsymbol{y}, z, \theta) = 0$, in other words, $\tilde{F}(\boldsymbol{x}, \boldsymbol{y}, z, \upsilon(\boldsymbol{x}, \boldsymbol{y}, z)) \equiv 0$.

The function $\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ in (9) conveys full information about the marginal rates of substitution between variable inputs but only partially so for outputs and quasi-fixed inputs. This is again analogous to the situation in consumption theory when one analyzes only a subset of the goods purchased and consumed. This can be shown by applying the implicit function theorem to $\tilde{F}$, which gives

$$\begin{aligned}
\nabla_{\boldsymbol{x}}\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) &= -\frac{\nabla_{\boldsymbol{x}}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))}{\nabla_{\theta}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))},\\
\nabla_{\boldsymbol{y}}\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) &= -\frac{\nabla_{\boldsymbol{y}}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))}{\nabla_{\theta}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))},\\
\nabla_{\boldsymbol{z}}\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) &= -\frac{\nabla_{\boldsymbol{z}}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))}{\nabla_{\theta}\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))}.
\end{aligned} \tag{A.11}$$

This demonstrates that υ conveys full information on marginal rates of substitution between variable inputs,

$$\frac{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial x_i}{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial x_j} = \frac{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))/\partial x_i}{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}))/\partial x_j} = \frac{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial x_i}{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial x_j},\ \forall\ i,j=1,\cdots,n_x, \tag{A.12}$$

but only partial information on marginal rates of product transformation between outputs,

$$\begin{aligned}
\frac{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial y_i}{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial y_j} &= \frac{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial y_i + \partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial\theta\cdot\partial\theta(\boldsymbol{y},\boldsymbol{z})/\partial y_i}{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial y_j + \partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial\theta\cdot\partial\theta(\boldsymbol{y},\boldsymbol{z})/\partial y_j}\\
&\neq \frac{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial y_i}{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial y_j},\ \forall\ i,j=1,\cdots,n_y,
\end{aligned} \tag{A.13}$$

and marginal rates of substitution between quasi-fixed inputs,

$$\begin{aligned}
\frac{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial z_i}{\partial F(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial z_j} &= \frac{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial z_i + \partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial\theta\cdot\partial\theta(\boldsymbol{y},\boldsymbol{z})/\partial z_i}{\partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial z_j + \partial\tilde{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\theta(\boldsymbol{y},\boldsymbol{z}))/\partial\theta\cdot\partial\theta(\boldsymbol{y},\boldsymbol{z})/\partial z_j}\\
&\neq \frac{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial z_i}{\partial\upsilon(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})/\partial z_j},\ \forall\ i,j=1,\cdots,n_z.
\end{aligned} \tag{A.14}$$

This background leads directly to the following result.

***Proposition* 1:** *The following functional structures are equivalent:*

$$\boldsymbol{x} = \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}) \equiv \tilde{\boldsymbol{X}}(\boldsymbol{w}, c, \boldsymbol{z}); \tag{A.15}$$

$$c = C(\boldsymbol{w}, \boldsymbol{y}, \boldsymbol{z}) \equiv \tilde{C}(\boldsymbol{w}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})); \tag{A.16}$$

and

$$0 = F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \equiv \tilde{F}(\boldsymbol{x}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})). \tag{A.17}$$

Proof: (A.16) $\Rightarrow$ (A.15). Differentiating (A.16) with respect to $\boldsymbol{w}$, Shephard's Lemma implies,

$$\boldsymbol{x} = \nabla_{\boldsymbol{w}} \tilde{C}. \tag{A.18}$$

$\tilde{C}$ is strictly monotonic in and has a unique inverse with respect to θ, say $\theta = \tilde{\gamma}(\boldsymbol{w}, \boldsymbol{z}, c)$. Substituting this into (A.18) obtains

$$\boldsymbol{x} = \nabla_{\boldsymbol{w}} \tilde{C}(\boldsymbol{w}, \boldsymbol{z}, \tilde{\gamma}(\boldsymbol{w}, \boldsymbol{z}, c)) \equiv \tilde{\boldsymbol{X}}(\boldsymbol{w}, c, \boldsymbol{z}). \tag{A.19}$$

(A.17) $\Rightarrow$ (A.15) $\Rightarrow$ (A.16). If the representation of technology has the separable structure in (A.17), then

$$\arg\min \left\{ \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x} : \tilde{F}(\boldsymbol{x}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})) \le 0,\ \boldsymbol{x} \ge \boldsymbol{0} \right\} \equiv \tilde{\boldsymbol{X}}(\boldsymbol{w}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})). \tag{A.20}$$

This implies that the variable cost function has the separable structure

$$\boldsymbol{w}^{\mathsf{T}} \tilde{\boldsymbol{X}}(\boldsymbol{w}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})) \equiv \tilde{C}(\boldsymbol{w}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z})). \tag{A.21}$$

(A.16) $\Rightarrow$ (A.17). Given (A.16), the quasi-production transformation function satisfies

$$\tilde{\upsilon}(\boldsymbol{x}, \boldsymbol{z}) \equiv \min_{\boldsymbol{w} \ge \boldsymbol{0}} \left\{ \tilde{\gamma}(\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{w}^{\mathsf{T}} \boldsymbol{x}) \right\}. \tag{A.22}$$

This implies that

$$\theta(\boldsymbol{y}, \boldsymbol{z}) \equiv \gamma(\boldsymbol{x}, \boldsymbol{z}, \tilde{C}(\boldsymbol{x}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z}))) \ge \tilde{\upsilon}(\boldsymbol{x}, \boldsymbol{z}), \tag{A.23}$$

for all interior, feasible $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$, with the boundary of the closed and convex production possibilities set defined by equality on the far right. Since $\tilde{\upsilon}$ is independent of $\boldsymbol{y}$, equations (A.11) and (A.13) imply

$$\frac{\partial F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) / \partial y_i}{\partial F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) / \partial y_j} = \frac{\partial \theta(\boldsymbol{y}, \boldsymbol{z}) / \partial y_i}{\partial \theta(\boldsymbol{y}, \boldsymbol{z}) / \partial y_j}, \ \forall \ i, j = 1, \cdots, n_y. \tag{A.24}$$

Hence, the marginal rates of transformation between outputs are independent of variable inputs,

$$\frac{\partial}{\partial x_k}\left(\frac{\partial F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) / \partial y_i}{\partial F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) / \partial y_j}\right) = \frac{\partial}{\partial x_k}\left(\frac{\partial \theta(\boldsymbol{y}, \boldsymbol{z}) / \partial y_i}{\partial \theta(\boldsymbol{y}, \boldsymbol{z}) / \partial y_j}\right) = 0, \ \forall \ i, j = 1, \cdots, n_y, \ \forall \ k = 1, \cdots, n_x, \tag{A.25}$$

Thus, $\boldsymbol{y}$ is separable from $\boldsymbol{x}$ in the joint production transformation function (Goldman and Uzawa 1964, Lemma 1), that is, $F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \tilde{F}(\boldsymbol{x}, \boldsymbol{z}, \theta(\boldsymbol{y}, \boldsymbol{z}))$. ■

Appendix B: Sufficiency Algebra for Proposition 2

Define the function $\omega : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ by

$$\omega(x, y) = y + \int_0^x [\lambda(s) + \omega(s, y)^2] ds, \tag{B.1}$$

where $\lambda : \mathbb{R} \to \mathbb{R}$ is an arbitrary smooth function and w is subject to the pair of initial conditions, $w(0, y) = y$ and $\partial w(0, y)/\partial x = y^2$, to ensure that the definition is unique and smooth. Given two arbitrary smooth functions $\eta : \mathbb{R}^{n_x}_{++} \to \mathbb{R}_+$ and $\theta : \mathbb{R}^{n_z} \times \mathbb{R}^{n_y} \to \mathbb{R}$, by Leibniz' Rule of differentiation, we have

$$\frac{\partial \omega(\eta(\boldsymbol{w}), \theta(\boldsymbol{z}, \overline{\boldsymbol{y}}))}{\partial \boldsymbol{w}} = \left[\lambda(\eta(\boldsymbol{w})) + \omega(\eta(\boldsymbol{w}), \theta(\boldsymbol{z}, \overline{\boldsymbol{y}}))^2 \right] \frac{\partial \eta(\boldsymbol{w})}{\partial \boldsymbol{w}}. \tag{B.2}$$

Given a monotonic, smooth function $f : \mathbb{R}_{++} \to \mathbb{C}$, $f' \neq 0$, define the relationship between f and ω by $f = (\alpha\omega + \beta)/(\gamma\omega + \delta)$, $\alpha, \beta, \gamma, \delta : \mathbb{R}^{n_x}_{++} \to \mathbb{C}$, and $\alpha\delta - \beta\gamma \equiv 1$. Let the cost function be $c : \mathbb{R}^{n_x}_{++} \times \mathbb{R}^{n_z}_{++} \times \mathbb{R}^{n_q}_{++} \to \mathbb{R}_{++}$ and denote an arbitrary positive-valued, 1° homogeneous, increasing, and concave deflator by $\pi : \mathbb{R}^{n_x}_{++} \to \mathbb{R}_{++}$. The projective transformation group representation of any exactly aggregable ex ante cost function is

$$f\left(\frac{c(\boldsymbol{w}, \boldsymbol{z}, \overline{\boldsymbol{y}})}{\pi(\boldsymbol{w})} \right) = \frac{\alpha(\boldsymbol{w})\omega(\eta(\boldsymbol{w}), \theta(\boldsymbol{z}, \overline{\boldsymbol{y}})) + \beta(\boldsymbol{w})}{\gamma(\boldsymbol{w})\omega(\eta(\boldsymbol{w}), \theta(\boldsymbol{z}, \overline{\boldsymbol{y}})) + \delta(\boldsymbol{w})}. \tag{B.3}$$

Hereafter, suppress all arguments of all functions and use bold italics subscripts to denote vector-valued partial derivatives. For example, rewrite (B.2) compactly as $\omega_{\boldsymbol{w}} = (\lambda + \omega^2)\eta_{\boldsymbol{w}}$.

The inverse of (B.3) with respect to ωw is $\omega = (\delta f - \beta)/(-\gamma f + \alpha)$. Combine this with the identification normalization $\alpha\delta - \beta\gamma \equiv 1$ to obtain the following:

$$\gamma\omega+\delta=\gamma\left(\frac{\delta f-\beta}{-\gamma f+\alpha}\right)+\delta=\frac{\gamma\delta f-\beta\gamma-\gamma\delta f+\alpha\delta}{-\gamma f+\alpha}=\frac{1}{-\gamma f+\alpha}, \tag{B.4}$$

or equivalently, $-\gamma f+\alpha=1/(\gamma\omega+\delta)$. Multiply each side of this by the corresponding side of equation (B.3) to obtain $(-\gamma f+\alpha)f=(\alpha\omega+\beta)\big/(\gamma\omega+\delta)^2$. These relationships are used in what follows to simplify expressions.

Our task is to differentiate (B.3) with respect to ***w***, combine terms, and rewrite the expression that results so that the elements of $\{1, f, f^2\}$ appear on the right. Differentiating gives

$$\begin{aligned} f'\cdot\left(\frac{c_w}{\pi}-\frac{c\pi_w}{\pi^2}\right)&=\frac{(\alpha_w\omega+\alpha\omega_w+\beta_w)}{(\gamma w+\delta)}-\frac{(\alpha\omega+\beta)(\gamma_w\omega+\gamma\omega_w+\delta_w)}{(\gamma w+\delta)^2} \\ &=(-\gamma f+\alpha)[\alpha_w\omega+\alpha(\lambda+\omega^2)\eta_w+\beta_w]-(-\gamma f+\alpha)f[\gamma_w\omega+\gamma(\lambda+\omega^2)\eta_w+\delta_w]. \end{aligned} \tag{B.5}$$

The second line follows from $1/(\gamma\omega+\delta)=-\gamma f+\alpha$, $(\alpha\omega+\beta)\big/(\gamma\omega+\delta)^2=(-\gamma f+\alpha)f$, and $\omega_w=(\lambda+\omega^2)\eta_w$. Group terms in ω on the second line of (B.5) to obtain

$$\begin{aligned} f'\cdot\left(\frac{c_w}{\pi}-\frac{c\pi_w}{\pi^2}\right)&=(-\gamma f+\alpha)\left[\beta_w+\alpha\lambda\eta_w-(\delta_w+\gamma\lambda\eta_w)f\right] \\ &\quad+(-\gamma f+\alpha)(\alpha_w-\gamma_w f)\omega+(-\gamma f+\alpha)^2\eta_w\omega^2. \end{aligned} \tag{B.6}$$

Substituting $\omega=(\delta f-\beta)/(-\gamma f+\alpha)$ into the second line of (B.6) now leads to

$$\begin{aligned} f'\cdot\left(\frac{c_w}{\pi}-\frac{c\pi_w}{\pi^2}\right)&=(-\gamma f+\alpha)\left[\beta_w+\alpha\lambda\eta_w-(\delta_w+\gamma\lambda\eta_w)f\right] \\ &\quad+(-\gamma f+\alpha)(\alpha_w-\gamma_w f)\left(\frac{\delta f-\beta}{-\gamma f+\alpha}\right)+(-\gamma f+\alpha)^2\eta_w\left(\frac{\delta f-\beta}{-\gamma f+\alpha}\right)^2 \\ &=(-\gamma f+\alpha)\left[\beta_w+\alpha\lambda\eta_w-(\delta_w+\gamma\lambda\eta_w)f\right]+(\alpha_w-\gamma_w f)(\delta f-\beta)+\eta_w(\delta f-\beta)^2. \end{aligned} \tag{B.7}$$

Expanding the quadratic forms and grouping terms in *f* in the last line of (B.7) gives

$$
\begin{aligned}
f' \cdot \left(\frac{c_w}{\pi} - \frac{c\pi_w}{\pi^2} \right) &= -\gamma f \left[\beta_w + \alpha\lambda\eta_w - (\delta_w + \gamma\lambda\eta_w) f \right] \\
&\quad + \alpha \left[\beta_w + \alpha\lambda\eta_w - (\delta_w + \gamma\lambda\eta_w) f \right] \\
&\quad - \alpha_w \beta + (\alpha_w \delta + \gamma_w \beta) f - \gamma_w \delta f^2 \\
&\quad + \eta_w (\beta^2 - 2\beta\delta f + \delta^2 f^2) \\
&= \alpha(\beta_w + \alpha\lambda\eta_w) - (\gamma\beta_w + \alpha\delta_w + 2\alpha\gamma\lambda\eta_w) f + \gamma(\delta_w + \gamma\lambda\eta_v) f^2 \\
&\quad - \alpha_w \beta + \eta_w \beta^2 + (\alpha_w \delta + \gamma_w \beta - 2\beta\delta\eta_w) f + (-\gamma_w \delta + \eta_w \delta^2) f^2 \\
&= \alpha\beta_w - \beta\alpha_w + (\alpha^2\lambda + \beta^2)\eta_w \\
&\quad - \left[\alpha\delta_w - \delta\alpha_w + \gamma\beta_w - \beta\gamma_w + 2(\alpha\gamma\lambda + \beta\delta)\eta_w \right] f \\
&\quad + \left[\gamma\delta_w - \delta\gamma_w + (\gamma^2\lambda + \delta^2)\eta_w \right] f^2 .
\end{aligned}
\tag{B.8}
$$

Grouping terms in η_w as well gives

$$
\begin{aligned}
f' \cdot \left(\frac{c_w}{\pi} - \frac{c\pi_w}{\pi^2} \right) &= \alpha\beta_w - \beta\alpha_w - (\alpha\delta_w - \delta\alpha_w + \gamma\beta_w - \beta\gamma_w) f + (\gamma\delta_w - \delta\gamma_w) f^2 \\
&\quad + \left[(\delta f - \beta)^2 + \lambda(-\gamma f + \alpha)^2 \right] \eta_w .
\end{aligned}
\tag{B.9}
$$

Finally, solving for $c_w = \boldsymbol{x}$ gives

$$
\begin{aligned}
\boldsymbol{x} &= \frac{\pi_w}{\pi} c + \pi \left\{ \left[\alpha\beta_w - \beta\alpha_w + (\alpha^2\lambda + \beta^2)\eta_w \right] \frac{1}{f'} \right. \\
&\quad - \left[\alpha\delta_w - \delta\alpha_w + \gamma\beta_w - \beta\gamma_w + 2(\alpha\gamma\lambda + \beta\delta)\eta_w \right] \frac{f}{f'} \\
&\quad \left. + \left[\gamma\delta_w - \delta\gamma_w + (\gamma^2\lambda + \delta^2)\eta_w \right] \frac{f^2}{f'} \right\} \\
&= \frac{\pi_w}{\pi} c + \frac{\pi}{f'} \left\{ \alpha\beta_w - \beta\alpha_w - (\alpha\delta_w - \delta\alpha_w + \gamma\beta_w - \beta\gamma_w) f + (\gamma\delta_w - \delta\gamma_w) f^2 \right. \\
&\quad \left. + \left[(\delta f - \beta)^2 + (-\gamma f + \alpha)^2 \lambda \right] \eta_w \right\} .
\end{aligned}
\tag{B.10}
$$

Appendix C. Specifying the Cost Function

The first $n_x - 1$ variable input prices, $\boldsymbol{w}$, and total variable cost, c, are normalized by the average wage rate for hired farm labor, w_{n_x}. We consider the following transformation of normalized variable cost, which nests the PIGLOG and PIGL class of models,

$$f(c) = (c^{\kappa} + \kappa - 1)/\kappa, \;\; f'(c) = c^{\kappa}, \;\; f''(c) = (\kappa - 1)c^{\kappa - 2}, \;\; \kappa \in \mathbb{R}_{+}.$$

This includes all of the real-valued Gorman functional forms, with $f(c) = c$ when $\kappa = 1$, and $\lim_{\kappa \to 0} f(c) = 1 + \ln c$. Therefore, the highest rank that the variable input demands can achieve is three (Gorman 1981; Lewbel 1987; LaFrance and Pope 2009).

Previous empirical work considered translated Box-Cox functions of input prices, $(w_i^{\lambda} + \lambda - 1)/\lambda$, $\lambda \in [0,1]$, $i = 1, \cdots, n_x - 1$, to nest models with that have log prices, power functions of prices, and are linear prices. In the national model $\lambda = 1$ is optimal on this interval and for our data set. Hence, we restrict attention here to normalized input prices. Our previous empirical results using this data at state- and national-levels of aggregation and various levels of aggregation across inputs, suggests quite strongly that rank three over-parameterizes this data set (Ball, et al., 2010). Hence, we focus here on rank two:

$$\begin{aligned}
& f(c(\boldsymbol{w}, A, K, \boldsymbol{a}, \overline{\boldsymbol{Y}})) = \alpha(\boldsymbol{w}, A, K) + \beta(\boldsymbol{w})\theta(A, K, \boldsymbol{a}, \overline{\boldsymbol{Y}}), \\
& \Leftrightarrow \tilde{\theta}(\boldsymbol{w}, c, A, K) = \frac{f(c) - \alpha(\boldsymbol{w}, A, K)}{\beta(\boldsymbol{w})}, \\
& \alpha(\boldsymbol{w}, A, K) = (\alpha_{10} + \boldsymbol{\alpha}_1^{\mathsf{T}}\boldsymbol{w})A + (\alpha_{20} + \boldsymbol{\alpha}_2^{\mathsf{T}}\boldsymbol{w})K, \\
& \beta(\boldsymbol{w}) = \sqrt{\boldsymbol{w}^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w} + 2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w} + 1}, \\
& \overline{\boldsymbol{Y}} = \overline{\boldsymbol{y}} \bullet \boldsymbol{a}_t = [\overline{y}_1 a_1 \; \cdots \; \overline{y}_{n_y} a_{n_y}]^{\mathsf{T}},
\end{aligned} \tag{C.1}$$

where $\overline{y}_i$ is the expected (planned) yield for the ith crop, a_i is the acreage planted to this

crop, and the symbol • denotes the Hadamard/Schur product for matrices and vectors.

This appendix identifies restrictions on the parameters in (C.1) that are necessary and sufficient for economic regularity of the variable cost function.

Monotonicity in $\boldsymbol{w}$*:*

$$\begin{aligned} c^{\kappa-1}\frac{\partial c}{\partial \boldsymbol{w}} &= \boldsymbol{\alpha}_1 A + \boldsymbol{\alpha}_2 K + \frac{\theta}{\beta}(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma}) \geq \mathbf{0} \\ \Leftrightarrow \tilde{\boldsymbol{x}} = c^{1-\kappa}&\left[\boldsymbol{\alpha}_1 A + \boldsymbol{\alpha}_2 K + \left(\frac{f-\alpha}{\beta^2}\right)(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})\right] \geq \mathbf{0}, \end{aligned} \tag{C.2}$$

where $\tilde{\boldsymbol{x}} = [x_1 \cdots x_{n_x-1}]^{\mathsf{T}}$ is the (n_x–1)–vector of the first n_x–1 input quantities, excluding labor.

Concavity in $\boldsymbol{w}$*:*

$$\begin{aligned} (\kappa-1)c^{\kappa-2}\frac{\partial c}{\partial \boldsymbol{w}}\frac{\partial c}{\partial \boldsymbol{w}^{\mathsf{T}}} + c^{\kappa-1}\frac{\partial^2 c}{\partial \boldsymbol{w}\partial \boldsymbol{w}^{\mathsf{T}}} &= \frac{\theta}{\beta}\boldsymbol{B} - \frac{\theta}{\beta^3}(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})^{\mathsf{T}} \\ \Leftrightarrow \frac{\partial^2 c}{\partial \boldsymbol{w}\partial \boldsymbol{w}^{\mathsf{T}}} &= \left(\frac{1-\kappa}{c}\right)\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^{\mathsf{T}} + c^{1-\kappa}\left(\frac{f-\alpha}{\beta^2}\right)\left[\boldsymbol{B} - \frac{(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})^{\mathsf{T}}}{\beta^2}\right], \end{aligned} \tag{C.3}$$

The first matrix on the right-hand-side of the second line is rank and is negative semi-definite if and only if $\kappa \geq 1$. The matrix in square brackets on the far right of the second line will be positive semi-definite if $\boldsymbol{B} = \boldsymbol{L}\boldsymbol{L}^{\mathsf{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathsf{T}}$, where $\boldsymbol{L}$ is a triangular matrix with nonzero main diagonal elements. This makes the following $n_x \times n_x$ matrix positive definite:

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L} & \boldsymbol{\gamma} \\ \mathbf{0}^{\mathsf{T}} & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{L}^{\mathsf{T}} & \mathbf{0}^{\mathsf{T}} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}\boldsymbol{L}^{\mathsf{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathsf{T}} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^{\mathsf{T}} & 1 \end{bmatrix}, \tag{C.4}$$

since it give a Choleski factorization of the matrix on the left. It follows from this that

$\boldsymbol{w}^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}+2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}+1=[\boldsymbol{w}^{\mathsf{T}}\ 1]\begin{bmatrix}\boldsymbol{B} & \boldsymbol{\gamma}\\ \boldsymbol{\gamma}^{\mathsf{T}} & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{w}\\ 1\end{bmatrix}>0\ \forall\ \boldsymbol{w}\in\mathbb{R}_{++}^{n_x-1}$, and $\left[\boldsymbol{B}-\dfrac{(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})(\boldsymbol{B}\boldsymbol{w}+\boldsymbol{\gamma})^{\mathsf{T}}}{\boldsymbol{w}^{\mathsf{T}}\boldsymbol{B}\boldsymbol{w}+2\boldsymbol{\gamma}^{\mathsf{T}}\boldsymbol{w}+1}\right]$ is positive semi-definite, by the Cauchy-Schwartz inequality in n_x–dimensional Euclidean space. Given this, the second term on the right-hand-side of the second line of equation (12) will be negative semi-definite if and only if $f\le\alpha$.

Constant returns to scale (CRS):

$$\theta\equiv\frac{\partial\theta}{\partial A}A+\frac{\partial\theta}{\partial K}K+\frac{\partial\theta}{\partial\boldsymbol{a}^{\mathsf{T}}}\boldsymbol{a}+\frac{\partial\theta}{\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\bar{\boldsymbol{Y}}. \tag{C.5}$$

We believe that we have a much more accurate measure of capital than we do of land. Hence, we normalize θ by the value of capital rather than land in farms.

Monotonicity in $(A,K,\boldsymbol{a},\bar{\boldsymbol{Y}})$:

$$\begin{gathered}c^{\kappa-1}\frac{\partial c}{\partial A}=\alpha_{10}+\boldsymbol{\alpha}_1^{\mathsf{T}}\boldsymbol{w}+\beta\frac{\partial\theta}{\partial A}\le 0, c^{\kappa-1}\frac{\partial c}{\partial K}=\alpha_{20}+\boldsymbol{\alpha}_2^{\mathsf{T}}\boldsymbol{w}+\beta\frac{\partial\theta}{\partial K}\le 0,\\ c^{\kappa-1}\frac{\partial c}{\partial\boldsymbol{a}}=\beta\frac{\partial\theta}{\partial\boldsymbol{a}}\le\boldsymbol{0}, c^{\kappa-1}\frac{\partial c}{\partial\bar{\boldsymbol{Y}}}=\beta\frac{\partial\theta}{\partial\bar{\boldsymbol{Y}}}\le\boldsymbol{0}.\end{gathered} \tag{C.6}$$

Joint Convexity in $(A,K,\boldsymbol{a},\bar{\boldsymbol{Y}})$:

$$\begin{bmatrix}\frac{\partial^2 c}{\partial A^2} & \frac{\partial^2 c}{\partial A\partial K} & \frac{\partial^2 c}{\partial A\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2 c}{\partial A\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2 c}{\partial K\partial A} & \frac{\partial^2 c}{\partial K^2} & \frac{\partial^2 c}{\partial K\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2 c}{\partial K\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2 c}{\partial\boldsymbol{a}\partial A} & \frac{\partial^2 c}{\partial\boldsymbol{a}\partial K} & \frac{\partial^2 c}{\partial\boldsymbol{a}\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2 c}{\partial\boldsymbol{a}\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2 c}{\partial\bar{\boldsymbol{Y}}\partial A} & \frac{\partial^2 c}{\partial K\partial\bar{\boldsymbol{Y}}} & \frac{\partial^2 c}{\partial\boldsymbol{a}\partial\bar{\boldsymbol{Y}}} & \frac{\partial^2 c}{\partial\bar{\boldsymbol{Y}}\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\end{bmatrix}=\frac{1-\kappa}{c}\begin{bmatrix}\frac{\partial c}{\partial A}\\ \frac{\partial c}{\partial K}\\ \frac{\partial c}{\partial\boldsymbol{a}}\\ \frac{\partial c}{\partial\bar{\boldsymbol{Y}}}\end{bmatrix}\begin{bmatrix}\frac{\partial c}{\partial A}\\ \frac{\partial c}{\partial K}\\ \frac{\partial c}{\partial\boldsymbol{a}}\\ \frac{\partial c}{\partial\bar{\boldsymbol{Y}}}\end{bmatrix}^{\mathsf{T}}+c^{1-\kappa}\beta\begin{bmatrix}\frac{\partial^2\theta}{\partial A^2} & \frac{\partial^2\theta}{\partial A\partial K} & \frac{\partial^2\theta}{\partial A\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2\theta}{\partial A\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2\theta}{\partial K\partial A} & \frac{\partial^2\theta}{\partial K^2} & \frac{\partial^2\theta}{\partial K\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2\theta}{\partial K\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2\theta}{\partial\boldsymbol{a}\partial A} & \frac{\partial^2\theta}{\partial\boldsymbol{a}\partial K} & \frac{\partial^2\theta}{\partial\boldsymbol{a}\partial\boldsymbol{a}^{\mathsf{T}}} & \frac{\partial^2\theta}{\partial\boldsymbol{a}\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\\ \frac{\partial^2\theta}{\partial\bar{\boldsymbol{Y}}\partial A} & \frac{\partial^2\theta}{\partial K\partial\bar{\boldsymbol{Y}}} & \frac{\partial^2\theta}{\partial\boldsymbol{a}\partial\bar{\boldsymbol{Y}}} & \frac{\partial^2\theta}{\partial\bar{\boldsymbol{Y}}\partial\bar{\boldsymbol{Y}}^{\mathsf{T}}}\end{bmatrix}. \tag{C.7}$$

The first matrix on the right is rank one and will be positive semi-definite if and only if $\kappa\le 1$. Therefore, $c(\boldsymbol{w},A,K,\boldsymbol{a},\bar{\boldsymbol{Y}})$ will be concave in $\boldsymbol{w}$ and jointly convex in $(A,K,\boldsymbol{a},\bar{\boldsymbol{Y}})$

more than locally if and only if $\kappa = 1$. We estimated the rank two model using the Box-Cox transformation on cost. The NL3SLS/GMM point estimate for κ is 1.124 with a classical (Gaussian) asymptotic standard error of .152 and a White/Huber heteroskedasticity consistent standard error of .111. We can not reject a null hypothesis of $\kappa = 1$ in either case at the 25% significance level. Hence, in this paper we restrict our attention to $\kappa = 1$.

Given this restriction, the cost function will be jointly convex in $(A, K, \boldsymbol{a}, \bar{\boldsymbol{Y}})$ if and only if the Hessian matrix for θ,

$$\begin{bmatrix} \frac{\partial^2\theta}{\partial A^2} & \frac{\partial^2\theta}{\partial A\partial K} & \frac{\partial^2\theta}{\partial A\partial \boldsymbol{a}^{\mathrm{T}}} & \frac{\partial^2\theta}{\partial A\partial \bar{\boldsymbol{Y}}^{\mathrm{T}}} \\ \frac{\partial^2\theta}{\partial K\partial A} & \frac{\partial^2\theta}{\partial K^2} & \frac{\partial^2\theta}{\partial K\partial \boldsymbol{a}^{\mathrm{T}}} & \frac{\partial^2\theta}{\partial K\partial \bar{\boldsymbol{Y}}^{\mathrm{T}}} \\ \frac{\partial^2\theta}{\partial \boldsymbol{a}\partial A} & \frac{\partial^2\theta}{\partial \boldsymbol{a}\partial K} & \frac{\partial^2\theta}{\partial \boldsymbol{a}\partial \boldsymbol{a}^{\mathrm{T}}} & \frac{\partial^2\theta}{\partial \boldsymbol{a}\partial \bar{\boldsymbol{Y}}^{\mathrm{T}}} \\ \frac{\partial^2\theta}{\partial \bar{\boldsymbol{Y}}\partial A} & \frac{\partial^2\theta}{\partial K\partial \bar{\boldsymbol{Y}}} & \frac{\partial^2\theta}{\partial \boldsymbol{a}\partial \bar{\boldsymbol{Y}}} & \frac{\partial^2\theta}{\partial \bar{\boldsymbol{Y}}\partial \bar{\boldsymbol{Y}}^{\mathrm{T}}} \end{bmatrix}, \tag{C.8}$$

is positive semi-definite. Given these considerations, the specification for θ employed in the paper is

$$\theta(A_t, K_t, \boldsymbol{a}_t, \bar{\boldsymbol{Y}}_t) = -\theta_1 A_t - \theta_2 K_t - \boldsymbol{\theta}_3^{\mathrm{T}}\boldsymbol{a}_t + \boldsymbol{\theta}_4^{\mathrm{T}}\bar{\boldsymbol{Y}}_t + ½\left(\frac{\theta_5 A_t^2 + \boldsymbol{a}_t^{\mathrm{T}}\boldsymbol{\Theta}_6\boldsymbol{a}_t + \bar{\boldsymbol{Y}}_t^{\mathrm{T}}\boldsymbol{\Theta}_7\bar{\boldsymbol{Y}}_t}{K_t}\right), \tag{C.9}$$

where $\theta_1, \theta_2, \theta_5 > 0$, $\boldsymbol{\theta}_3, \boldsymbol{\theta}_4 > \mathbf{0}$, and $\boldsymbol{\Theta}_6, \boldsymbol{\Theta}_7$ are symmetric and positive semidefinite. The implied constraints for monotonicity can be written as

$$
\begin{aligned}
&\frac{\partial c_t}{\partial A_t} < 0 \;\forall\; t \Leftrightarrow \min_t\left(\theta_1 - \theta_5 \frac{A_t}{K_t}\right) > \max_t\left(\frac{\alpha_0 + \boldsymbol{\alpha}_1^{\mathsf{T}}\boldsymbol{w}_t}{\sqrt{\beta(\boldsymbol{w})}}\right),\\
&\frac{\partial c_t}{\partial K_t} < 0 \;\forall\; t \Leftrightarrow \min_t\left[\theta_2 + ½\left(\frac{\theta_5 A_t^2 + \boldsymbol{a}_t^{\mathsf{T}}\boldsymbol{\Theta}_6\boldsymbol{a}_6 + \bar{\boldsymbol{Y}}_t^{\mathsf{T}}\boldsymbol{\Theta}_7\bar{\boldsymbol{Y}}}{K_t^2}\right)\right] > \max_t\left(\frac{\alpha_2 + \boldsymbol{\alpha}_3^{\mathsf{T}}\boldsymbol{w}_t}{\sqrt{\beta(\boldsymbol{w})}}\right),\\
&\frac{\partial c_t}{\partial \boldsymbol{a}_t} < \mathbf{0} \;\forall\; t \Leftrightarrow \boldsymbol{\theta}_3 > \max_t\left(\frac{\boldsymbol{\Theta}_6\boldsymbol{a}_6}{K_t}\right),\\
&\frac{\partial c_t}{\partial \bar{\boldsymbol{Y}}_t} > \mathbf{0} \;\forall\; t \Leftrightarrow \boldsymbol{\theta}_4 + \min_t\left(\frac{\boldsymbol{\Theta}_7\bar{\boldsymbol{Y}}_t}{K_t}\right) > \mathbf{0}.
\end{aligned}
\tag{C.10}
$$

These can be imposed iteratively in estimation, if necessary (LaFrance 1991). In this paper, we checked for the monotonicity conditions at each data point given the parameter estimates obtained without imposing monotonicity.

Also, given that $K_t > 0,$ the implied curvature conditions are that the matrix

$$
\begin{bmatrix}
\theta_5 K_t^2 & -\theta_5 A_t K_t & \mathbf{0}_{n_y}^{\mathsf{T}} & \mathbf{0}_{n_y}^{\mathsf{T}}\\
-\theta_5 A_t K_t & (\theta_5 A_t^2 + \boldsymbol{a}_t^{\mathsf{T}}\boldsymbol{\Theta}_6\boldsymbol{a}_6 + \bar{\boldsymbol{Y}}_t^{\mathsf{T}}\boldsymbol{\Theta}_7\bar{\boldsymbol{Y}}_t) & -K_t\boldsymbol{a}_t^{\mathsf{T}}\boldsymbol{\Theta}_6 & -K_t\bar{\boldsymbol{Y}}_t^{\mathsf{T}}\boldsymbol{\Theta}_7\\
\mathbf{0}_{n_y} & -K_t\boldsymbol{\Theta}_6\boldsymbol{a}_6 & \boldsymbol{K}_t^2\boldsymbol{\Theta}_6 & \mathbf{0}_{n_y\times n_y}\\
\mathbf{0}_{n_y} & -K_t\boldsymbol{\Theta}_7\bar{\boldsymbol{Y}}_t & \mathbf{0}_{n_y\times n_y} & \boldsymbol{K}_t^2\boldsymbol{\Theta}_7
\end{bmatrix}
\tag{C.11}
$$

is positive semidefinite. This can be imposed during estimation with the Choleski factors, $\boldsymbol{\Theta}_6 = \boldsymbol{L}_6\boldsymbol{L}_6^{\mathsf{T}}$ and $\boldsymbol{\Theta}_7 = \boldsymbol{L}_7\boldsymbol{L}_7^{\mathsf{T}},$ with $\boldsymbol{L}_6$ and $\boldsymbol{L}_7$ lower triangular Choleski factors for $\boldsymbol{\Theta}_6$ and $\boldsymbol{\Theta}_7,$ respectively, and the inequality $\theta_5 > 0.$ In this paper, only the matrix $\boldsymbol{\Theta}_7 = \boldsymbol{L}_7\boldsymbol{L}_7^{\mathsf{T}}$ is estimated as part of the arbitrage conditions.

Appendix D. Specification Errors and Parameter Stability Tests

Many diagnostic procedures for testing parameter stability and model specification errors have been developed. Few are designed for large systems of nonlinear simultaneous equations in small samples. These properties preclude using recursive-forecast residuals or Chow tests based on sequential sample splits to analyze specification errors or non-constant parameters Brown, Durbin, and Evans 1975; Harvey 1990, 1993; Hendry 1995). It is desirable to test whether the data are consistent with the model specification and constant parameters. LaFrance (2008) derived a set of specification and parameter stability diagnostics for this class of problems. These test statistics rely on the estimated in-sample residuals and have power against a range of alternatives, including non-constant parameters and specification errors. The purpose of this section is to discuss briefly the main ideas that underpin this class of test statistics.

If the model is stationary and the errors are innovations, then consistent estimates of the model parameters can be found in any number of ways. Given consistent parameter estimates, the estimated errors converge in probability (and therefore, in distribution) to the true errors, $\hat{\boldsymbol{\varepsilon}}_t \xrightarrow{P} \boldsymbol{\varepsilon}_t$. Therefore, for each $i = 1, \cdots, n_x - 1$, by the central limit theorem for stationary Martingale differences, we have

$$\frac{1}{\sqrt{T}\sigma_i} \sum_{t=1}^{T} \varepsilon_{it} \xrightarrow{D} N(0,1), \tag{D.1}$$

where $\sigma_i^2 = E(\varepsilon_{it}^2)$ is the variance of the residual for the ith demand equation. Moreover, for any given proportion of the sample, uniformly in $z \in [0,1]$,

$$\frac{1}{\sqrt{T}\sigma_i}\sum_{t=1}^{[zT]}\varepsilon_{it} \xrightarrow{D} N(0,z)\,, \tag{D.2}$$

where $[zT]$ is the largest integer that does not exceed zT. The variance is z because we sum $[zT]$ independent terms each with variance $1/T$. Multiplying (D.1) by z and subtracting from (D.2) then gives

$$\frac{1}{\sqrt{T}\sigma_i}\sum_{t=1}^{[zT]}\left(\varepsilon_{it}-\bar{\varepsilon}_i\right) \xrightarrow{D} W(z)-zW(1)\equiv B(z)\,, \tag{D.3}$$

where $W(z)$ is a standard Brownian motion on the unit interval, with $W(z)\sim N(0,z)$, and $B(z)$ is a standard Brownian bridge, or tied Brownian motion. For all $z \in [0,1]$, $B(z)$ has an asymptotic Gaussian distribution, with mean zero and standard deviation $\sqrt{z(1-z)}$ (Bhattacharya and Waymire, 1990). For a given z – that is, to test for a break point in the model at a fixed and known date – an asymptotic 95% confidence interval for $B(z)$ is $\pm 1.96\sqrt{z(1-z)}$. To check for an unknown break point, a statistic based on the supremum norm,

$$Q_T = \sup_{z\in[0,1]}\left|B_T(z)\right| \tag{D.4}$$

has an asymptotic 5% critical value of 1.36 (Ploberger and Krämer, 1992).

We can use consistently estimated residuals and consistently estimated standard errors to obtain sample analogues to these asymptotic Brownian bridges. This gives

$$B_{iT}(z)\equiv\frac{1}{\sqrt{T}\hat{\sigma}_i}\sum_{t=1}^{[zT]}(\hat{\varepsilon}_{it}-\bar{\hat{\varepsilon}}_i) \xrightarrow{D} B(z)\,, \tag{D.5}$$

also uniformly in $z\in[0,1]$, so long as the model specification is correct and the parame-

ters are constant across time periods. This statistic is a single equation first-order specification/parameter stability statistic since it is based on the first-order moment conditions, $E(\varepsilon_{it}) = 0 \ \forall \ i,t$. A system-wide first-order specification/parameter stability statistic can be defined by

$$B_T(z) \equiv \frac{1}{\sqrt{T}} \sum_{t=1}^{[zT]} \left[\frac{1}{\sqrt{n_x}} \sum_{i=1}^{n_q} (\hat{\xi}_{it} - \bar{\hat{\xi}}) \right] \xrightarrow{D} B(z) , \tag{D.6}$$

where $\hat{\boldsymbol{\xi}}_t = \hat{\boldsymbol{\Sigma}}^{-1/2} \hat{\boldsymbol{\varepsilon}}_t$ is the tth estimated standardized error vector and $\bar{\hat{\xi}} \equiv \sum_{t=1}^{T} \sum_{i=1}^{n_q} \hat{\xi}_{it} \big/ n_x T$.

Similar methods apply to second-order stationarity and parameter staibility. We focus on system-wide statistics. Let $\boldsymbol{\Sigma}$ be factored into $\boldsymbol{LL}^{\mathrm{T}}$, where $\boldsymbol{L}$ is lower triangular and nonsingular. Define the random vector $\boldsymbol{\xi}_t$ by $\boldsymbol{\varepsilon}_t = \boldsymbol{L}\boldsymbol{\xi}_t$. In addition to the assumptions above, add $\sup_{i,t} E(\varepsilon_{it}^4) < \infty$. Estimate the within-period average sum of squared standardized residuals by

$$\hat{\upsilon}_t = \tfrac{1}{n} \hat{\boldsymbol{\xi}}_t^{\mathrm{T}} \hat{\boldsymbol{\xi}}_t = \tfrac{1}{n} \hat{\boldsymbol{\varepsilon}}_t^{\mathrm{T}} \hat{\boldsymbol{\Sigma}}^{-1} \hat{\boldsymbol{\varepsilon}}_t , \tag{D.7}$$

where $\hat{\boldsymbol{\varepsilon}}_t$ is the vector of consistently estimated residuals in period t and $\hat{\boldsymbol{\Sigma}} = \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t^{\mathrm{T}} \big/ T$ is the associated consistently estimated error covariance matrix. The mean of the true υ_t is one for each t, and the martingale difference property of $\boldsymbol{\varepsilon}_t$ is inherited by $\upsilon_t - 1$. A consistent estimator of the asymptotic variance of υ_t is

$$\hat{\sigma}_\upsilon^2 = \frac{1}{T} \sum_{t=1}^{T} (\hat{\upsilon}_t^2 - 1) . \tag{D.8}$$

A system wide second-order specification/parameter stability test statistic is obtained by calculating centered and standardized partial sums of $\hat{\upsilon}_t$,

$$B_T(z) = \frac{1}{\sqrt{T}\hat{\sigma}_\upsilon} \cdot \sum_{t=1}^{[zT]} (\hat{\upsilon}_t - 1) \xrightarrow[T\to\infty]{D} B(z), \tag{D.9}$$

uniformly in $z \in [0,1]$, where the limiting distribution on the far right follows from the identity $\bar{\hat{\upsilon}} \equiv \sum_{t=1}^{T} \hat{\upsilon}_t / T \equiv 1$.