The Equilibrium Real Funds Rate: Past, Present and Future

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ABSTRACT

We examine the behavior, determinants, and implications of the equilibrium level of the real federal funds rate, interpreted as the long run or steady state value of the real funds rate. We draw three main conclusions. First, the uncertainty around the equilibrium rate is large, and its relationship with trend GDP growth much more tenuous than widely believed. Our narrative and econometric analysis using cross-country data and going back to the 19th century supports a wide range of plausible estimates for the current level of the equilibrium rate, from a little over 0% to the pre-crisis consensus of 2%. Second, a bivariate vector error correction model that looks only to U.S. and world real rates well captures the behavior of U.S. real rates. The model treats real rates as cointegrated unit root processes. As of the end of our sample (2014), the model forecasts the real rate in the U.S. will asymptote to an equilibrium value of a little less than half a percent by 2021. Consistent with our first point, however, confidence intervals around this point estimate are huge. Third, the uncertainty around the equilibrium rate argues for more "inertial" monetary policy than implied by standard versions of the Taylor rule. Our simulations using the Fed staff's FRB/US model show that explicit recognition of this uncertainty results in a later but steeper normalization path for the funds rate compared with the median "dot" in the FOMC's Summary of Economic Projections.

1. Introduction

What is the steady-state value of the real federal funds rate? Is there a new neutral, with a low equilibrium value for the foreseeable future?

A consensus seems to be building that the answer to the second question is yes. Starting in 2012 FOMC members have been releasing their own estimates of the “longer run” nominal rate in the now somewhat infamous “dot plot.” As Exhibit 1.1 shows, the longer run projection for PCE inflation has remained steady at 2.0%, but longer run projections for both the GDP and the nominal funds rate projections have dropped 50 bp. The implied equilibrium real rate has fallen from 2.0% to 1.5% and the current range among members is between 1 and 2%. Indeed, going back to January 2012, the first FOMC projections for the longer run funds rate had a median of 4.25%, suggesting an equilibrium real rate of 2.25%. Forecasters at the CBO, OMB, Social Security Administration and other longer term official forecasts show a similar cut in the assumed equilibrium rate, typically from 2% to 1.5%.

The consensus outside official circles points to an even lower equilibrium rate. A hot topic of recent discussion is whether the U.S. has drifted into “secular stagnation,” a period of chronically low equilibrium rates due to a persistent weak demand for capital, rising propensity to save and lower trend growth in the economy (see Summers (2013b,2014)). A similar view holds that there is a "new neutral" for the funds rate of close to zero in real terms (see McCulley (2003) and Clarida (2014)). The markets seem to agree, with the yield on 10-year Treasury Inflation Protected Securities having spent much of the last 5 years below 0.5%.

The equilibrium real rate is sometimes interpreted as the value of the intercept in a Taylor rule. In dynamic stochastic general equilibrium models, the equilibrium real rate is sometimes thought of as the value associated with the deterministic steady-state growth path, which is a function of the discount factor and growth rates of technology and population. If these are constant, we should observe a tendency of the real rate to return to a fixed long-run average over time. In richer models the equilibrium real rate is sometimes defined as the value that would be observed in the absence of monetary frictions. Empirical estimates of this magnitude often exhibit huge variation over time; see for example Justiniano and Primiceri (2010), Barsky et al. (2014) and Cúrdia et al. (2015). Laubach and Williams (2003) obtained empirical estimates of the equilibrium rate assuming that trend growth was a central determinant, and also found tremendous variation.

In this paper we evaluate some reduced-form evidence on this question. We focus on long run or steady state values, examining the behavior of the real interest rate over long periods of time and its empirical relation to factors such as the economic growth rate. We consider evidence from a large number of countries, though our primary focus will be on the United States. In Section 2 we describe the data and procedures that we will use to construct the ex-ante real rates used in our analysis. These go back as far as two centuries for some countries, and also include more detailed data on the more recent experience of OECD economies. We also note the strategy we often use to make empirical statements about the equilibrium rate: for the most part we will look to averages or moving averages of our measures of real rates; at no point will we estimate a structural model.

Section 3 summarizes and interprets some of the existing theoretical and empirical work and highlights the theoretical basis for anticipating a relation between the equilibrium real rate and the trend growth rate. In this and the next section, we look to moving averages as (noisy) measures of the equilibrium rate and the trend growth rate. Using both long time-series observations for the United States as well as the experience across OECD countries since 1970, we investigate the relation between safe real rates and trend output growth. We uncover some evidence that higher trend growth rates are associated with higher average real rates. However, that finding is sensitive to the particular sample of data that is used. And even for the samples with a positive relation, the correlation between growth and average rates is modest. We conclude that factors in addition to changes in the trend growth rate are central to explaining why the equilibrium real rate changes over time.

In Section 4 we briefly discuss the secular stagnation hypothesis in particular. We suggest that advocates of this view are misinterpreting the delayed recovery from the Great Recession as evidence of chronically weak aggregate demand. Our analysis suggests that the current cycle could be similar to the previous two, with a delayed normalization of both the economy and the funds rate. A comparison with previous cycles, which is detailed in Hamilton et al. (2015), suggests that the equilibrium rate may have fallen, but not as much as the secular stagnation hypothesis would imply. This comparison is one source of the 0 to 2 percent range for the equilibrium rate given above.

In Section 5 we perform some statistical analysis of the long-run U.S. data and find, consistent with our narrative history as well as with empirical results found by other researchers in postwar datasets, that we can reject the hypothesis that the real interest rate converges over time to some fixed constant. We do find a relation that appears to be stable. The U.S. real rate is cointegrated with a measure that is similar to the median of a 30-year-average of real rates around the world. When the U.S. rate is below that long-run world rate (as it was as of the beginning of 2015), we could have some confidence that the U.S. rate is going to rise. The model forecasts the U.S. and world long-run real rate settling down at a value around a half a percent within about six years. However, because the world rate itself is also nonstationary with no clear tendency to revert to a fixed mean, the uncertainty associated with this forecast is not only large, but grows larger the farther we try to look into the future.

More generally, the picture that emerges from our analysis is that the determinants of the equilibrium rate are manifold and time varying. We are skeptical of analysis that puts growth of actual or potential output at the center of real interest rate determination. The link with growth is weak. Historically, that link seems to have been buried by effects from other factors influencing the real rate. We conclude that reasonable forecasts for the equilibrium rate will come with large confidence intervals.

We close the paper in Section 6 by considering the policy implications of uncertainty about the equilibrium rate. Orphanides and Williams (2002, 2006, 2007) have noted that if the Fed does not have a good estimate of what the equilibrium real rate should be, it may be preferable to put more inertia into policy than otherwise. We use simulations of the FRB/US model to gauge the relevance of this concern in the current setting. We conclude that, given that we do not know the equilibrium real rate, there may be benefits to waiting to raise the nominal rate until we actually see some evidence of labor market pressure and increases in inflation. Relative to the "shallow glide path" for the funds rate that has featured prominently in recent Fed communications, our findings suggest that the funds rate should start to rise later but—provided the recovery does gather pace and inflation picks up—somewhat more steeply.

To conclude, we think the long-run equilibrium U.S. real interest rate remains positive, and forecasts that the real rate will remain stuck at or below zero for the next decade appear unwarranted. But we find little basis in the data for stating with confidence exactly what the value of the equilibrium real rate is going to be. In this respect our policy recommendation shares some common ground with the stagnationists—it pays for the Fed to be cautious about raising the nominal interest rate in the current environment until we see more evidence from the behavior of the economy and inflation that such increases are clearly warranted.

2. The real interest rate across countries and across time

Our focus is on the behavior of the real interest rate, defined as the nominal short-term policy rate minus expected inflation. The latter is of course not measured directly, and we follow the common approach in the literature of inferring expected inflation from the forecast of an autoregressive model fit to inflation. However, we differ from most previous studies in that we allow the coefficients of our inflation-forecasting relations to vary over time. We will be making use of both a very long annual data set going back up to two centuries as well as a quarterly data set available for more recent data. The countries we will be examining are listed in Exhibit 2.1. In this section we describe these data and our estimates of real interest rates.

2A. A very long-run annual data set

Our long-run analysis is based on annual data going as far back as 1800 for 17 different countries. Where available we used the discount rate set by the central bank as of the end of each year. For the Bank of England this gives us a series going all the way back to 1801, while for the U.S. we spliced together values for commercial paper rates over 1857-1913, the Federal Reserve discount rate over 1914-1953, and the average fed funds rate during the last month of the year from 1954 to present.[1] Our interest rate series for these two countries are plotted in the top row of Exhibit 2.2 and for 15 other countries in the panels of Exhibit 2.3.[2] The U.S. nominal rate shows a broad tendency to decline through World War II, rise sharply until 1980, and decline again since. The same broad trends are also

[1] Values of the 3 separate U.S. series are very close to each other at the dates at which they were spliced together.
[2] Our data set is largely identical to Hatzius et. al (2014) and mainly comes from the Global Financial Data Inc. database, supplemented with information from Haver Analytics. In most cases, the short-term interest rate series is a central bank discount rate (known as bank rate in UK parlance) or an overnight cash or repo rate. When more than one series is used for the same country because of changes over time in definitions and market structure, we splice the series using the discount rate as the basis.

seen in most other countries. However, there are also dramatic differences across countries as well, such as the sharp spike in rates in Finland and Germany following World War I.

We also assembled estimates of the overall price level for each country. For the U.S., we felt the best measure for recent data is the GDP deflator which is available since 1929. We used an estimate of consumer prices for earlier U.S. data and all other countries. The annual inflation rates are plotted in the second row of Exhibit 2.2 for the U.S. and U.K. and for 15 other countries in the panels of Exhibit 2.4. There is no clear trend in inflation for any country prior to World War I, suggesting that the downward trend in nominal rates prior to that should be interpreted as a downward trend in the real rate. Inflation rose sharply in most countries after both world wars, with hyperinflations in Germany and Finland following World War I and Japan and Italy after World War II. But the postwar spike in inflation was in every case much bigger than the rise in nominal interest rates.

How much of the variation in inflation would have been reasonable to anticipate ex ante? Barsky (1987) argued that U.S. inflation was much less predictable in the 19th century than it became later in the 20th century. More generally, we might expect predictions of inflation to be very different for episodes characterized by a gold standard or fixed exchange rates compared to a floating exchange rate regime. To deal with these issues we estimated a time-varying first-order autoregression to predict the inflation rate in country *n* for year *t*:

$$\pi_{nt} = c_n + \phi_n \pi_{n,t-1} + \varepsilon_{nt} \tag{2.1}$$

To allow for variation over time in inflation persistence, we estimated equation (2.1) by ordinary least squares using a sample of thirty years of data ending in each year *T*. The resulting estimates of the persistence of inflation for country *n* in year *T*, $\hat{\phi}_{nT}$, are plotted as a function of *T* for the U.S. and U.K. in row 3 of Exhibit 2.2 and for other countries in the panels of Exhibit 2.5. There is indeed little persistence in realized inflation for most countries during the 19th century, implying that changes in the nominal rate should be viewed as changes in the ex-ante real rate. However, by World War I there is a fair amount of persistence in most countries, suggesting that at least some degree of war-related inflation should have been anticipated at the time. People knew there had been a war and that last year there had been significant inflation. To maintain that they nevertheless anticipated stable prices for the following year in such a setting seems an unlikely hypothesis.

In the last row of Exhibit 2.2 and the panels in Exhibit 2.6 we plot the value for the ex-ante real interest rate that is implied by the above forecasting model, that is, we plot

$$r_{nt} = i_{nt} - \hat{c}_{nt} - \hat{\phi}_{nt} \pi_{nt} \tag{2.2}$$

where $\hat{c}_{nt}$ and $\hat{\phi}_{nt}$ are the estimated intercept and slope for a regression estimated using 30 years of data for that country ending at date *t*. These suggest that ex-ante real rates were typically higher in the 19th century than they have been over the last half century. For example, a real rate above 4% was fairly often observed in the United States prior to 1900 but has been much less common since 1960. Clark

(2005) argued that a downward trend in interest rates since the industrial revolution might be attributed to increasing life expectancy, and Reinhart and Rogoff (2009) documented sovereign debt defaults prior to World War II that could add a risk premium in the earlier data. Note that for readability the numbers in Exhibit 2.6 have been truncated at $\pm 20\%$. The actual inferences can reach spectacular negative values, such as -1853% for Germany in 1922, -1376% for Italy in 1944, and -583% for Japan in 1946. Though not quite as spectacular as these, there are strongly negative real rates for almost all countries during both world wars, with negative real rates in many countries extending well into the 1950s, for which Reinhart and Rogoff (2009, p. 106) offered this explanation:

> During the post-World War II era, many governments repressed their financial markets, with low ceilings on deposit rates and high requirements for bank reserves, among other devices, such as directed credit and minimum requirements for holding government debt in pension and commercial bank portfolios.)

Although one could arrive at different estimates of the ex-ante real rate using a different specification of expected inflation, the above broad conclusions are fundamentally tied to what we see in the raw interest rate and inflation data and would be unlikely to be changed under any reasonable specification of inflation expectations. There is undeniably a downward trend in the nominal rate in the first half of the sample that is not matched by any clear trend in inflation, and the wartime inflations far outstripped anything seen in nominal rates and could not have been completely unanticipated. The preliminary impression is that an economy can be quite far from the equilibrium rate for extended periods and that the equilibrium rate itself is likely changing over time.

2B. Postwar quarterly data

We will also be making use of more recent, higher frequency data. For the U.S. we use the average fed funds rate over the last month of the quarter for the measure of the policy rate (available since 1954:3) and 400 times the log difference of the GDP deflator (available since 1947:1) as our series for inflation. For other countries we use the short-term interest rate (generally 3-month LIBOR or Eurocurrency rates) and the GDP deflator, as reported by the IMF World Economic Outlook and the OECD Economic Outlook database. Sample periods for which our constructed real rates are available vary across countries, as indicated in column (4) in Exhibit 2.1. For all countries but the U.S., the quarterly data end in 2013:2.

For quarterly data we replaced the forecasting equation (2.1) with a fourth-order autoregression:

$$\pi_{nt} = c_n + \phi_{n1}\pi_{n,t-1} + \phi_{n2}\pi_{n,t-2} + \phi_{n3}\pi_{n,t-3} + \phi_{n4}\pi_{n,t-4} + \varepsilon_{nt}. \tag{2.3}$$

Note that using four quarterly lags in (2.3) corresponds to the single lag in (2.1) using annual data—in each case the forecast is based on what was observed over the previous year. Because of the limited sample we begin the estimation using only 40 observations and then let the number of observations grow until we get to 80. For example, our first price-level observation for the U.S. is the value of the

GDP deflator for 1947:1. Our first available estimate of expected inflation, $E_{1958:1}\pi_{US,1958:2}$, thus comes from the coefficients estimated on a sample estimated for *t* = 1948:2 to 1958:1, from which we get the 1958:1 real interest rate from $r_{US,1958:1} = i_{US,1958:1} - E_{1958:1}\pi_{US,1958:2}$. We then add one more observation (without dropping the initial data point) to infer the 1958:2 real interest rate using a sample of 41 observations and the 1958:3 real rate using 42 observations. Once we get past 1968:1, we start to drop the observation at the start of the previous sample so that each estimate from then on uses a 20-year sample.[3]

Our series for the real interest rate constructed from annual and quarterly U.S. data align quite closely (see Exhibit 2.7). We also see from Exhibit 2.8 that our quarterly series for expected inflation aligns quite well with the subsequent realized inflation, with a correlation of 0.95. Exhibit 2.9 plots the postwar U.S. series for nominal and real interest rates.

Exhibit 2.10 presents some summary statistics for the U.S. The use of rolling regressions means that one could in principle have rather different means for inflation and expected inflation; in fact the two are quite similar. Use of rolling regressions also means that expected inflation need not be less variable than actual inflation. But our series for expected inflation is indeed less variable.

2C. Real rate vs. equilibrium rate

We close with a note on terminology**.** A prominent monetary policy maker (Ferguson (2004, p2)) once complained about the multiplicity of terms for the equilibrium real interest rate:

> Economists famously cannot agree on much. In this case, we cannot even agree on the name of the benchmark concept that I have just described. The real interest rate consistent with the eventual full utilization of resources has been called the equilibrium real federal funds rate, the natural rate of interest, and the neutral real rate. I prefer the first name, the equilibrium real federal funds rate, because, by using the word ‘equilibrium’, it reminds us that it is a concept related to the clearing of markets.

We follow Ferguson and use “equilibrium.” As well, we substitute “safe rate” or “policy rate” for “federal funds rate” when we reference data from outside the U.S. or from distant dates in the U.S. To state the obvious, the equilibrium real federal funds rate is distinct from the equilibrium real or nominal rate of return on business capital, on equities, on long term government debt, or on short or long term consumer or corporate debt, though of course those returns are related to the equilibrium real federal funds rate.

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[3] In preliminary work, we also experimented with keeping the window size fixed at 40 quarters through the whole sample. This led to a very similar series; the correlation was 0.98 between the expected inflation series with a 40 quarter window and the equation (2.3) version that we actually used.

In empirical application, we generally use measures that will capture a long run or steady state value of the safe real rate. In the next section, we use averages over a cycle or longer moving averages to capture the equilibrium rate. In section 5, we use a reduced form time series forecast to get a handle on the steady state, or long run, value we are likely to see going forward. This notion of an equilibrium real rate as a steady state value is consistent with the equilibrium rate being the rate we see with output at potential and stable inflation. Of course, over the cycle, there may be time variation in the rate that sets output at potential or inflation at target. In much of our discussion we abstract from such variations, since we look at averages over a cycle or longer moving averages.

3. The real rate and aggregate growth

What could account for the dramatic changes over time in real rates seen in the long term data in Exhibits 2.2 and 2.6 or the shorter recent sample in Exhibit 2.7? Much scholarly and blog discussion has tied interest rates to growth in output or potential output. This is central to the much cited paper by Laubach and Williams (2003). It is also central to discussions of secular stagnation. Gordon (2012, 2014) has argued that the trend rate of growth will be lower, which, given a presumed link between real rates and growth, suggests lower real rates. Summers (2013a) argues that in the near term, interest rates might have to be negative if output is to be at potential.

This section considers the link between the safe real rate and aggregate growth. In section 3A we review a standard theoretical reason for the safe real rate to be tied to consumption and output growth. In section 3B, we review existing evidence suggesting that, historically, the link between the real rate and consumption growth is weak. We then present new evidence of a weak link to output growth using US (section 3C)[4] and cross-country (section 3D) data. Finally, section 3E summarizes the empirical results in sections 3C and 3D.

3A. Growth and the real rate of interest in the New Keynesian model

A basic building block in macro models used in scholarly and policy work is one that links real interest rates with consumption. We do not exploit that relationship in our quantitative work. But we do think it necessary to both motivate the relationship and, in the next section 3B, explain why we did not think it productive to make such a relationship a key part of our empirical work. We do so in the context of the basic New Keynesian model, in part so that we can also briefly link the equilibrium rate that is our focus to the natural rate of New Keynesian models.

In New Keynesian models, the basic building block referenced in the previous paragraph is a dynamic IS equation that relates the intertemporal marginal rate of substitution in consumption to the real interest rate. We exposit this relationship in its simplest and very familiar form.

[4] Using a different approach, Leduc and Rudebusch (2014) also conclude the link in the U.S. is weak.

The dynamic IS equation is a formal statement of the following condition: the consumer cannot expect to be made better off by consuming one fewer unit this period, investing in the nominally safe asset, and consuming the proceeds next period. To exposit this textbook relationship, let

i_t - π_{t+1} = ex-post real return on a nominally safe asset, (3.1)

i_t = safe nominal rate, π_{t+1} = realized inflation,

$\pi_{t+1} = \ln(P_{t+1}/P_t)$, P_t = price level;

$r_t \equiv i_t - E_t\pi_{t+1}$ = ex-ante real rate, with E_t denoting conditional expectation

C_t = consumption in period t,

$c_t = \ln(C_t)$;

β = consumer's per period discount factor = $\frac{1}{1+\delta}$,

(e.g., δ=0.04 and β=.96 if data are yearly);

$U(C_t)$ = per period utility;

σ_p^2, σ_c^2, σ_{pc} = conditional variances of inflation and of consumption growth, and conditional covariance between inflation and of consumption growth, assumed constant; for example, $\sigma_p^2 = E_t(\pi_{t+1}-E_t\pi_{t+1})^2$.

For the moment, let utility be isoelastic,

$$U(C_t) = C_t^{1-\alpha}/(1-\alpha),\ \alpha>0. \tag{3.2}$$

Then after a second order loglinearization (or conditional lognormality of inflation and consumption growth)

$$r_t \equiv i_t - E_t\pi_{t+1} \approx \delta + \alpha E_t\Delta c_{t+1} - 0.5(\sigma_p^2 + \alpha^2\sigma_c^2 + 2\alpha\sigma_{pc}). \tag{3.3}$$

Write this as

$$r_t \approx \rho + \alpha E_t\Delta c_{t+1},$$
$$\rho = \delta - 0.5(\sigma_p^2 + \alpha^2\sigma_c^2 + 2\alpha\sigma_{pc}). \tag{3.4}$$

This intertemporal condition ties the ex-ante short rate to expected consumption growth each period: Higher expected consumption growth is associated with higher real rates.

To formally tie consumption growth to output and potential output, we follow Galí (2008, ch. 3), modulo the fact that we have second order terms in our definition of ρ and he does not.

Rearrange (3.4) so that c_t is on the left. Using the definition of r_t,

$$c_t = E_tc_{t+1} - \frac{1}{\alpha}(i_t - E_t\pi_{t+1} - \rho). \tag{3.5}$$

Next, in the baseline New Keynesian model,

$$\text{consumption} = \text{output}, \tag{3.6}$$

and in all New Keynesian models, baseline or not, output can deviate from the flexible price equilibrium. Let

$$y_t^n = \text{potential output} = \text{flexible price output}, \tag{3.7}$$
$$y_t = c_t - y_t^n = \text{output gap} = \text{deviation from flexible price equilibrium.}$$

Then (3.5) can be written

$$y_t = E_t y_{t+1} - \frac{1}{\alpha}[i_t - E_t\pi_{t+1} - r_t^n], \tag{3.8}$$
$$r_t^n \equiv \rho + \alpha E_t \Delta y_{t+1}^n = \text{natural rate of interest.} \tag{3.9}$$

Equation (2) in Laubach and Williams (2003) corresponds to our equation (3.9), with a shock added on by Laubach and Williams.

The natural rate of interest has normative properties; it may be desirable for the Fed to set the expected short rate to the natural rate (see Galí (2008)). But the empirical counterpart is model dependent (see below).

If the steady state, or average, value of the output gap is zero, then in this baseline model the average value of the real interest rate (3.4) and the natural rate (3.9) are the same. But once one departs from the baseline model there may no longer be a simple connection between (1) growth of actual or potential output and (2) the real rate or the natural rate of interest. Expression (3.9) was derived assuming that consumption = output. That may be a fine simplification in some contexts but perhaps not here. The theoretical implications if consumption ≠ output are simply stated when the only departure from the baseline model is to allow two kinds of goods, one of which is imported. Then Clarida et al. (2002, p890) conclude that when, as well, $\alpha \neq 1$, the natural rate of interest is a weighted sum of the growth of potential output in (1) the home country, and (2) the rest of the world, with the weight on rest of world proportional to the share of imported goods in consumption.

$$r_t^n \equiv \rho + \omega_1 E_t \Delta y_{t+1}^n + \omega_2 E_t \Delta y_{t+1}^*, \tag{3.10}$$

where Δy_{t+1}^* is the growth rate of potential in the rest of the world and ω_1 and ω_2 are parameters that depend on the intertemporal elasticity α and the share of imported goods in consumption.

In the U.S., an adjustment of imported goods would likely be quantitatively small. The point is that (3.9) holds only in very special circumstances. Adjustments for other departures, such as fixed capital and wage and price markup shocks, come in various forms, and are quantitatively substantial. See Barsky et al. (2014), for example.

Hence the New Keynesian model does not give a strong a priori reason for a tight short-run relation between the real rate or the natural rate on the one hand and growth of potential or actual output on the other.

3B. Mean consumption growth and the equilibrium rate

The New Keynesian model does, however, provide an a priori reason for a tight link between the real rate and consumption growth, in the form of Equation (3.4): this equation does require that utility be of the form (3.2) but is agnostic about the presence or absence of capital, imports, wage and price shocks, etc. And equation (3.4) has some intuitively appealing implications.

●Higher uncertainty about either inflation or consumption growth (as indexed by the variance terms) lowers the safe real rate. This is consistent with stories about flight to safety.

●The more one discounts the future (higher δ) the higher the safe real rate, which again makes sense—if you are very impatient, a high return is what makes you cut back on consumption today so that you can consume tomorrow.

Unfortunately, a huge literature has documented that (3.4) does not work well empirically. See Kocherlakota (1996) and Mehra and Prescott (2003) for surveys. Given our topic, the most salient failure of the model relates to its implications for the average or equilibrium level of the real rate. The second order terms are small compared to the other terms (see, for example, Table 1 in Kocherlakota (1996)). So for quantitative purposes ignore them for the moment, setting $\rho \approx \delta$. Expressing things at annual rates: average per capita consumption growth is about .02; we generally put annual discount rates at something like δ=0.04. With α=1 (log utility), that implies an average value of the safe rate of .06—an implausibly high value. Since Weil (1989), the fact that this widely used model implies an implausibly high risk free rate is called the "risk free rate puzzle."

The huge literature referenced in the previous paragraph has examined various solutions to the puzzle. These efforts include among others varying the discount factor δ, varying risk aversion α, varying the utility function, and dropping the representative agent/complete markets paradigm. In New Keynesian models rich enough to be used quantitatively in monetary policy analysis, there usually is a representative agent, the discount factor and risk aversion are generally similar to what is above, but the utility function often incorporates what is called habit persistence.

It is our reading that habit persistence does not deliver a reasonable value for the equilibrium rate, though the evidence is a bit mixed. Habit can be modeled as internal or external. Internal persistence means utility this period depends on consumption this period relative to one's own consumption in the previous period. Internal habit is used in the influential Smets and Wouters (2003) or Christiano et al. (2005) models. External persistence means one's consumption this period is compared instead to aggregate consumption the previous period. External habit appears in papers such as de Paoli and Zabczyk (2013). In either case, let

X_t = habit level of consumption, (3.11)

$U(C_t\text{-}X_t) = (C_t\text{-}X_t)^{1\text{-}\alpha}/(1\text{-}\alpha)$, $\alpha>0$.

Then X_t varies either with one's own consumption (internal habit) or aggregate consumption (external habit).

Dennis (2009, equations (6), (7), (11) and (12)) supplies the first order analogues to (3.3) when utility is (a) of the form (3.11), or (b) when habit is multiplicative rather than additive. It follows from Dennis's expressions that neither internal nor external habit substantially affects the mean level of the safe rate when parameters are varied within the plausible range. Specifically, for additive habit, such as in (3.11) above, it follows analytically from Dennis's (11) and (12) that variation in habit has no effect on the mean safe rate. For multiplicative habit we have solved numerically for a range of plausible parameters and find habit has little effect on the mean rate. (Dennis's expressions are log linearized around a zero growth steady state. We have derived the log linearization in the presence of nonzero growth in one case (additive external habit), and the conclusion still holds.)

Time variation in second moments (assumed away for simplicity in (3.1) can be important in understanding the relationship between returns and consumption. Campbell and Cochrane (1999) let conditional second moments vary over time. They assume that the conditional variance of what they call "surplus consumption" rises as consumption C_t approaches habit X_t. They parameterize this in a way that delivers an equilibrium real rate that is indeed plausibly low on average. The model, however, implies counterfactual relations between nominal and real rates (Canzoneri et al. (2007)).

Hence our review of existing literature leads us to conclude that it is unlikely to be productive to focus on consumption when modeling the real rate, despite the strong theoretical presumption of a link between consumption growth and the real rate. The remaining parts of this section focus on GDP growth instead.

3C. Output growth and the real rate in the U.S.

There are theoretical reasons to expect a long-run relation between the real rate and GDP growth. In a model with balanced growth, consumption will, in the long run, grow at the same rate as output and potential output. Thus the combination of the intertemporal condition (3.4) and balanced growth means that over long periods of time, the average short real rate will be higher when the growth rate of output is higher and lower when output growth is lower. Perhaps there is a clear long-run relationship between output and the real rate, despite the weak evidence of such a relationship between consumption and the real rate. In this section we use our long-run U.S. dataset to investigate the correlation, over business cycles or over 10 year averages, between GDP growth and real rates. Our focus is on the sign of the correlation between average GDP growth and average real rates. We do not attempt to rationalize or interpret magnitudes. We generally refer to "average real rate" rather than equilibrium real rate. But of course our view is that we are taking averages over a long enough period that the average rate will closely track the equilibrium rate.

Real rate data were described in Section 2. We now describe our output data. Our U.S. GDP data runs from 1869 to the present. Balke and Gordon (1989) is the source for 1869-1929, FRED the source for 1929-present. Quarterly dates of business cycle peaks are from NBER. When we analyze annual data, quarterly turning points given by NBER were assigned to calendar years using Zarnowitz (1997, pp732-33). Zarnowitz's work precedes the 2001 and 2007 peaks so we assigned those annual dates ourselves. When, for robustness, we briefly experiment with potential output instead of GDP, the CBO is our source.

As just noted, we focus on the sign of the correlation between average GDP growth and average real rates. We find that this sign is sensitive to sample, changing sign when one or two data points are removed. We did not decide ex-ante which data points to remove. Rather, we inspected plots presented below and noted outliers whose removal might change the sign of the correlation. Ex-post, one might be able to present arguments for focusing on samples that yield a positive correlation, and thus are consistent with the positive relation suggested by theory. But one who does not come to the data with a prior of such a relation could instead conclude that there is little evidence of a positive relation.

Peak to peak results

Peak to peak results are in Exhibits 3.1-3.4. Our baseline set of data points for the peak to peak analysis are the 7 (quarterly) or 29 (annual) pairs of (GDP growth, *r*) averages presented in Exhibit 3.1. Here is an illustration of how we calculated peak to peak numbers. In our quarterly data, the last two peaks are 2001:1 and 2007:4. Our 2007:4 values are 2.52 for GDP growth and 0.45 for the real interest rate. Here, 2.52 is average GDP growth over the 27 quarters from 2001:2 (that is, beginning with the quarter following the previous peak) through 2007:4, with 0.45 the corresponding value for the real rate.

Let us begin with quarterly data (Exhibit 3.2, and rows (1)-(4) in Exhibit 3.4). A glance at the scatterplot Exhibit 3.2 suggests the following. First, the correlation between average GDP growth and the average real rate is negative, at -0.40 it so happens. (See line (1), column (6) of Exhibit 3.4. That exhibit reports this and other peak-to-peak correlations that we present here in the text.) Second, the negative correlation is driven by 1981:3. If we drop that observation—which, after all, reflects a cycle lasting barely more than a year (1980:2-1981:3), and is sometimes considered part of one long downturn (e.g., Mulligan (2009) and Angry Bear (2009))—the correlation across the remaining six peak to peak averages is indeed positive, at +0.32 (line (2) of Exhibit 3.4)). If we continue to omit the 1981:3 peak, but substitute CBO potential output for GDP (line (3)) or ex-post interest rates for our real rate series (line (4)), the correlation falls to -0.01 or 0.17.

Of course, such sensitivity to sample or data may not be surprising when there are only six or seven data points. But that sensitivity remains even when we turn to the much longer time series available with annual data, although the baseline correlation is now positive.

The averages computed from annual data in columns (5) and (6) in Exhibit 3.1 are plotted in Exhibit 3.3. A glance at the scatterplot in that exhibit reveals the positive correlation noted in the previous paragraph, at 0.23 it so happens (line (5) of Exhibit 3.4). That correlation stays positive, with a value of 0.30 (line (6) of Exhibit 3.4) if we drop 1981, the peak found anomalous in the analysis of quarterly data.

However, for annual data, one's eyes are drawn not only to 1981 but also to points such as 1918, 1920, 1944 and 1948. One can guess that the correlation may be sensitive to those points. To illustrate: Let us restore 1981, but remove the postwar 1920 and 1948 peaks, the correlation across the remaining 27 peak to peak averages is now negative, at -0.23 (line (7)). If we instead drop the three peaks that reflect the Great Depression or World War II, the correlation is again positive at 0.29 (line (8)).

The remaining rows of Exhibit 3.4 indicate that the annual data give results congruent with the quarterly data when the sample period is restricted (lines (9) and (10)) and that the annual results are not sensitive to the measure or timing aggregate output (Romer (1989) and year ahead data in lines (11) and (12)).

We defer interpretation of sensitivity until we have also looked at backward moving averages of U.S. data, and cross-country results.

Ten-year averages

We consider 40-quarter (quarterly data) or 10-year (annual data) backwards moving averages. Ten years is an arbitrary window intended to be long enough to average out transient factors and presumably will lead to reasonable alignment between average GDP growth and growth of potential output. Using annual data, we also experimented with a 20-year window, finding results similar to those about to be presented.

Numerical values of correlations are given in column (6) of Exhibit 3.5, with scatterplots presented in Exhibits 3.6 and 3.7. In Exhibit 3.6, the fourth quarter of each year is labeled with the last two digits of the year. We see in Exhibit 3.6 that for quarterly data, the correlation between the 40-quarter averages is positive, at 0.39 it so happens (line (1) in Exhibit 3.5). This is consistent with the quarterly peak-to-peak correlation of 0.32 when 1981:3 is removed (line (2) of Exhibit 3.4)). The result is robust to use of ex-post real rates (line (3)). But, as is obvious from Exhibit 3.6, if we remove the post-2007 points, which trace a path to the southwest, the correlation becomes negative, at -0.19 (line (2)). We see in Exhibit 3.7 that for annual data, the correlation between 10-year averages is negative, at -0.25 it so happens (line (4) in Exhibit 3.5). The postwar sample yields a positive correlation (line (5)). Omitting 1930-1950, so that the Depression years fall out of the sample, turns the correlation positive (line (6)). The value of 0.31 is consistent with 0.29 figure in line (8) of peak-to-peak results in Exhibit 3.4, which also removed Depression and post-World War II years.

3D. Cross-country results

Our GDP data come from the OECD. The source data were real, quarterly and seasonally adjusted. Sample coverage is dictated by our real rate series that were described in Section 2. Our real rate series for all countries had a shorter span than our GDP data. Our longest sample runs from 1971:2-2014:2.

We compute average values of GDP growth and of the real interest rate over samples of increasing size, beginning with roughly one decade (2004:1-2014:2, to be precise) and then move the start date backwards. The sample for averaging increases to approximately two (1994:1-2014:2), then three (1984:1-2014:2), and finally four (1971:2-2014:2) decades. Some countries drop out of the sample as the start of the period for averaging moves back from 2004 to 1971.

Exhibit 3.8 presents the resulting values. Exhibit 3.9 presents scatterplots of the data in Exhibit 3.8. Note that the scale of the 2004:1-2014:2 scatterplot is a little different than that of the other three scatterplots.

As suggested by the scatterplots and confirmed by the numbers presented in the "corr" row of Exhibit 3.8, the correlation between average GDP growth and average real rates is positive in all four samples, and especially so in the 20 year sample. However, the sign of the correlation is sensitive to inclusion of one or two data points. For example, in the 1984-2014 sample, if Australia is omitted, the correlation turns negative.

3E. Summary and interpretation

Both our U.S. and our international data yield a sign for the correlation between average GDP growth and the average real interest rate that is sensitive to sample, with correlations that are numerically small in almost all samples.[5] However, the theoretical presumption that there is a link between aggregate growth and real rates is very strong.[6] One could make an argument to pay more attention to the samples that yield a positive correlation—for example, dropping 1980-81 from the set of full U.S. expansions or dropping 1930-1950 from the 10-year U.S. averages—and deduce that there is modest evidence of a modestly positive relationship between the two. For our purposes, we do not need to finely dice the results to lean either towards or against such an argument. Rather, we have two conclusions. First, if, indeed, we are headed for stagnation for supply side reasons (Gordon (2012, 2014)), any such slowdown should not be counted on to translate to a lower equilibrium rate over periods as short as a cycle or two or a decade. Second, the relation between average output growth and average real rates is so noisy that other factors play a large, indeed dominant, role in determination of average real rates.

4. The secular stagnation hypothesis[7]

[5] This is consistent with the formal econometric work of Clark and Kozicki (2005,p403), who conclude that the link between trend growth and the equilibrium real rate is "quantitatively weak."
[6] Indeed, in section 6 we use a model that relies on a conventional link between real rates and growth.
[7] A much more detailed version of the argument in this section is available in Hamilton et al. (2015).

A number of prominent economists have recently suggested that the modern economy could be caught in a long-term stagnation with a very low equilibrium real rate. Krugman, Dominguez, and Rogoff (1998) noted that if the equilibrium real interest rate is negative, an economy could get stuck at suboptimal growth and deflation as a result of the zero lower bound on nominal interest rates. Summers (2013b) expressed the hypothesis this way:

> Suppose that the short-term real interest rate that was consistent with full employment had fallen to negative two or negative three percent in the middle of the last decade. Then … we may well need, in the years ahead, to think about how we manage an economy in which the zero nominal interest rate is a chronic and systemic inhibitor of economy activity, holding our economies back below their potential.

Summers (2014) argued that the stagnation goes back to the 1990s, which he claimed would have been even weaker in the absence of the stock-market bubble, while Krugman (2013) argued that the real-estate bubble played a similar role in the following decade. Martin and Ventura (2012) investigated the contribution of these bubbles to the observed economic growth.

But the timing of the stock-market bubble does not fit Summers's interpretation. The big surge in the NASDAQ did not begin until late 1998 (see Exhibit 4.1), yet the unemployment rate had already dropped to 4.7% by 1997. And while it's true that the housing bubble made a positive contribution to growth over 2003-2007 (Feroli et al. (2012)), there were also some important headwinds countering this. The trade deficit increased by 2.4% of GDP from 2001Q4 to 2006Q1, subtracting 0.5 percentage point per year from growth. Big increases in energy prices also held the economy back despite the housing boom (Hamilton, 2009).

It's also worth noting that unemployment ended up below the CBO estimate of the natural rate in the later part of both the 1991-2000 and 2001-2007 expansions (see Exhibit 4.2). Likewise, inflation ended up above the Fed's 2% target (Exhibit 4.3). These facts are inconsistent with the claim that the U.S. has chronically been performing below potential for the last two decades. The Great Recession was unusually severe and, consistent with other downturns that were associated with financial turmoil (Reinhart and Rogoff, 2014), took quite long to recover from. It was only at the end of 2015 that unemployment returned close to the natural rate and the Fed began a new tightening cycle. If we were to look at other expansions at a comparable point—the date when unemployment first returned to the natural rate or the tightening cycle first began—we would very often observe a real rate at the time that was still quite low, reflecting the depressed investment demand that typically characterizes such conditions—see Exhibits 4.4 and 4.5. Nevertheless, by the end of each of the previous expansions, the real rate would return to robust levels. To conclude that the U.S. has entered a prolonged period of a chronically low equilibrium real rate is in our opinion misinterpreting an unusually severe business cycle downturn as a chronic long-term condition.

5. Long-run tendencies of the real interest rate

In this section we provide a formal econometric investigation of whether there is some constant value to which the equilibrium real rate tends eventually return. We will first present evidence of nonstationarity of the U.S. ex-ante real interest rate and then develop a bivariate vector error correction model relating U.S. rates to global factors.

A number of studies have documented instability over time in postwar measures of the real interest rate. Although Garcia and Perron (1996) and Ang and Bekaert (2002) modeled these as shifts between possibly recurrent regimes, Caporale and Grier (2000) and Bai and Perron (2003) found these were better captured as permanent breaks, with Rapach and Wohar (2005) finding statistically significant structural breaks in postwar data for each of the 13 countries they examined. Since one of the striking features in our long-run data set is the apparent higher real rate in the 19th century, here we test for stability of the mean real interest rate over our full long-run data set.[8]

5A. Nonstationarity of the real interest rate

Consider a second-order autoregression[9] fit to annual levels of the U.S. real interest rate with a possible shift in the level beginning at some date t_0:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \kappa\delta(t \geq t_0) + \varepsilon_t. \qquad (5.1)$$

Here $\delta(t \geq t_0)$ takes the value of one if date *t* comes after some date t_0 and is zero for earlier values. A finding that $\kappa < 0$ would mean that real interest rates tended to be lower in the second part of the sample than in the first. We estimated equation (5.1) for $y_t = r_{US,t}$ over the 1861 to 2014 sample and used White's (1982) heteroskedasticity-robust test of the null hypothesis $\kappa = 0$ for every possible break date t_0 between 1900 and 1976. The resulting $\chi^2(1)$ statistic is plotted as function of t_0 in the top panel of Exhibit 5.1, along with the 1% critical value for a $\chi^2(1)$ variable (shown in dashed green). The hypothesis that the real interest rate tends to revert to a fixed constant over time would be rejected for any break date before World War I.

Of course, when one looks at a number of different possible break dates (as we have done here), the largest observed value for the test statistic no longer has a $\chi^2(1)$ distribution, but instead has an asymptotic distribution calculated by Andrews (1993, 2003), the 1% critical value for which is

[8] Elliott and Müller (2006) noted that departures from stationarity, whether in the form of structural breaks, unit roots, or time-varying parameters, can be embedded into a single unifying framework, and that, from the perspective of efficient tests of the null hypothesis, the seemingly different approaches are in fact equivalent. In most of what follows we will be using a finite number of structural breaks as specification of the alternative hypothesis against which the null of stationarity is being tested.

[9] Using a second-order autoregression for levels allows us to include as a special case a first-order autoregression for growth rates as one of the possible specifications that we will be considering.

plotted in solid blue in Exhibit 5.1. The evidence against a constant average real rate remains quite convincing.

Several alternative tests could also be used to test the null hypothesis that the real rate is a stationary process that tends over time to revert to a fixed constant. The KPSS test of Kwiatkowski et al. (1992) leads us to reject the null hypothesis of stationarity at the 5% level.[10] We also applied the approach suggested by Bai and Perron, looking for the possibility of more than one break in all the coefficients of (5.1). That approach identifies two breaks occurring at 1915 and 1921, interpreting the pre-World War I, World War I, and post-World War I episodes as three different regimes.[11] However, given the striking heteroskedasticity in these data, our preference is to use White's (1982) heteroskedasticity-robust test with a single break point, which will be the test used in the subsequent analysis.

If the real interest rate itself is nonstationary, can we identify a stable relation that can consistently describe a century and a half of U.S. and world data? This is the task we undertake next.

5B. A stable representation of long-run dynamics

Although apparently nonstationary, the real interest rate does exhibit a form of mean reversion in that episodes with real interest rates above 5% or below -5% proved to be temporary. We tried to capture this idea by fitting a first-order autoregression to 30-year rolling windows of data on each country's ex-ante real rate:

$$r_{nt} = b_n + \psi_n r_{n,t-1} + \varepsilon_{nt}. \tag{5.2}$$

In order to keep these estimates from being unduly influenced by outliers such as the extreme wartime observations, we allowed the variance of the residuals to vary over time according to an ARCH(2) process:

$$\begin{aligned} \varepsilon_{nt} &= h_{nt} v_{nt} \\ v_{nt} &\sim N(0,1) \\ h_{nt}^2 &= \kappa_n + \alpha_{n1}\varepsilon_{n,t-1}^2 + \alpha_{n2}\varepsilon_{n,t-2}^2. \end{aligned}$$

We estimated the parameters by maximizing the likelihood for 30 years of observations on country *n* ending in year *t*, and calculated the long-run value for the real rate implied by those estimates:

$$\ell_{nt} = \frac{\hat{b}_{nt}}{(1-\widehat{\psi}_{nt})} \; .$$

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[10] The test statistic using 5 lags and a constant is 0.55 for a *p*-value of 0.025.

[11] The null hypothesis of a single structural break is rejected in favor of the alternative of two structural breaks with a *p* value less than 0.01 using the test described in Table 2 of Bai and Perron (1998).

We then took the median value across countries as an estimate of the long-run world real rate associated with every year *t* in the sample:

$$\ell_t = \underbrace{median}_{n} \ell_{nt}.$$

The resulting series for the world long-run rate ℓ_t is plotted along with the actual U.S. ex-ante real rate $r_{US,t}$ in Exhibit 5.2.[12] The world long-run rate captures the broad trends noted in Section 2, such as the persistent tendency for real rates to be higher in the 19th century and a long episode of low real rates from World War II through the mid-1970s. World real rates rose through the mid-1980s and have been declining again since then.

Evidence that real rates do not show a tendency to revert to some constant value is even more dramatic when we use ℓ_t in place of $r_{US,t}$, as seen in the second panel of Exhibit 5.1.

However, it is interesting to note that the gap between the U.S. real rate and our series for the long-run real rate appears to be stationary. The third panel of Exhibit 5.1 performs the same stability test on $y_t = r_{US,t} - \ell_t$, consistent with the conclusion that the following relation is stable over time (robust standard errors in parentheses):

$$r_{US,t} - \ell_t = \underset{(0.204)}{-0.174} + \underset{(0.133)}{0.935}(r_{US,t-1} - \ell_{t-1}) - \underset{(0.103)}{0.357}(r_{US,t-2} - \ell_{t-2}) + e_t. \tag{5.3}$$

Note that with the sum of the autoregressive coefficients of only 0.58, we reject the null hypothesis of a unit root with a Dickey-Fuller robust *t* test of -4.89. A KPSS test likewise fails to reject the null hypothesis that $r_{US,t} - \ell_t$ is stationary. In other words, although neither the U.S. nor the world rate appear to return to a single stable value over time, the two series are cointegrated and tend not to stay too far apart. Expression (5.3) implies a long-run tendency for the U.S. rate to be on average 41 basis points below the world rate $(-0.174/(1-0.935+0.357) = -0.41)$ but this is poorly estimated even with a century and a half of data and cannot be distinguished statistically from zero.

An error-correction VAR for the U.S. and long-run world rate also appears to be stable:

$$\Delta r_{US,t} = \underset{(0.206)}{-0.195} + \underset{(0.104)}{0.382}\,\Delta r_{US,t-1} - \underset{(0.625)}{0.790}\,\Delta \ell_{t-1} - \underset{(0.085)}{0.403}(r_{US,t-1} - \ell_{t-1}) + e_t$$

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[12] Our measure of the long-run real rate is very similar to an average value of *r* over the preceding 30 years. In fact, if (5.2) were estimated by OLS instead of with ARCH(2) residuals, and if the interest rate in year *t* was identical to the interest rate in year *t* -30, our estimate would be identical to the average rate over the 30 years ending in *t*. One can see this from the OLS identity that $\bar{r} = \hat{b} + \hat{\psi}\bar{r}_{-1}$ and the fact that if $r_t = r_{t-30}$ then $\bar{r} = \bar{r}_{-1}$ meaning that $\hat{b}/(1-\hat{\psi}) = \bar{r}$. If $r_t > r_{t-30}$ then ℓ_t will tend to be a little above the average rate over the 30 years ending in *t*. Also, since we estimate (5.2) allowing for ARCH errors, our estimate tends to down-weight observations that appear to be outliers in calculating the long-run real rate.

$$\Delta \ell_t = -\underset{(0.025)}{0.020} + \underset{(0.010)}{0.026}\,\Delta r_{US,t-1} - \underset{(0.082)}{0.322}\,\Delta \ell_{t-1} + \underset{(0.009)}{0.016}(r_{US,t-1} - \ell_{t-1}) + e_t.$$

We find no evidence of a changing intercept in either of these equations, and readily accept the hypothesis that lagged levels of $r_{US,t-1}$ or ℓ_{t-1} do not enter as additional explanatory variables, supporting the conclusion that $r_{US,t}$ and ℓ_t are cointegrated with cointegrating vector (1,-1)′ and error-correction term $r_{US,t} - \ell_t$. If the U.S. rate is 100 basis points above the long-run world rate, the U.S. rate would be predicted to fall by 40 basis points the following year (with a *p*-value below 10^{-5}) and the world rate to rise by 1.6 basis points, though the latter is insignificantly different from zero (*p* = 0.07).

Note that this conclusion that a difference between U.S. and long-run world rates is mostly closed by adjustment of the U.S. rate is not inconsistent with the claims by Rey (forthcoming) that events in the United States are an important driver of world developments. Our variable ℓ_t summarizes not just the current world situation but also the long-run trends in U.S. and global data. Our error-correction specification thus is capturing both a tendency for the U.S. to converge to the rest of the world as well as for U.S. and global rates to revert to recent trend values.

The intercepts in both of the equations above are statistically insignificant. Dropping these gives the following more parsimonious representation of the error-correction VAR:

$$\Delta r_{US,t} = \underset{(0.103)}{0.379}\,\Delta r_{US,t-1} - \underset{(0.623)}{0.775}\,\Delta \ell_{t-1} - \underset{(0.085)}{0.396}(r_{US,t-1} - \ell_{t-1}) + e_{US,t} \qquad \widehat{\sigma}_{US} = 2.57 \tag{5.4}$$

$$\Delta \ell_t = -\underset{(0.082)}{0.321}\,\Delta \ell_{t-1} + \underset{(0.010)}{0.026}\,\Delta r_{US,t-1} + \underset{(0.009)}{0.017}(r_{US,t-1} - \ell_{t-1}) + e_t \qquad \widehat{\sigma}_{\ell} = 0.31. \tag{5.5}$$

Note that although we find that the U.S. and long-run world rate are cointegrated, in any given year they could differ enormously. The standard deviation of the shock in (5.4) is over 250 basis points, meaning in any given year the U.S. rate could easily be hundreds of basis points away from the long-run world rate. But what the relation says is that such deviations are likely to prove to be temporary.

5C. Implications for future real rates

It is interesting to look at what the system (5.4)-(5.5) would imply for the behavior of future interest rates. Exhibit 5.3 plots the forecasts for the U.S. and long-run world rate as of the beginning of 2015 that are implied by (5.4) and (5.5). The U.S. rate is expected to rise relatively quickly and the long-run world rate to fall very slightly, with both around 40 basis points within three years. Note that although the forecast levels of both rates converge to the same constant, the process for $(r_t, \ell_t)'$ implied by (5.4)-(5.5) is nonstationary. Although we expect the two to return to values close to each other as a result of the apparent cointegration between them, the particular level at which they do so becomes more uncertain the farther we look into the future, as illustrated by the continuing growth in the confidence interval for ℓ_t plotted in Exhibit 5.3. Note furthermore that the possible values for the

US rate for any given year could be in a very wide interval around these points, given the standard error of the estimate in (5.4) that is over 250 basis points.

Another implication of the unit root in real rates is that the implied forecast would be different if we used a different starting point to form the forecast. The formulation implies that there is a permanent component to any shock and therefore a permanent implication of initial conditions in any forecast. Again the expanding confidence intervals as one looks farther into the future from any given initial conditions are a necessary consequence of this property.

We should also note that this is a reduced-form model that makes no use of information such as the current state of the business cycle. Our analysis in Section 4 would argue that this is a reason to think the U.S. rate will rise faster and farther than implied by the reduced-form forecasts, which again is perfectly consistent with the confidence intervals.

It's nevertheless interesting to compare these predictions for the U.S. real rate with values implied by the term structure of Treasury Inflation Protected Securities (TIPS). By simultaneously buying a 3-year TIPS and selling a 2-year TIPS one can lock in the yield for a 1-year bond that will be purchased in two years, a rate of return known as the 2-year forward rate. Forward rates implied by the current yield curve are plotted in blue in Exhibit 5.4 along with the level for the US real rate predicted by the system (5.4)-(5.5).[13] If the U.S. real rate converges to the long-run world rate with the adjustment speed typically seen of the last century and a half, real rates will start rising at about the same rate implied by the current forward curve, though will end up at a rate somewhat below current longer-term forward rates. We nevertheless again caution that the standard errors associated with our model's forecasts are enormous.

To summarize, although it is commonly assumed in economic models that the real interest rate should eventually revert to an equilibrium or neutral value, there is little evidence for this in the data. Plausible measures of real rates were very negative in most countries for lengthy periods around the two world wars and have gone through other long periods of very high or very low values. Real rates were substantially higher in the 19th century than they have been since the middle of the 20th century. We do find some tendency for the U.S. real rate not to diverge for too long from a measure of the long-run world real rate. The fact that the latter is significantly above current U.S. real rates at present suggests a likelihood that U.S. real rates will rise over the next several years.

6. Implications for monetary policy

[13] The forward rates implied by yields as of December 20, 2014 were calculated as in Gürkaynak, Sack, and Wright (2010) using the spreadsheet available at http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html. Note that the forward rate cannot be reliably calculated by this method for a horizon less than 2 years into the future, which is why the plotted curve only begins in 2016. We do not attempt to correct for risk premia and use these calculations only as a rough guideline.

A key conclusion from our analysis is that the uncertainty around the equilibrium interest rate (which we denote here by r^*) is very considerable. This conclusion is also consistent with Laubach and Williams (2003). They estimate a standard error for r^* that ranges from 109bp to 258bp depending on the specification (see the bottom of Table 1 of their paper). Indeed, this uncertainty may be even greater at this point in history as policy makers try to gauge the size and persistence of a variety of economic headwinds. And predicting how r^* will change in the future is even harder, both because the linkage between r^* and its fundamental drivers such as potential GDP growth is tenuous and because these fundamental drivers themselves are difficult to forecast. In this section, we explore the implications of uncertainty about r^* for the normalization of the federal funds rate in coming years.

The study by Orphanides and Williams (2002, henceforth OW) provides a good starting point. Using a small estimated model of the US economy, they consider the policy rule

$$i_t = a_0 i_{t-1} + (1-a_0)\left(r_t^* + \pi_t\right) + a_1\left(\pi_t - \pi^*\right) + a_2\left(u_t - u^*\right) \tag{6.1}$$

where i is the nominal short-term interest rate, r^* is the real equilibrium rate, π is inflation, π^* is the central bank's inflation target, u is the unemployment rate, u^* is the structural unemployment rate, and a_0, a_1, and a_2 are parameters of the monetary policy reaction function. This model nests two extremes. If a_0 is equal to zero, it collapses to the familiar Taylor rule in which the level of short-term rates depends on the inflation and unemployment gap. But if a_0 is equal to one it collapses to a "difference rule" in which the *change* in the short-term interest rate depends on the inflation and unemployment gap. OW then derive the optimal choices of a_0, a_1, and a_2 as functions of the uncertainty around r^*.[14]

Their main conclusion is if the Fed acts as though it knows r^* with certainty, Fed reactions to false signals can themselves introduce an additional destabilizing influence on the economy. They find that uncertainty around r^* should push Fed officials toward a more "inertial" policy rule, in which the current funds rate depends more on the lagged funds rate and less on the Fed's (uncertain) estimate of the equilibrium rate. In terms of equation (6.1) they show that greater uncertainty raises the optimal value of a_0, up to a value of 1 in the limiting case where the Fed knows nothing about r^*. In this limiting case, the OW analysis suggests that Fed officials should adopt a difference rule, i.e. hike rates when the economy is "too hot"—i.e. inflation is too high and/or unemployment is too low—and lower rates when the economy is "too cold."

6A. Calibrating the baseline in the FRB/US model

To assess the relevance and implications of these insights for the current monetary policy outlook, we turn to FRB/US. This is a large-scale econometric model developed and maintained by the Federal Reserve Board staff. The model has a neoclassical core based on optimizing behavior by forward-looking households and firms but contains substantial detail on the components of GDP and various measures of inflation and interest rates; for more detailed documentation see Brayton, Laubach, and Reifschneider (2014). Since FRB/US is a rich model of the US economy that can be benchmarked to

[14] OW also consider uncertainty around u^* and therefore also include a potential response to the change in the unemployment rate. We set aside this issue in our formal analysis but touch on it at the end of this section.

the economic and interest rate projections in the FOMC's Summary of Economic Projections (SEP), we believe it is more useful for providing insights into current US monetary policy issues than a small and more stylized model. Note that FRB/US builds in a link between real rates and growth—a link whose steady state strength we questioned earlier in the paper. Thus in this section we defer to our theoretical priors that such a link exists.

We use FRB/US to answer two questions. First, how should Fed officials adjust their policy reaction function if they worry about uncertainty around the true value of r^*? And second, in the current context, what does this imply for the normalization of the funds rate in terms of the timing of liftoff, the pace of rate hikes, and the peak level of the funds rate?

We begin with a baseline path for r^* that is consistent with public perceptions of the Federal Reserve's economic assessment as of the start of 2015, starting from an initial value of -25 basis points. We assume that the Fed subsequently intended to adjust the interest rate according to a Taylor (1999) rule of the form

$$i_t = r_t^* + \pi_t + 0.5(\pi_t - \pi^*) - 2(u_t - u^*) \tag{6.2}$$

where notation is as before. We set $\pi^*=2$ and u^* equal to the structural unemployment rate in FRB/US (which converges to the SEP long-run expectation for u). Given the median SEP trajectories for i and π, we calculated the path for r^* that is implied by equation (6.2). This path is plotted as the solid curve in Exhibit 6.1. This path is consistent with statements by Fed officials that they regarded r^* as depressed and only expected a slow increase back to the longer-term level of 1¾% implied by the SEP.[15] This path also broadly matches the estimate from the model constructed by Thomas Laubach and John Williams. Both of these authors at the time held senior positions in the Federal Reserve System, Laubach as the Director of the Division of Monetary Affairs and Secretary of the FOMC and Williams as the President of the San Francisco Fed.

We then considered two alternative scenarios, the first in which the Fed incorrectly perceived r^* to lie 150bp above its true level, and a second scenario in which r^* was perceived to be 150bp below the true level. Note this potential perception error of +/-150bp is well within the range of estimates in Laubach and Williams (2003). However, we also consider the implications of a smaller perception error of +/-50bp as well as a larger error of +/-250bp. We assume that the error is quite long-lasting but ultimately temporary, i.e. that Fed officials eventually converge to the true r^*.

Admittedly, there is an arbitrary aspect to the way we generated the baseline path for r^* because we assume that the median FOMC participant uses equation (6.2) to generate his or her funds rate path. A different baseline path would result if we had assumed a different reaction function, e.g. one with different weights on the inflation and unemployment gap or one that already incorporates some inertia. But the precise baseline path for r^* does not matter much in terms of our main goal,

[15] For example, Chairman Bernanke said in the September 2013 FOMC press conference that the equilibrium rate would likely still be depressed by the end of 2016—the point at which the committee expected to hit its mandate at that time—and that it "…looks like it will be lower for a time because of these headwinds that will be slowing aggregate demand growth."

which is to illustrate what happens to the policy rule if Fed officials become more uncertain about *r**. Our formal results below show that the implication of this uncertainty is an increase in the amount of inertia in the preferred policy rule, from zero to a significantly positive amount.

6B. Fed policy without inertia

Armed with our perceived paths for *r**, we can simulate the behavior of the economy using FRB/US and assuming, for starters, that Fed officials always use equation (6.2) to set the funds rate. In our main set of simulations, we use the version of FRB/US in which the private sector forms its expectations by using a backward-looking vector autoregression; later on we also explore the alternative version in which expectations are formed in in a forward-looking or model consistent fashion.

The results are shown in Exhibit 6.2. In the case where Fed officials correctly perceive *r**, both the economy and the funds rate evolve in line with the median projection in the SEP as of the beginning of 2015. In particular, the first rate hike would have occurred in the middle of 2015, the pace of rate hikes would peak at 140bp per year, and the funds rate would converge smoothly to its terminal rate of 3.75% without overshooting.

However, in the case where Fed officials incorrectly perceive *r** to be higher than it really is, they would have hiked much earlier and more aggressively. The premature tightening would have aborted the decline in the unemployment rate, resulting in a failure to return inflation back to the 2% target. In turn, the weaker economy would prompt the FOMC to slow—and ultimately partly reverse—the normalization of the funds rate.

Conversely, in the case where Fed officials incorrectly perceive *r** to be lower than it really is, they hike much later. This leads to a bigger drop in unemployment to a level well below its structural rate, a slight overshooting of inflation above 2%, and a sizable overshooting of the funds rate as the FOMC reacts to the overheating of the economy. The upshot of these simulations is that perception errors around *r** introduce significant volatility into the economy's performance and the path of the funds rate.

6C. Fed policy with inertia

So what happens if Fed officials become more aware of the risk of perception errors and therefore introduce inertia into their reaction function? To gain intuition for the basic story, we first look at a simple example in which the FOMC decides to set the inertia term a_0 in equation (6.1) equal to 0.6 but keep all the other parameters of the reaction function unchanged. We later derive the optimal setting of a_0 based on a monetary policy loss function.

Exhibit 6.3 shows the consequences of setting a_0=0.6 in terms of the funds rate and the behavior of the economy, keeping everything else the same as in Exhibit 6.2. The paths for both the economy and the funds rate when the FOMC misperceives *r** are now more similar to the baseline case in which the FOMC correctly perceives *r**. This more robust performance is the direct consequence of the fact that the policy rule now puts less weight on *r**.

The other difference compared with Exhibit 6.2 is that the normalization for the funds rate now occurs later but ultimately more steeply. In effect, Fed officials wait until there is a stronger message from the behavior of inflation and employment that rate hikes are warranted. This greater patience results in a bigger drop in the unemployment rate and more upward pressure on inflation, and ultimately a greater need to tighten policy in order to limit the overheating of the economy. To make the contrast even clearer, Exhibit 6.4 compares the two paths of the funds rate in the benchmark case where Fed officials are correct in their perception of *r**. Under the inertial policy rule, the first rate hike occurs about six months later than under the non-inertial rule, the pace of rate hikes is about one-third faster, and the funds rate overshoots its terminal value by about 40bp.

6D. The optimal degree of inertia

Our discussion suggests that inertia has costs and benefits. On the one hand, reduced reliance on *r** leads to more robust economic outcomes. On the other hand, more inertia generally implies some degree of overshooting in the funds rate as well as the economy, which is undesirable. This trade-off suggests that there is an "optimal" degree of inertia at which the marginal benefit from greater inertia (i.e. more robust outcomes) is equal to the marginal cost of greater inertia (i.e. more overshooting).

What is this optimal degree? To answer this question, we use the intertemporal monetary policy loss function

$$Loss = \sum_{t=0}^{T} \beta^t \left(\left(u_t - u^*\right)^2 + \left(\pi_t - \pi^*\right)^2 + 0.5 * \left(\Delta i_t\right)^2 \right) \qquad (6.3)$$

where β is the discount factor and all other notation is as before. Equation (6.3) specifies the loss from a particular policy based on the discounted sum of future deviations from the Fed's goals for inflation and unemployment as well as quarter-to-quarter changes in the federal funds rate. In our benchmark loss function, we assume that unemployment and inflation misses are equally costly from the Fed's perspective, an assumption dubbed the "balanced approach" by Fed Chair Janet Yellen. We also assume that a 50bp quarter-to-quarter change in the funds rate is as costly as a ¼-point miss on either unemployment or inflation.[16] Finally, in evaluating the optimal degree of inertia, we assume that each of the three cases in Exhibit 6.1—i.e. that Fed officials are correct, too high, or too low in their perception of *r**—is equally likely.

Under these assumptions, Exhibit 6.5 shows that the optimal degree of inertia is equal to a_0=0.61 if the perception error is 150bp. Moreover, the optimal degree of inertia rises with the size of the perception errors around *r**, from a relatively negligible 0.16 if the error is 50bp to a very substantial 0.76 if the error is 250bp.[17] We therefore conclude that the risk of perception errors around r* creates a

[16] This assumption lies in between the very low aversion to changes in the funds rate in OW and the higher aversion in Yellen (2012). We explore the implications of varying it below.

[17] A different baseline for *r** (e.g. resulting from a different assumption about the baseline policy reaction function) would generate different optimal values for a_0. However, it would not change the result that greater uncertainty around *r** increases the optimal value for a_0.

strong case for a more inertial rule and a later but steeper normalization of the funds rate relative to the current SEP baseline.[18]

6E. Robustness

How robust is this result? We focus on two aspects of this question. First, we vary the monetary policy loss function in equation (6.3) with respect to the cost of quarter-to-quarter changes in the federal funds rate. Exhibit 6.6 shows that if we increase this cost from 0.5 to 1, as in the 2012 "optimal control" simulations by then-Vice Chair Yellen, the optimal inertia coefficient a_0 declines to 0.53. This is intuitive because a greater inherent cost of funds rate changes penalizes overshooting by more relative to our baseline assumption. Conversely, if we reduce the weight to 0—i.e. if we assume that the Fed only cares about its dual employment and inflation mandate and not about volatility in the funds rate—the optimal inertia coefficient a_0 increases to 0.75. Our assessment of these numbers is that our basic results seem fairly robust to plausible variation in the cost of funds rate changes.

Second, we switch from backward-looking VAR expectations to forward-looking model consistent expectations. Even leaving the aside the issue of uncertainty around *r**, this change in the model setup already favors a greater amount of inertia in the policy reaction function. The reason is that the private sector now anticipates future monetary policy moves, which enables the Fed to wait longer before having to step on the brakes. Beyond this, however, the basic qualitative point of our analysis is unchanged—greater uncertainty around *r** favors an even more inertial approach. As shown in Exhibit 6.7, an increase in the potential error from +/-50bp to +/-150bp and on to +/-250bp raises the optimal value of a_0 substantially.

6F. Conclusions and extensions

Summing up, our analysis confirms the findings by OW that uncertainty around *r** provides a rationale for making the policy reaction function more inertial, i.e. for putting greater weight on the lagged level of the funds rate relative to the uncertain estimate of *r**. This result seems to be robust to changes in the loss function and the expectations formation process. In the context of monetary policy decisions in 2015 and 2016, our finding implies that policymakers who are concerned about *r** uncertainty may want to adopt a later but steeper path for normalizing the funds rate. Although such a later but steeper path tends to result in an overshooting of the funds rate, our welfare calculations suggest that such an overshooting is a price worth paying for a more robust approach to policy normalization that limits the risk of either exiting prematurely or belatedly because of misperceptions around the true value of *r**.

It is worth noting that uncertainty around r* is only one potential rationale for a later but steeper normalization path. In fact, the work of OW suggests that the policy implications of uncertainty around the structural unemployment rate *u**—or more broadly around the correct measure of labor market utilization—are similar to those of uncertainty around the equilibrium interest rate *r**. In both

[18] Haldane (2015) also concludes that delayed normalization is preferred.

cases, there is an incentive to keep interest rates low until the behavior of the economy—and particularly the behavior of inflation—sends a strong signal that tightening is appropriate.

Although it is not the focus of the present study, uncertainty around the size of the employment gap is also a very important issue at the current juncture. Some indicators of labor market utilization such as the headline unemployment rate and the job openings rate are already at or near normal levels, but others such as the broad underemployment rate U6 and the growth rate of hourly wages are still signaling substantial labor market slack. In our formal analysis, we assumed for simplicity that the gap between the headline unemployment rate and the structural unemployment rate embedded in FRB/US provides an accurate measure of how far the FOMC is from its goal of maximum employment. But this is probably not realistic, and introducing uncertainty into this assessment would likely reinforce our conclusion in favor of a later but steeper normalization path. Presumably the lack of policy ammunition for dealing with a downturn—the zero lower bound and the fiscal policy impasse in Washington—also argues in favor of a later but steeper path.

Another important question is whether there are other ways, besides introducing inertia, of making monetary policy rules more robust to uncertainty around the equilibrium rate. One possibility is to place some weight on changes in financial conditions. For example, if it is true that changes in r^* partly reflect variation in the marginal product of capital, and if the equity market "sniffs out" such variation from corporate earnings results before it becomes visible in the macroeconomic data, a change in equity prices might provide an early indication of a change in r^*. If Fed officials placed some weight on changes in financial conditions when setting the funds rate, they would automatically incorporate this information and might thereby improve upon a policy rule that relies on signals from the macroeconomic data alone. Of course, since the relationship between the equity market and r^* is likely to be a noisy one, there is also the risk that an excessively large weight on equity market moves will cause Fed officials to overreact to the fleeting ups and downs of the markets. Exploring the resulting cost-benefit calculations would be a useful avenue for future research.

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Exhibit 1.1. Economic projections of Federal Reserve Board members and Federal Reserve Bank presidents.

	2014	2015	2016	2017	Longer run
GDP					
December 2015	—	2.1	2.4	2.2	2.0
December 2014	2.35	2.80	2.75	—	2.15
December 2012	3.25	3.35	—	—	2.40
Unemployment rate					
December 2015	—	5.0	4.7	4.7	4.9
December 2014	5.80	5.25	5.10	—	5.35
December 2012	7.05	6.30	—	—	5.60
PCE inflation					
December 2015	—	0.4	1.6	1.9	2.0
December 2014	1.25	1.30	1.85	—	2.0
December 2012	1.75	1.85	—	—	2.0
Core PCE inflation					
December 2015	—	1.3	1.6	1.9	—
December 2014	1.55	1.65	1.85	—	—
December 2012	1.80	1.90	—	—	—
Fed funds rate					
December 2015	—	0.4	1.4	2.4	3.5
December 2014	0.13	1.13	2.50	—	3.75
December 2012	0.25	1.00	—	—	4.0

Note: Q4/Q4 percent changes, except unemployment (4Q average) and the fed funds rate (eop). Middle of the central tendency range, except fed funds rate (median).

Exhibit 2.1. Country mnemonics and start dates for real interest rate series.

	Mnemonic (1)	Country (2)	Sample Start, Annual Data (3)	Sample Start, Quarterly Data (4)
(1)	AUS	Australia	1893	1971:2
(2)	AUT	Austria	n.a.	1971:2
(3)	BEL	Belgium	n.a.	1971:2
(4)	CAN	Canada	1900	1971:2
(5)	CHE	Switzerland	1912	1981:2
(6)	DEU	Germany	1858	2002:2
(7)	DNK	Denmark	1864	1979:2
(8)	ESP	Spain	1874	1977:2
(9)	FIN	Finland	1946	1971:2
(10)	FRA	France	1861	1971:2
(11)	GBR	United Kingdom	1858	1971:2
(12)	IRL	Ireland	n.a.	2001:2
(13)	ITA	Italy	1893	1971:2
(14)	JPN	Japan	1900	1971:2
(15)	KOR	South Korea	n.a.	1981:2
(16)	NLD	Netherlands	1858	1971:2
(17)	NOR	Norway	1858	1971:2
(18)	PRT	Portugal	1960	1971:2
(19)	SWE	Sweden	1858	1982:2
(20)	USA	USA	1858	1958:1
(21)	NZL	New Zealand	1939	n.a.

Exhibit 2.2. U.S. and U.K. nominal interest rate, inflation rate, persistence of inflation, and ex-ante real rate, annual 1800-2014.

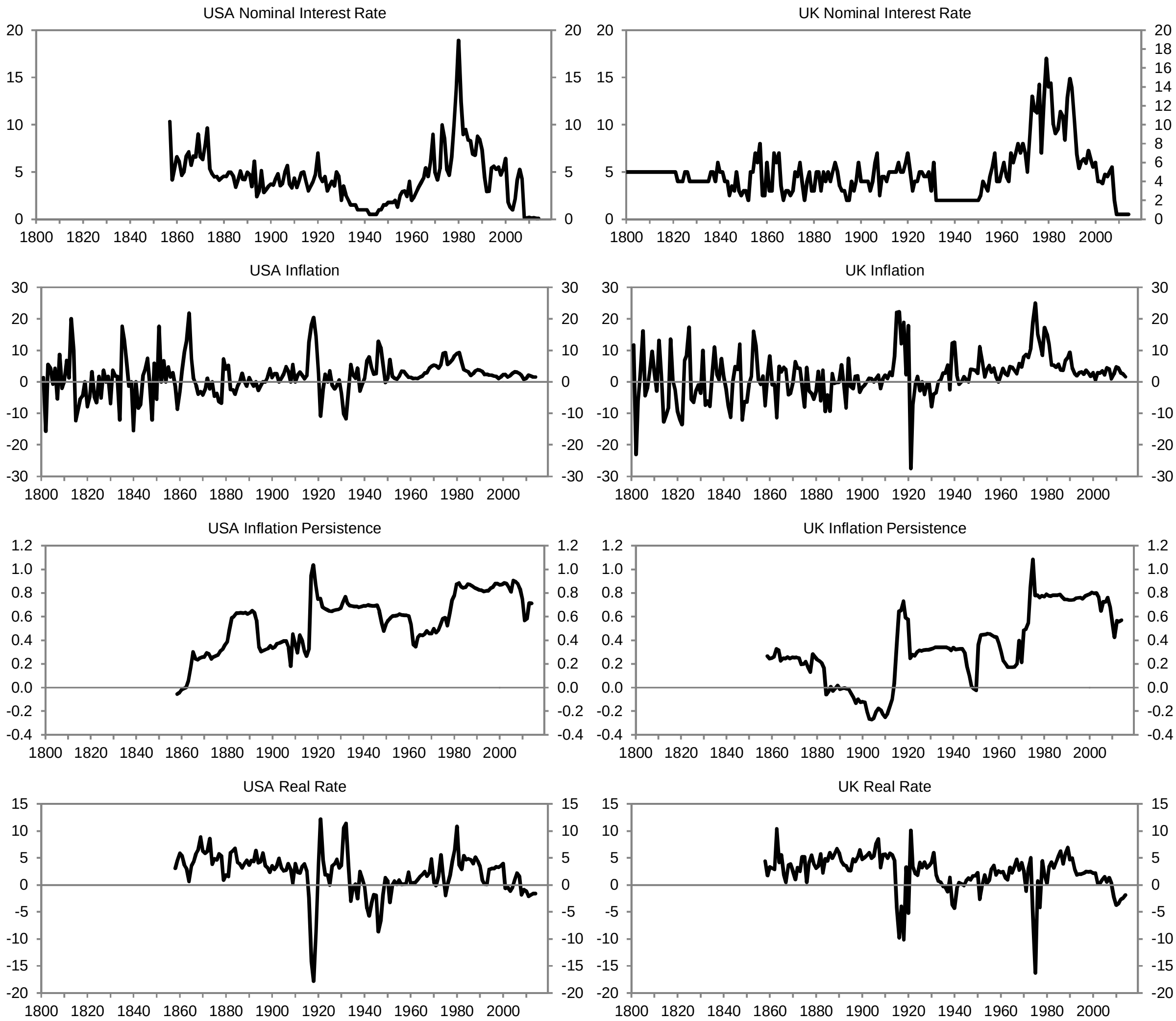


Exhibit 2.3. Nominal interest rates for 15 different countries, annual 1858-2014.

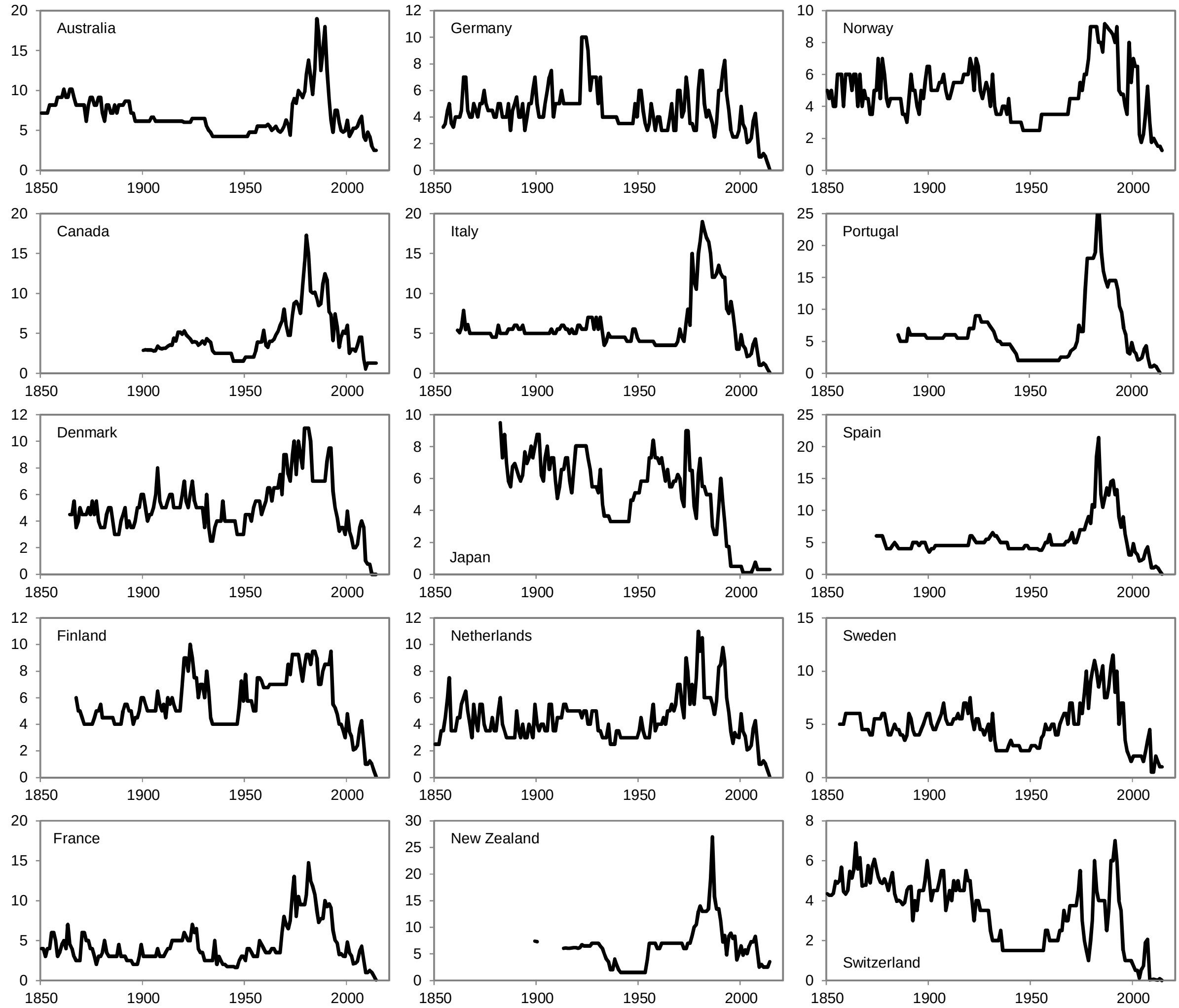


Exhibit 2.4. Inflation rate for 15 different countries, annual 1858-2014.

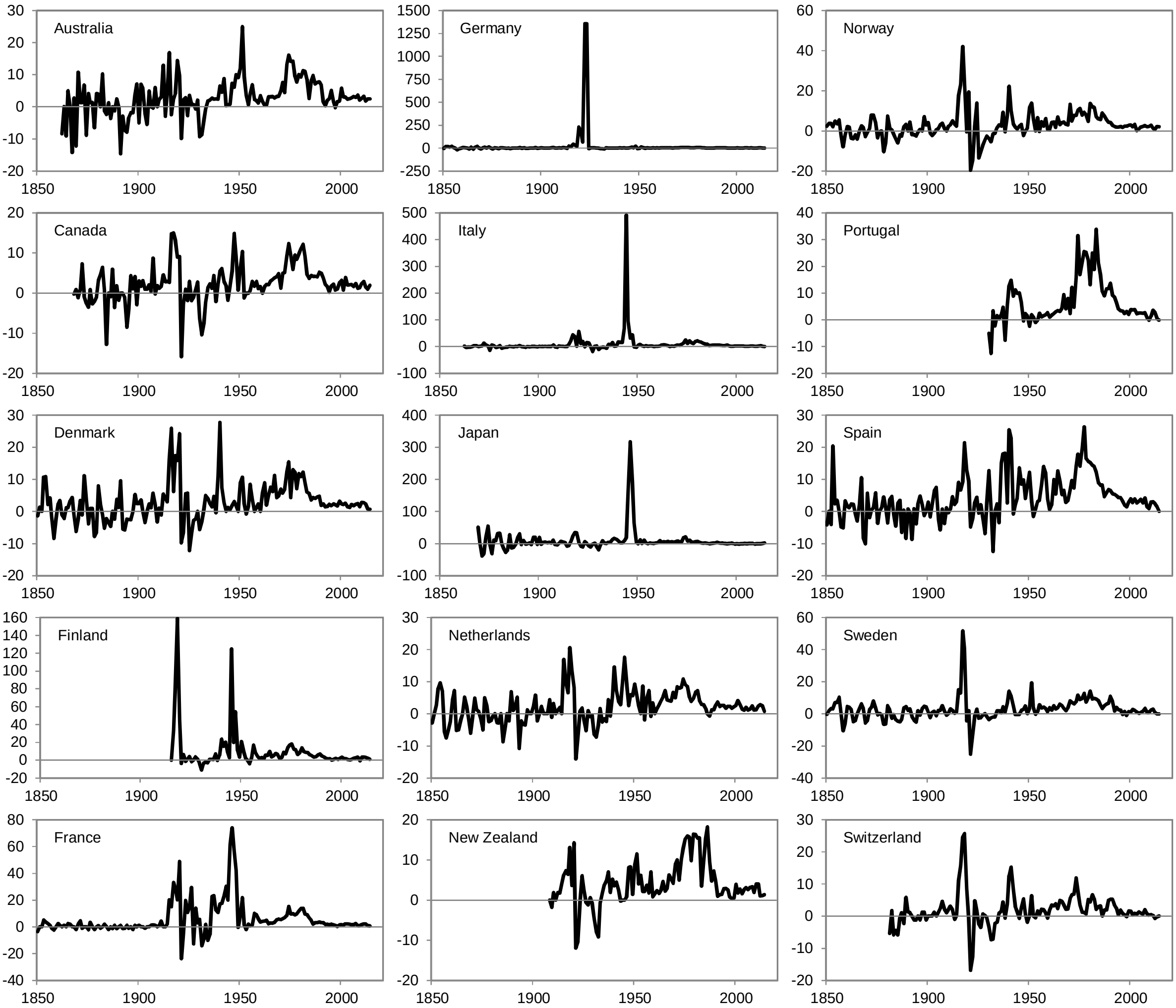


Exhibit 2.5. Inflation persistence for 15 different countries, annual 1858-2014.

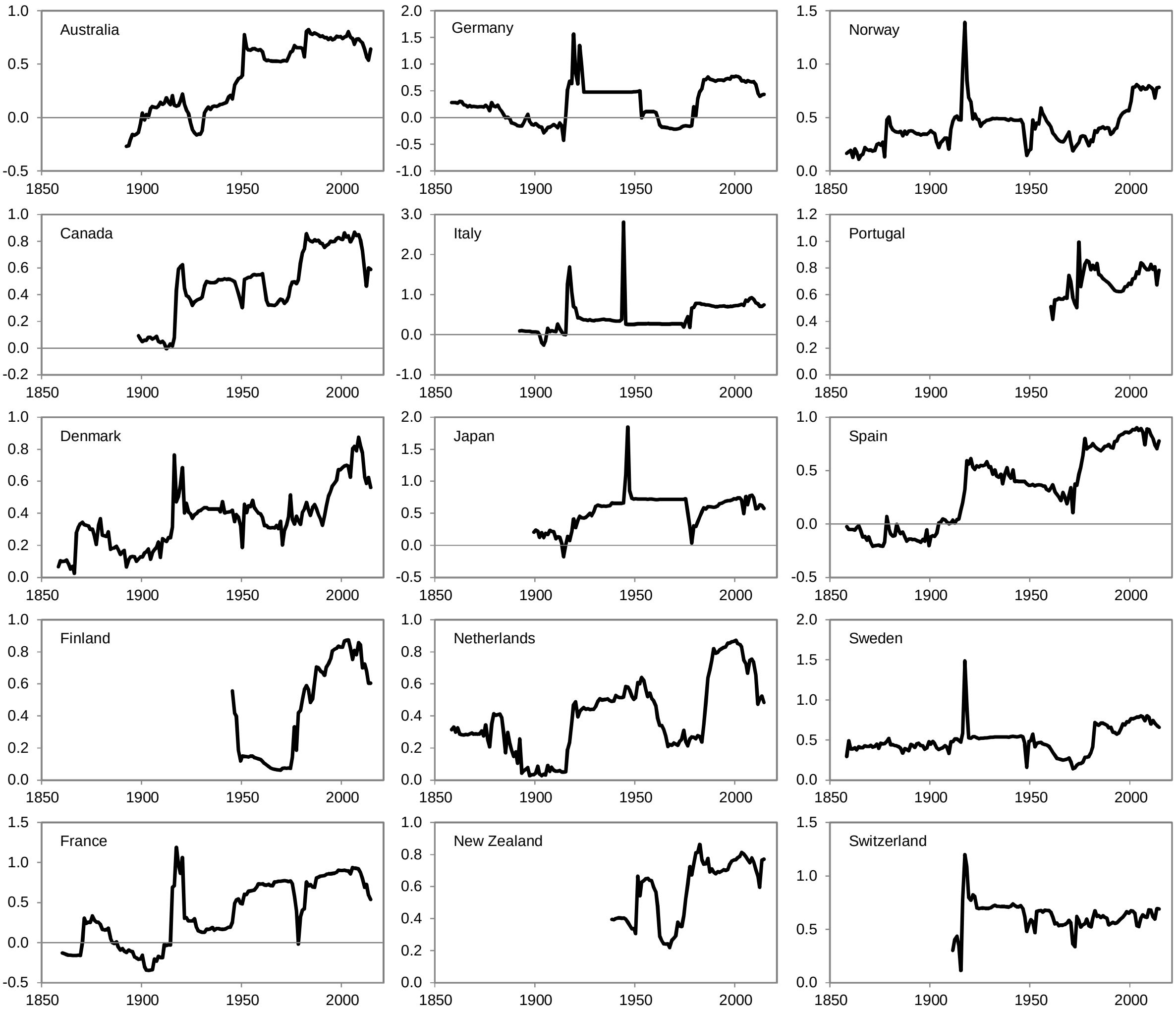


Exhibit 2.6. Ex-ante real interest rate for 15 different countries, annual 1858-2014.

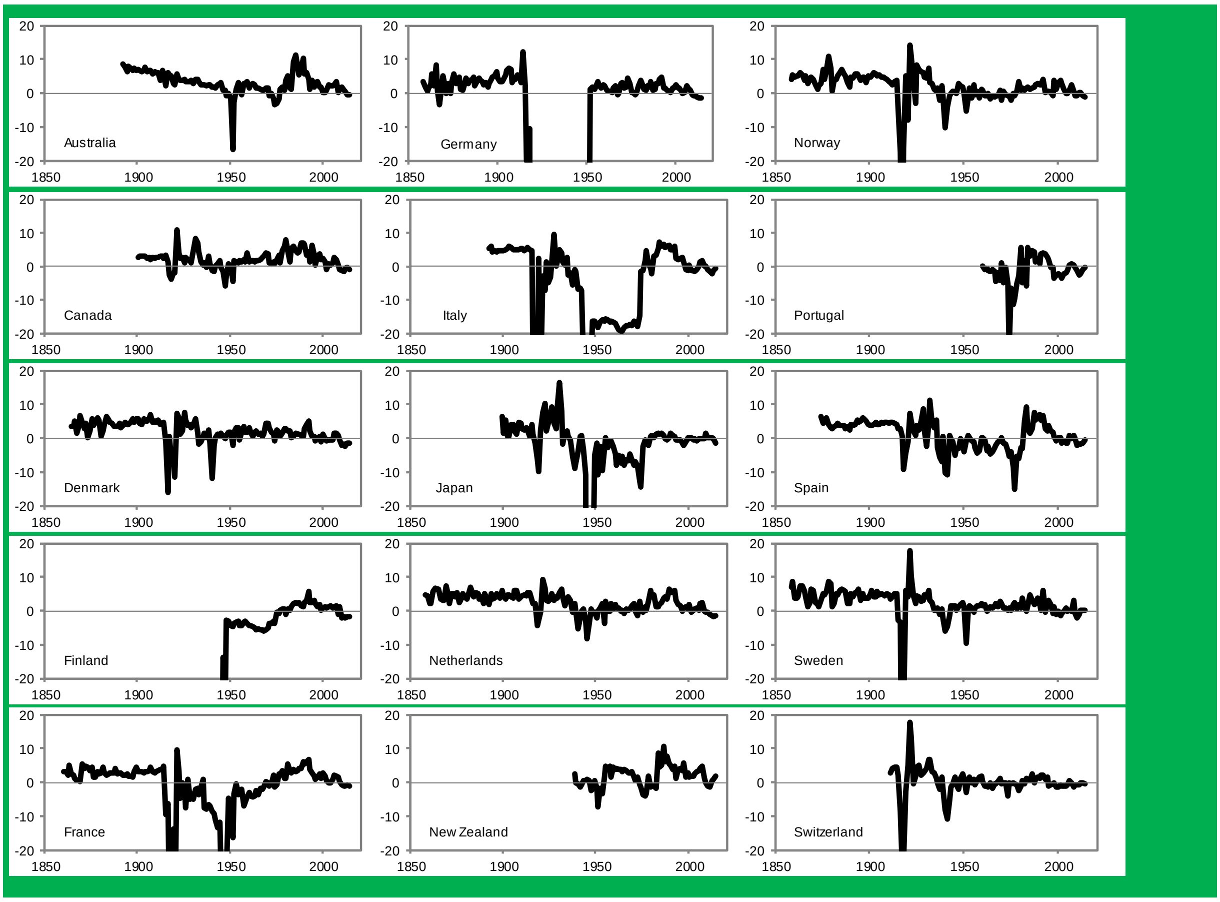


Exhibit 2.7. U.S. ex-ante real interest rate as inferred from annual and quarterly data.

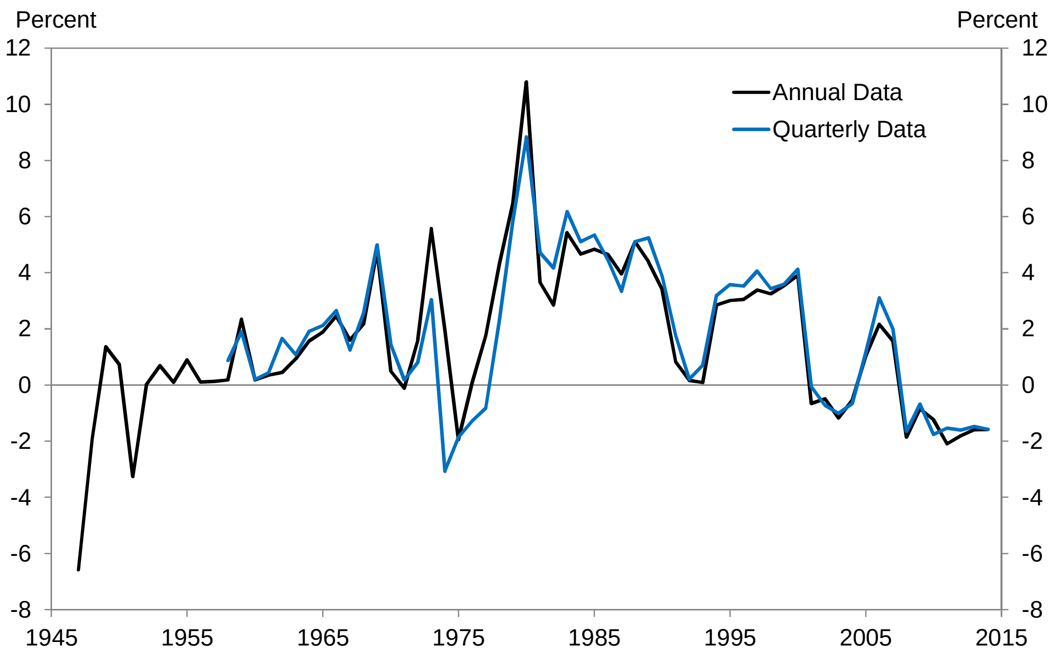


Exhibit 2.8. Expected and actual inflation for quarterly U.S. data, 1958:2 to 2014:3.

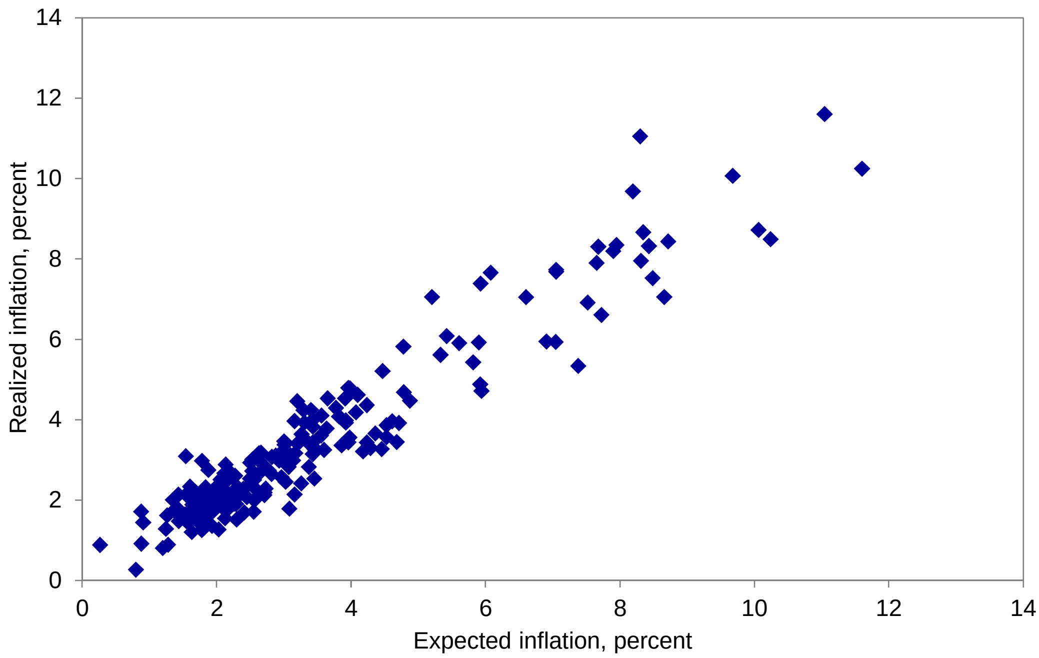


Exhibit 2.9. Nominal U.S. interest rate (average fed funds rate for last month of the quarter) and ex-ante real interest rate, 1958:1-2014:3.

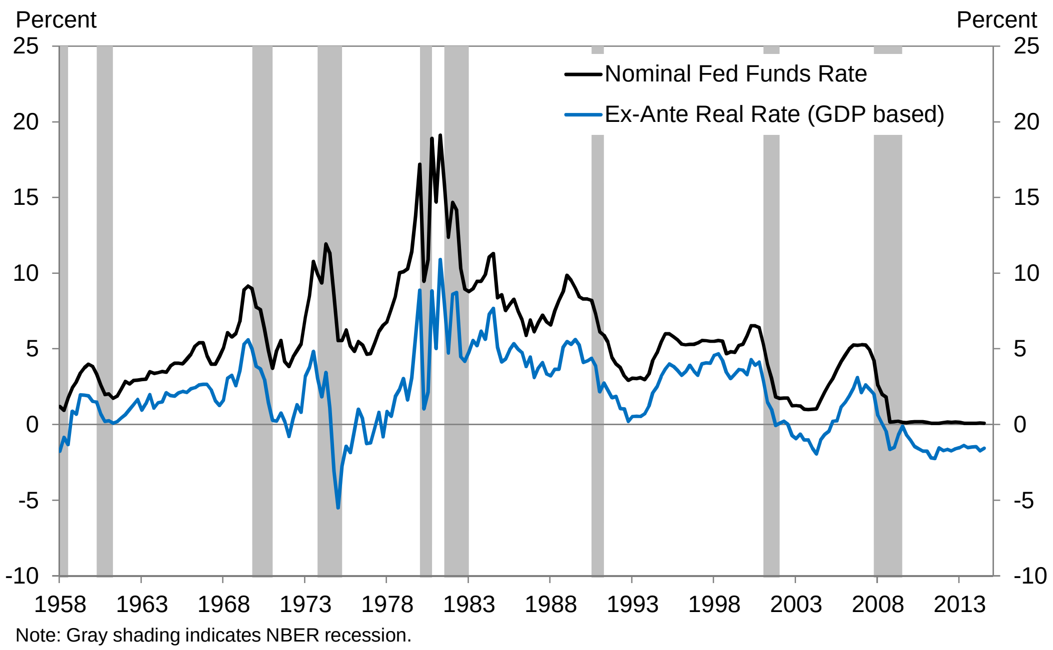


Exhibit 2.10. Basic statistics for U.S. quarterly ex-ante real rates, 1958:2-2014:3.

	r	i	$E_t\pi_{t+1}$	π
Mean	1.95	5.27	3.32	3.30
S.D.	2.55	3.60	2.12	2.32

Notes:

1. r=ex-ante real rate=i-$E_t\pi_{t+1}$, i=nominal rate, π=inflation, $E_t\pi_{t+1}$=expected inflation.
2. Expected inflation is computed from a univariate AR(4) in inflation estimated from rolling regressions. For t=1968:2 through t=2014:3, the rolling sample size is 80 quarters, with the most distant observation dropped each time a new observation is added to the end of the sample. For t=1958:2 to t=1967:4, the sample starts at 40 quarters and then grows to 79 quarters, with no observations dropped each time a new observation is added to the end of the sample.

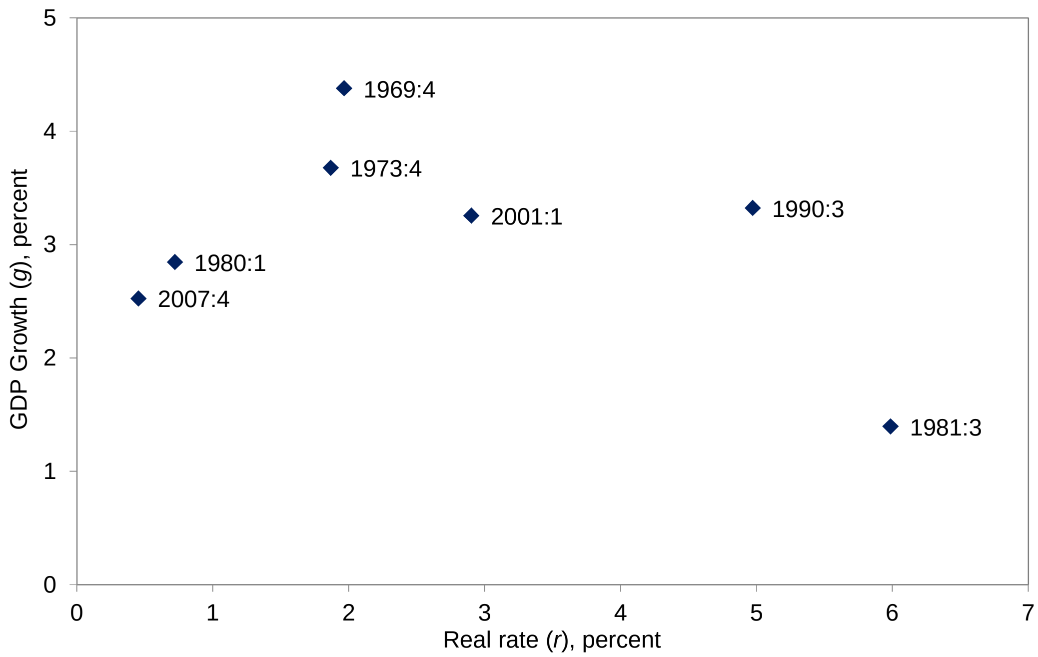
Exhibit 3.1. U.S. GDP growth and ex-ante real rate *r*, peak-to-peak averages.

	QUARTERLY			ANNUAL		
	Peak Date (1)	GDP growth (2)	*r* (3)	Peak Date (4)	GDP growth (5)	*r* (6)
(1)	2007:4	2.52	0.45	2007	2.66	0.43
(2)	2001:1	3.25	2.90	2001	3.16	2.13
(3)	1990:3	3.32	4.97	1990	3.36	4.36
(4)	1981:3	1.40	5.99	1981	1.16	7.23
(5)	1980:1	2.84	0.72	1979	2.93	2.10
(6)	1973:4	3.68	1.87	1973	3.52	1.89
(7)	1969:4	4.38	1.97	1969	4.62	1.80
(8)	1960:2	2.86	n.a.	1960	2.82	0.90
(9)	1957:3	2.43	n.a.	1957	2.63	0.30
(10)	1953:2	5.39	n.a.	1953	4.83	-0.09
(11)				1948	-2.58	-4.77
(12)				1944	9.97	-1.70
(13)				1937	0.67	2.89
(14)				1929	2.87	3.93
(15)				1926	3.56	1.79
(16)				1923	5.48	6.22
(17)				1920	-2.54	-2.35
(18)				1918	3.64	-5.60
(19)				1913	4.20	2.64
(20)				1910	1.95	2.39
(21)				1907	3.74	3.11
(22)				1903	4.51	3.68
(23)				1899	4.72	3.75
(24)				1895	2.69	4.90
(25)				1892	3.90	4.42
(26)				1890	2.33	4.07
(27)				1887	2.47	4.87
(28)				1882	5.04	3.84
(29)				1873	5.20	6.73

Notes:

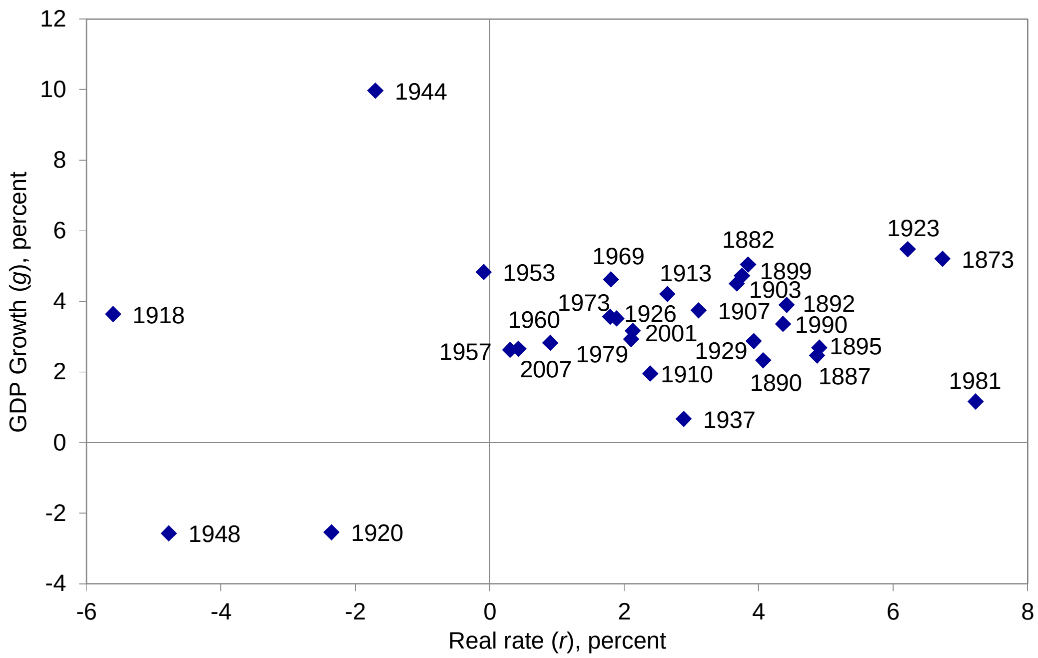
1. Each entry is a peak to peak average, expressed at annual rates. For example, the 2.52 figure in row (1), column (2), means that average GDP growth over the 27 quarters from 2001:2 through 2007:4 was 2.52%; the 0.45 figure in column (3) is the corresponding value for the ex-ante real rate *r* over this period.
2. The ex-ante real rate *r* is the short term nominal policy rate minus next quarter's (column (3)) or next year's (column (6)) expected inflation. Expected inflation is computed from an autoregression in log differences of the GDP deflator, using rolling samples. See Section 2 for details.
3. The nominal policy rate is end of period Federal funds rate from 1954 forward. See text for sources of nominal rates in earlier periods. Real GDP growth data from Balke and Gordon (1989) prior to 1930, from FRED afterwards. Business cycle peak dates from NBER and Zarnowitz (1997).

Exhibit 3.2. Peak-to-peak average real GDP growth versus average *r*, quarterly data, 1969:4-2007:4.



Note: This figure plots the quarterly data in Exhibit 3.1.

Exhibit 3.3. Peak-to-peak average real GDP growth versus average *r*, annual data, 1873-2007.



Note: This figure plots the annual data in Exhibit 3.1.

Exhibit 3.4. Correlation of U.S. GDP growth with *r*, peak-to-peak averages.

	Start (1)	End (2)	No. of Peaks (3)	Freq. (4)	Specification (5)	Correlation (6)
(1)	1969:4	2007:4	7	Q	Baseline (Exhibit 3.2)	-0.40
(2)	1969:4	2007:4	6	Q	Omit 1981:3	0.32
(3)	1969:4	2007:4	6	Q	Potential GDP growth, omit 1981:3	-0.01
(4)	1960:2	2007:4	7	Q	Ex-post real interest rate, omit 1981:3	0.17
(5)	1873	2007	29	A	Baseline (Exhibit 3.3)	0.23
(6)	1873	2007	28	A	Omit 1981	0.30
(7)	1873	2007	27	A	Omit 1920, 1948	-0.23
(8)	1873	2007	26	A	Omit 1937, 1944, 1948	0.29
(9)	1953	2007	9	A	Postwar, omit 1981	-0.04
(10)	1969	2007	6	A	Quarterly sample, omit 1981	0.18
(11)	1873	2007	29	A	Romer (1989) data used 1870-1929	0.21
(12)	1873	2007	29	A	Year ahead GDP growth	0.10

Notes:

1. In column (6), correlations are computed with a peak to peak average considered a single observation. In line (1), for example, the value of -0.40 is the sample correlation between the 7 values of GDP growth and of the real rate given in columns (2) and (3) of Exhibit 3.1.
2. In column (4), "A" denotes annual data, "Q" quarterly data.

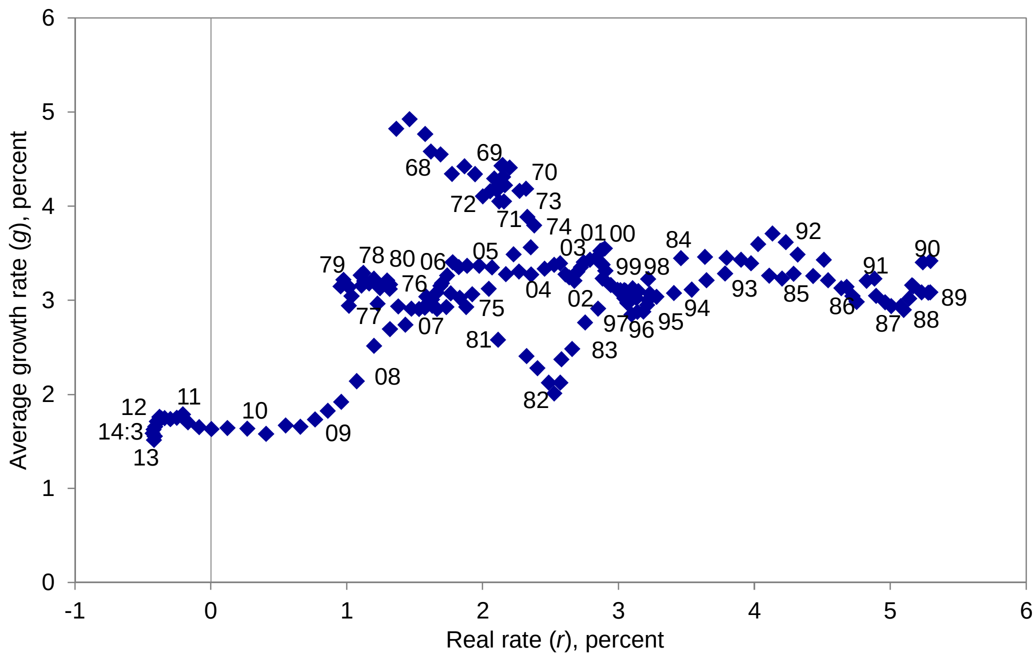
Exhibit 3.5. Correlation of U.S. GDP growth with *r,* overlapping 10-year averages.

	Start (1)	End (2)	No. of Peaks (3)	Freq. (4)	Specification (5)	Correlation (6)
(1)	1968:1	2014:3	187	Q	40Q backward averages (Exhibit 3.6)	0.39
(2)	1968:1	2007:4	160	Q	Omit 2008:1-2014:3	-0.19
(3)	1968:1	2014:2	186	Q	40 Q backward avgerages, ex-post r	0.27
(4)	1879	2014	136	A	10 year backward averages (Exhibit 3.7)	-0.25
(5)	1955	2014	60	A	Post-World War II	0.18
(6)	1889	2014	114	A	10 yr. backward avg, omit 1930-1950	0.31

Notes:

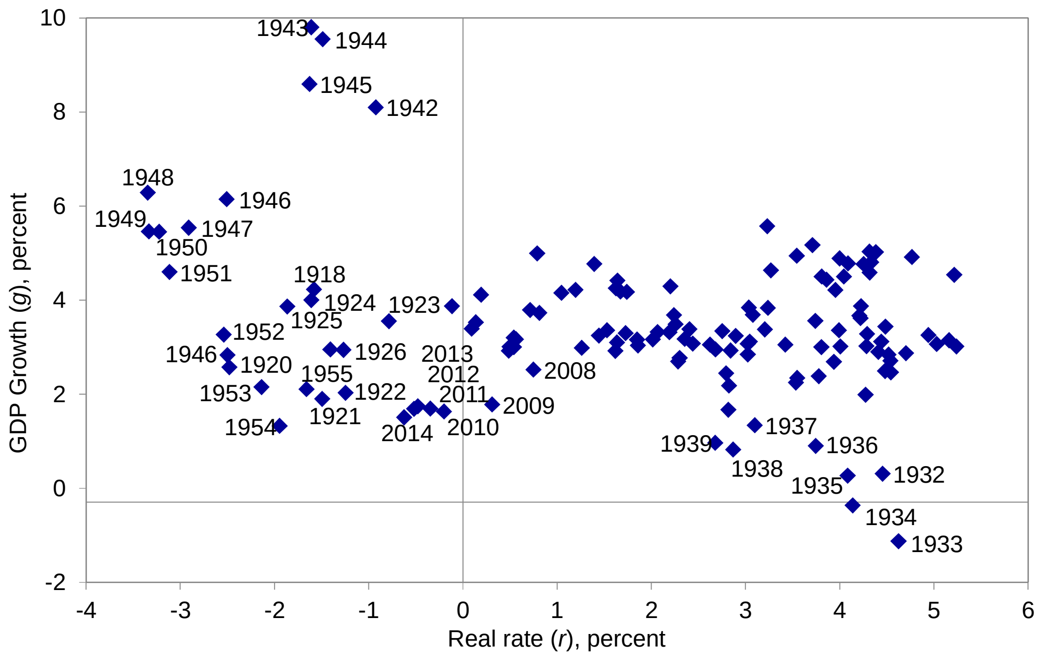
1. The units of observation are overlapping 10 year or 40 quarter averages. In row (1), for example, the first of the 187 observations is average GDP growth and average *r* over the 40 quarters from 1958:2-1968:1; the last is average GDP growth and average *r* over the 40 quarters from 2004:4-2014:3.
2. For annual data, 2014 data only use data from the first three quarters of 2014.

Exhibit 3.6. GDP growth versus *r*: 40-quarter backward moving averages, 1968:1-2014:3.



Note: The two digit label identifies the observation corresponding to the fourth quarter of that year. For example, “08” labels the 2008:4 observation: over the 40 quarter period 1999:1-2008:4, the average value of *r* and GDP growth were average *r*=1.08, average GDP growth=2.14. See notes to Exhibit 3.5.

Exhibit 3.7. GDP growth versus *r*: 10-year backward moving averages, 1879-2014.



Note: See notes to Exhibit 3.5.

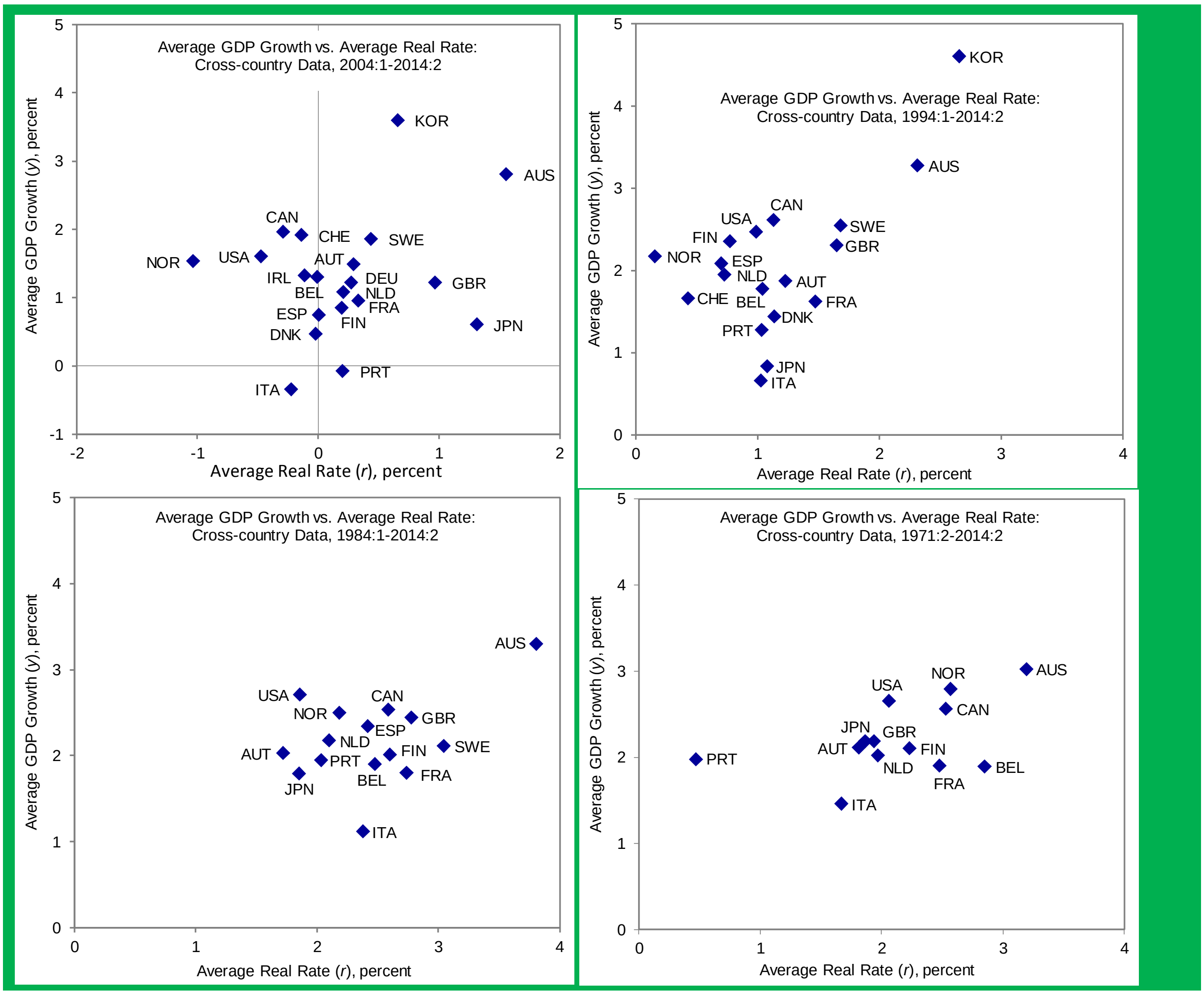
Exhibit 3.8. Cross-country GDP growth and *r*, multiyear averages.

	(1) Country	2004:1-2014:2 (2) GDP	(3) *r*	1994:1-2014:2 (4) GDP	(5) *r*	1984:1-2014:2 (6) GDP	(7) *r*	1971:2-2014:2 (8) GDP	(9) *r*
(1)	AUS	2.81	1.56	3.28	2.32	3.30	3.81	3.20	3.03
(2)	AUT	1.49	0.29	1.87	1.23	2.03	1.72	1.81	2.11
(3)	BEL	1.30	-0.01	1.78	1.04	1.90	2.48	2.85	1.89
(4)	CAN	1.97	-0.29	2.62	1.13	2.53	2.59	2.53	2.56
(5)	CHE	1.91	-0.14	1.67	0.43				
(6)	DEU	1.22	0.97						
(7)	DNK	0.47	-0.02	1.44	1.14				
(8)	ESP	0.75	0.01	2.09	0.70	2.34	2.42		
(9)	FIN	0.86	0.20	2.36	0.77	2.01	2.60	2.23	2.10
(10)	FRA	0.96	0.33	1.63	1.47	1.80	2.74	2.48	1.90
(11)	GBR	1.22	0.28	2.31	1.65	2.45	2.78	1.93	2.19
(12)	IRL	1.33	-0.11						
(13)	ITA	-0.34	-0.22	0.66	1.03	1.12	2.38	1.67	1.46
(14)	JPN	0.61	1.31	0.83	1.08	1.79	1.85	1.87	2.19
(15)	KOR	3.60	0.66	4.60	2.66				
(16)	NLD	1.08	0.21	1.95	0.73	2.18	2.10	1.97	2.02
(17)	NOR	1.53	-1.03	2.17	0.16	2.50	2.18	2.57	2.80
(18)	PRT	-0.08	0.20	1.28	1.03	1.95	2.03	0.47	1.97
(19)	SWE	1.86	0.44	2.54	1.69	2.12	3.05		
(20)	USA	1.60	-0.48	2.47	0.99	2.71	1.86	2.06	2.66
Corr		0.23		0.63		0.42		0.50	

Notes:

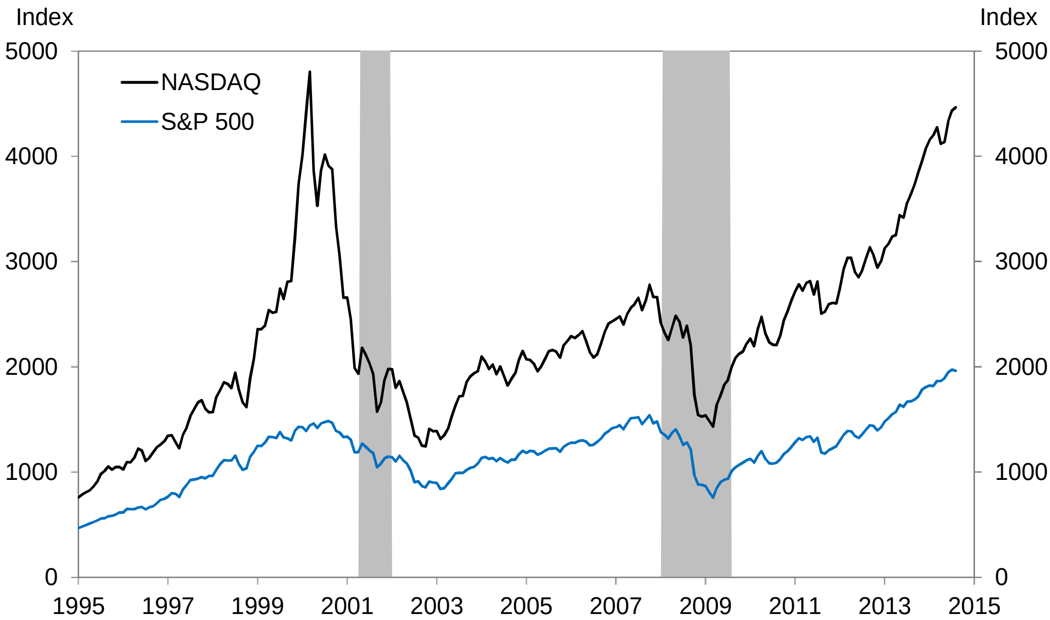
1. Each entry is an average of quarterly data over the indicated period. For example, the 2.81 figure in column (2), row (1), means that average GDP growth in Australia over the 42 quarters from 2004:1 through 2014:2 was 2.81%; the 1.56 figure in column (3) is the corresponding value for the ex-ante real rate *r* over this period.
2. The ex-ante real rate *r* is a nominal policy rate minus next quarter's expected inflation. Expected inflation is computed from an autoregression in inflation, using rolling samples. See text for details.
3. Real, quarterly GDP data is from the OECD. See text for sources of nominal policy rates and inflation.

Exhibit 3.9. Cross-country relations between GDP growth and *r* over selected samples.



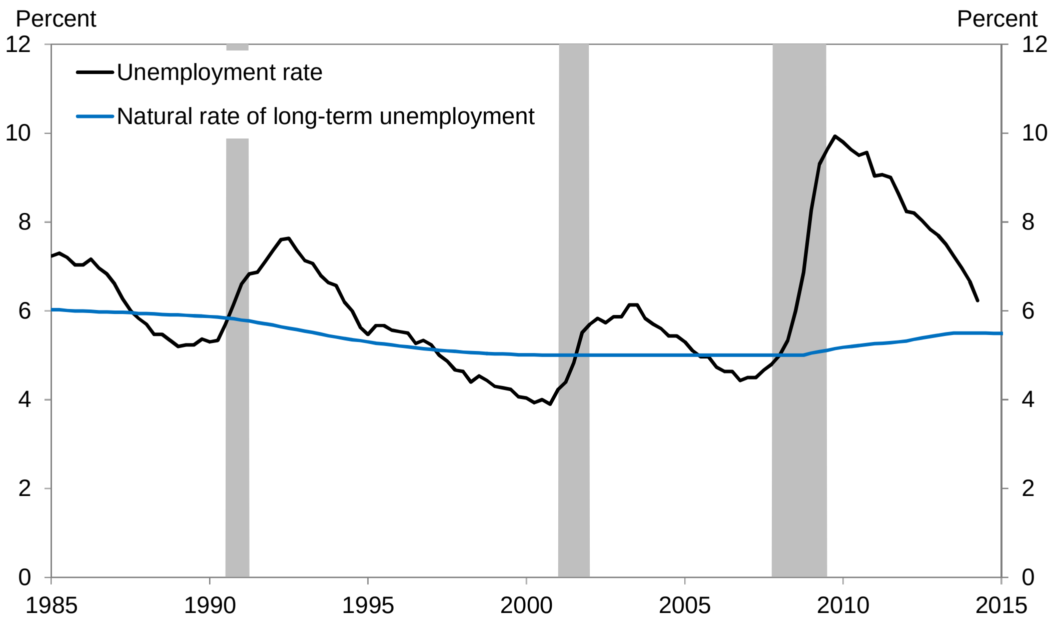
Note: This exhibit plots the data listed in Exhibit 3.8. See notes to Exhibit 3.8. Mnemonics for country names are given in Exhibit 2.1.

Exhibit 4.1. Stock prices, 1995-2014.



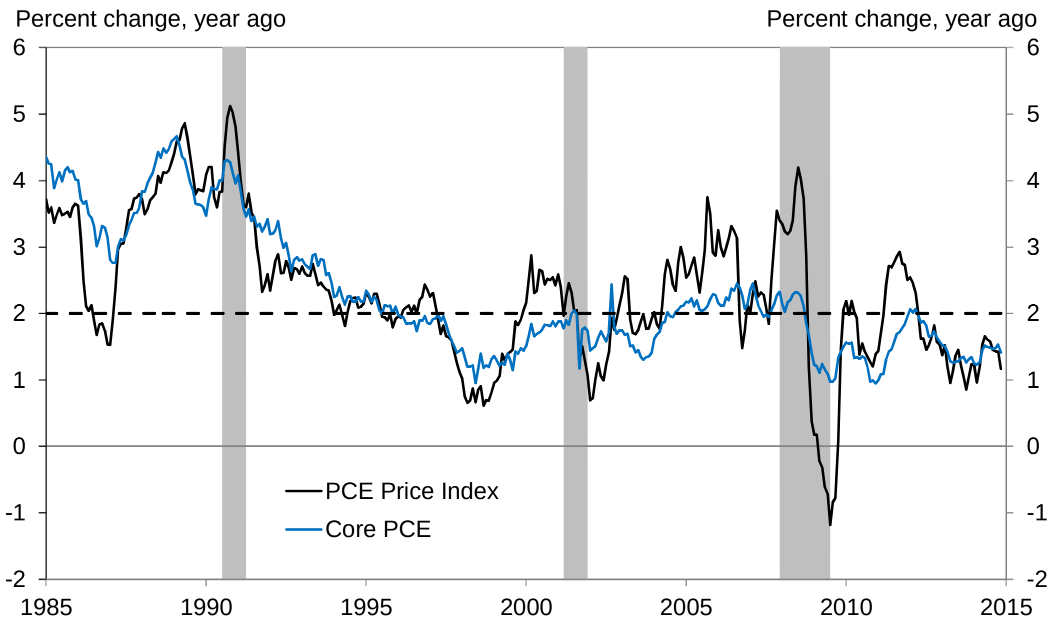
Note: Gray shading indicates NBER recession.

Exhibit 4.2. Unemployment rate, 1985-2014.



Note: Gray shading indicates NBER recession.

Exhibit 4.3. Inflation measures, 1985-2014.



Note: Gray shading indicates NBER recession.

Exhibit 4.4. Fed funds rate and inflation, 1980-2014.

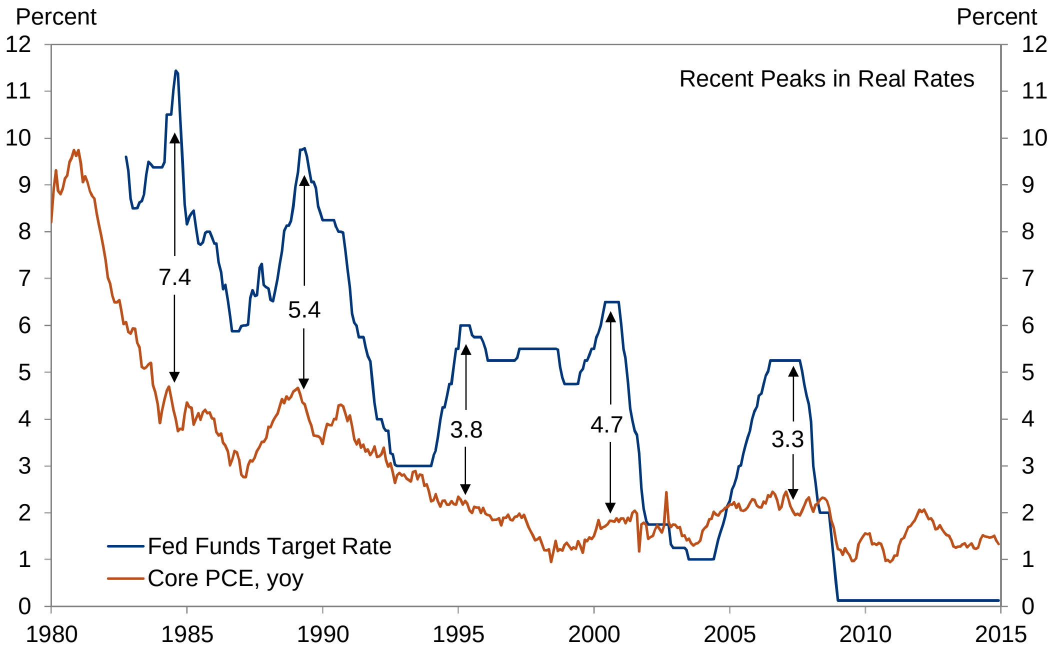
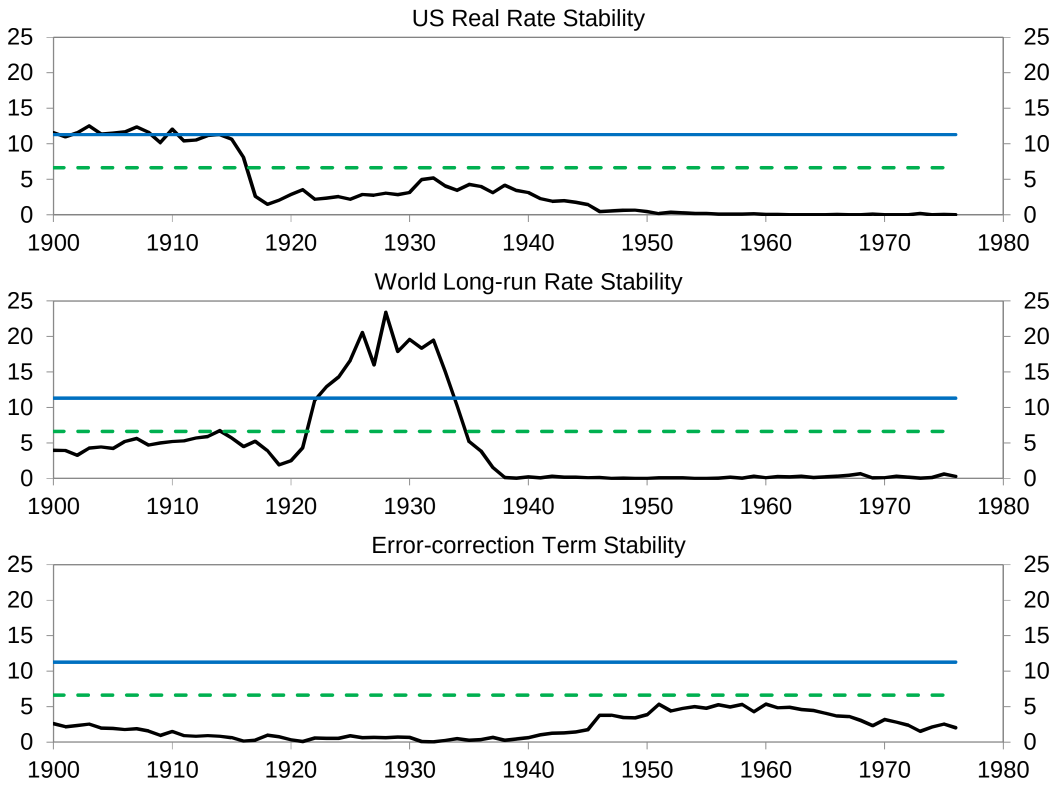


Exhibit 4.5. Nominal rates, inflation rates, and ex-ante real rates over postwar business expansions.

	Funds rate average over full cycle			At Full	Peak Real Funds
Cycle	Nominal	Inflation	Real	Employment	Rate
3Q 1960 - 4Q 1969	4.26	2.48	1.97	1.44	5.59
1Q 1970 - 4Q 1973	6.23	5.11	1.87	0.18	4.84
1Q 1974 - 3Q 1981	9.39	7.64	1.74	0.55	10.91
4Q 1981 - 3Q 1990	8.80	3.51	4.97	3.34	8.73
4Q 1990 - 1Q 2001	5.04	2.04	2.90	4.01	4.68
2Q 2001 - 4Q 2007	2.92	2.43	0.45	0.25	3.10

Exhibit 5.1. $\chi^2(1)$ statistic for test of null hypothesis of stability against the alternative of a break in the mean at the indicated date, 1900-1976.



Note: Dashed green: 1% critical value if test were only performed at a single date (6.63). Solid blue: 1% critical value if the maximal statistic over the range 1900-1976 were used (11.28, from Andrews (2003)). Top panel: U.S. real interest rate $r_{US,t}$; middle panel: long-run world rate ℓ_t; bottom panel: difference between U.S. real rate and long-run world rate $(r_{US,t} - \ell_t)$.

Exhibit 5.2. Long-run world real rate (ℓ_t, in blue) and U.S. ex-ante real rate ($r_{US,t}$, in black).

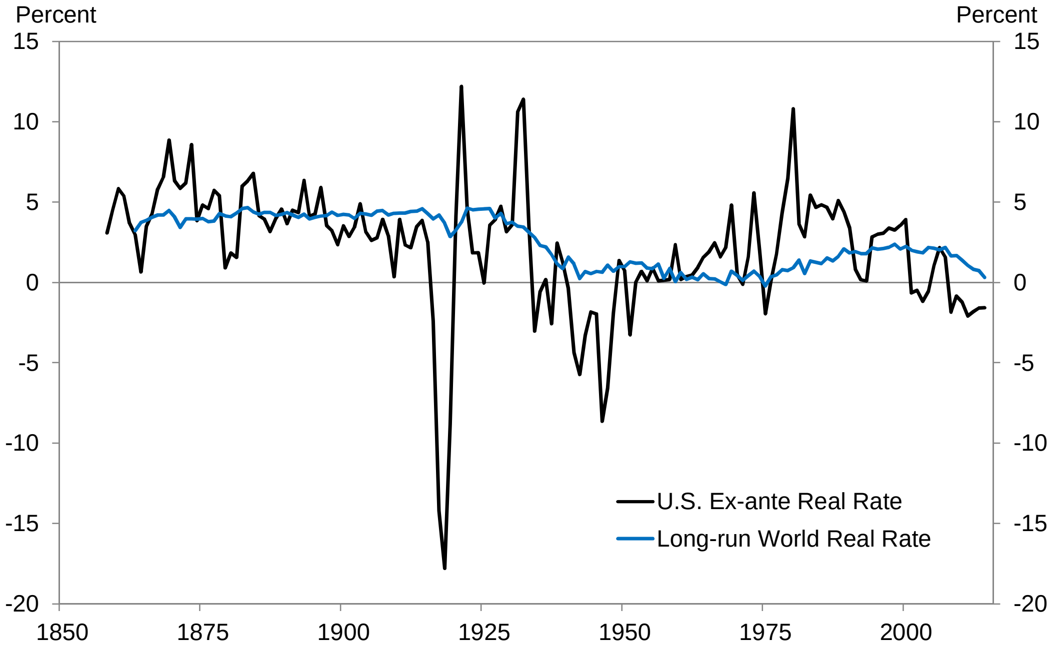


Exhibit 5.3. Forecasts for U.S. and long-run world real rates implied by (5.4) and (5.5) along with 90% confidence intervals for the latter.

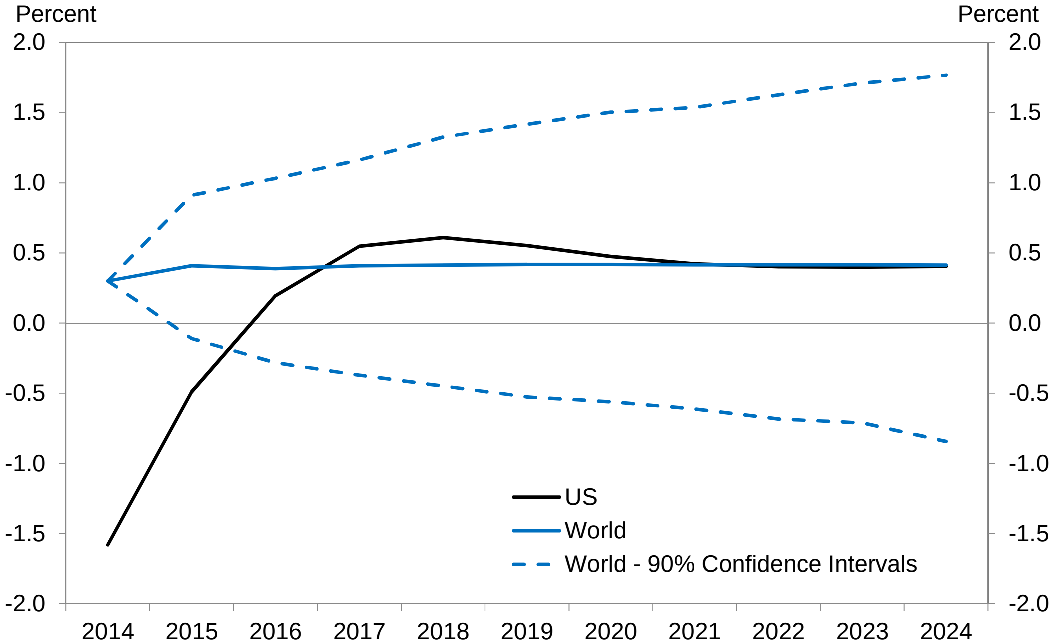


Exhibit 5.4. Predicted value (in black) for the U.S. real interest rate implied by equations (5.4)-(5.5) and forward rates (in blue) implied by the term structure of TIPS as of the end of 2014.

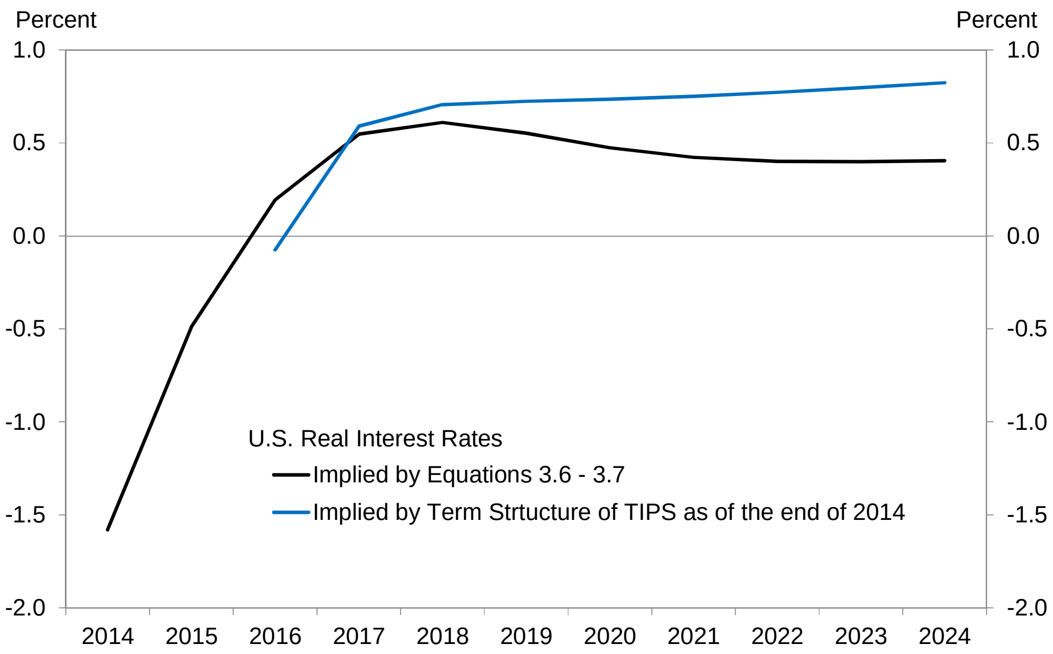


Exhibit 6.1. Baseline path around *r** and potential misperceptions.

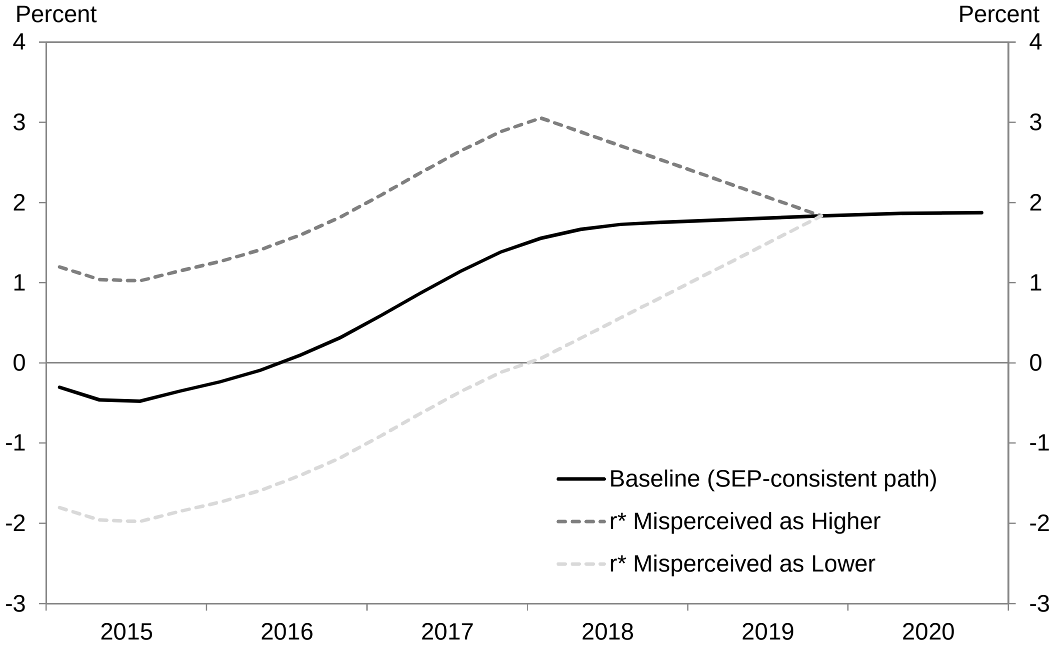


Exhibit 6.2. Behavior of the economy with perception errors but no inertia.

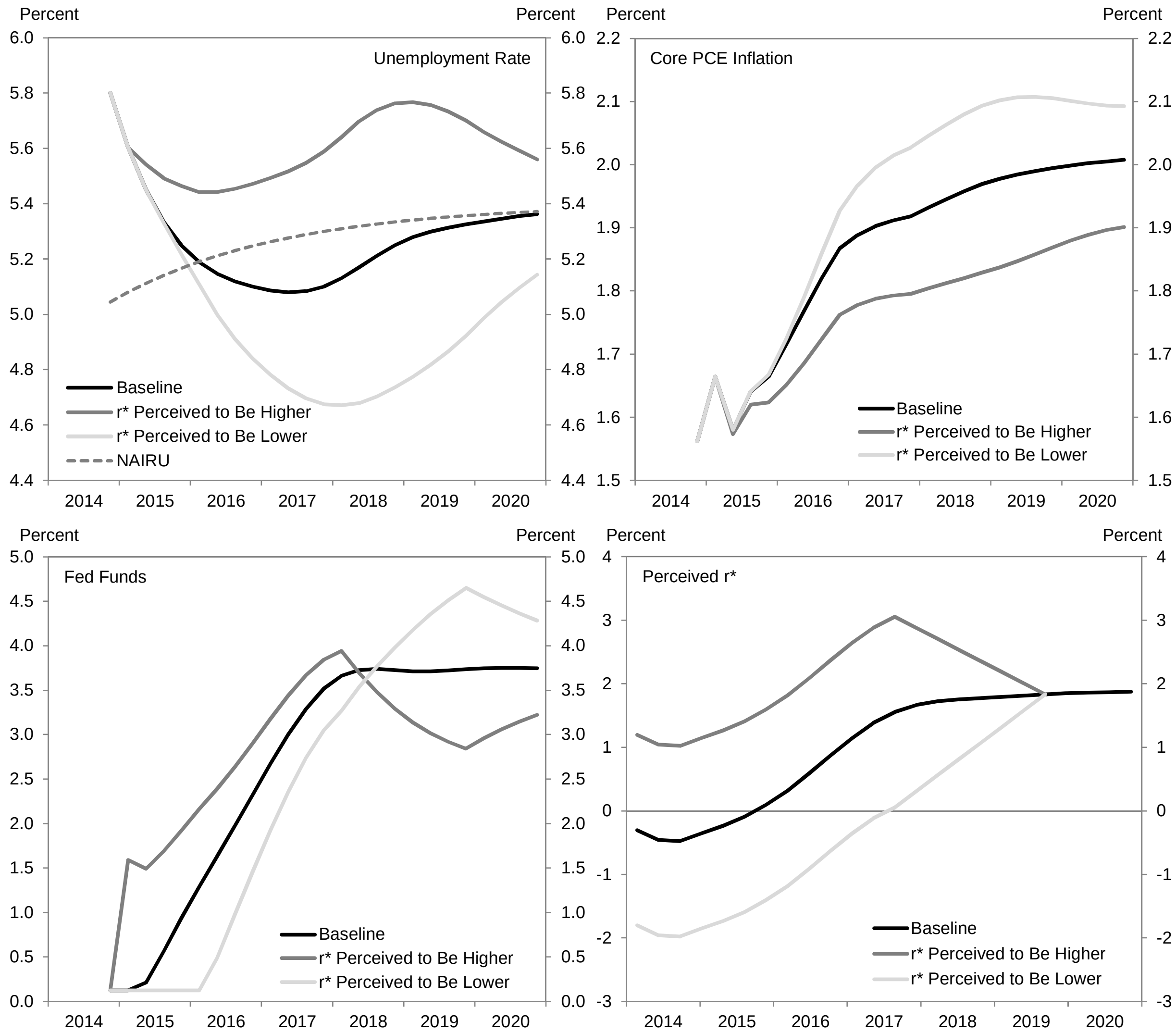


Exhibit 6.3. Behavior of the economy with perception errors and inertia.

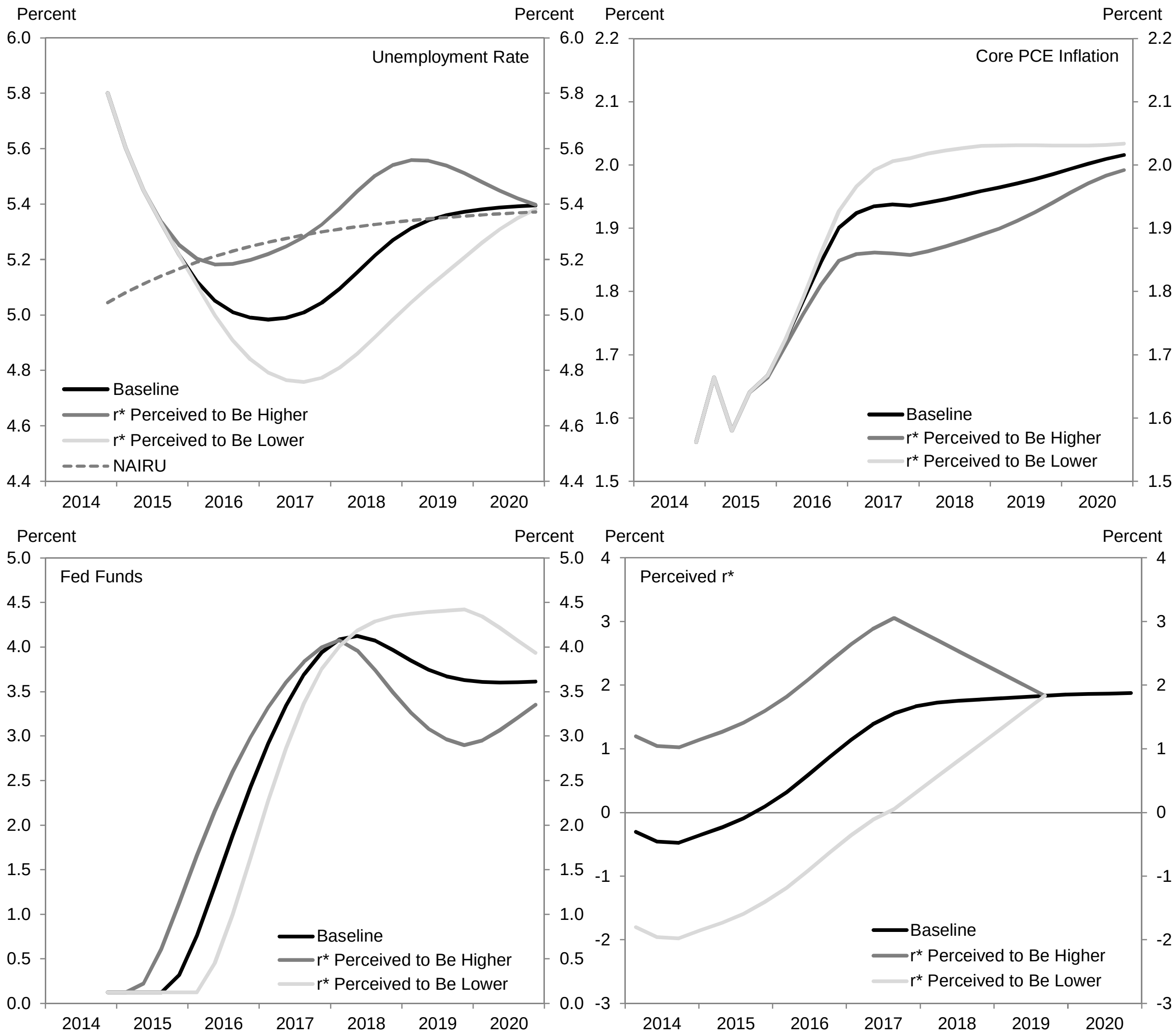


Exhibit 6.4. Funds rate paths with and without inertia.

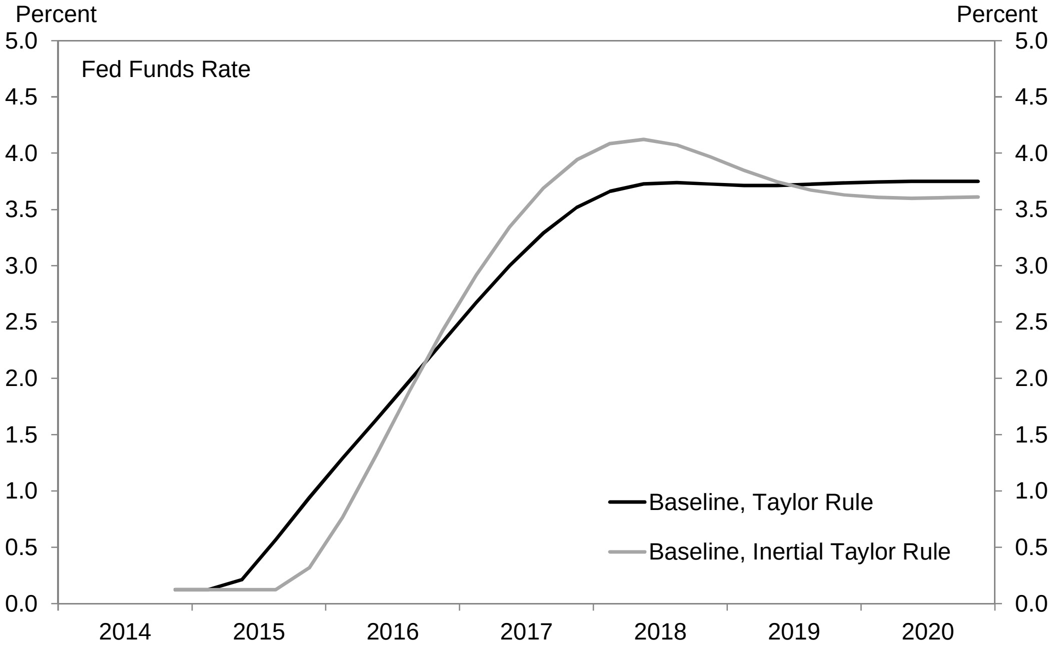
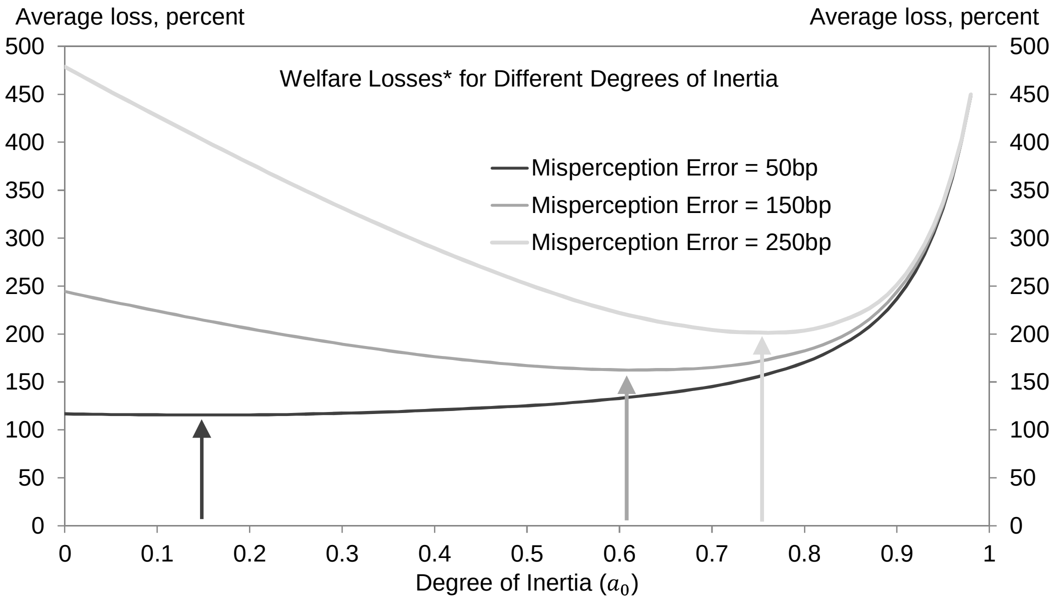
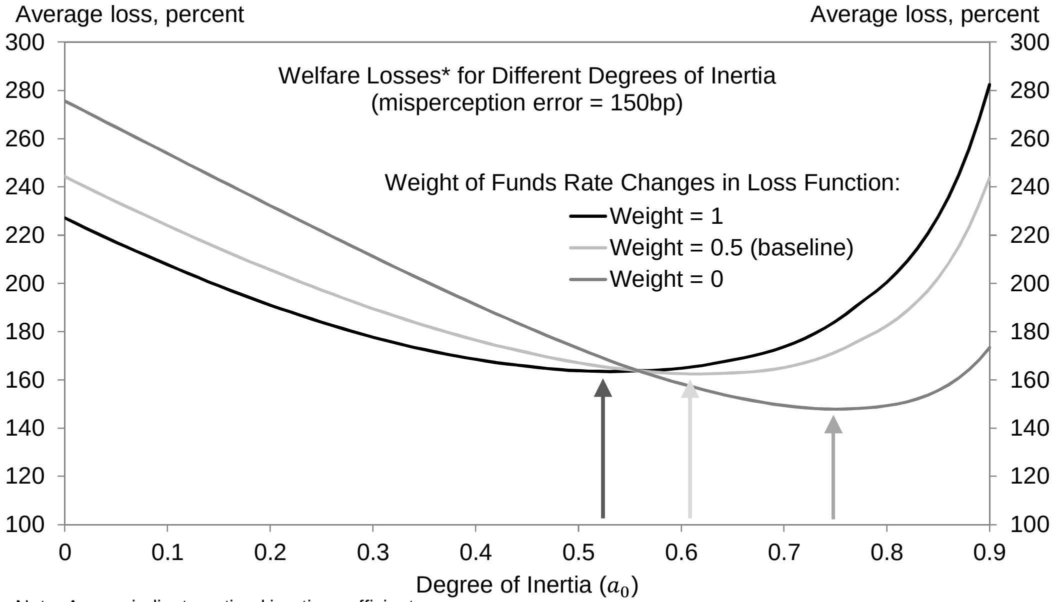


Exhibit 6.5. Optimal inertia curves for different r^* errors.



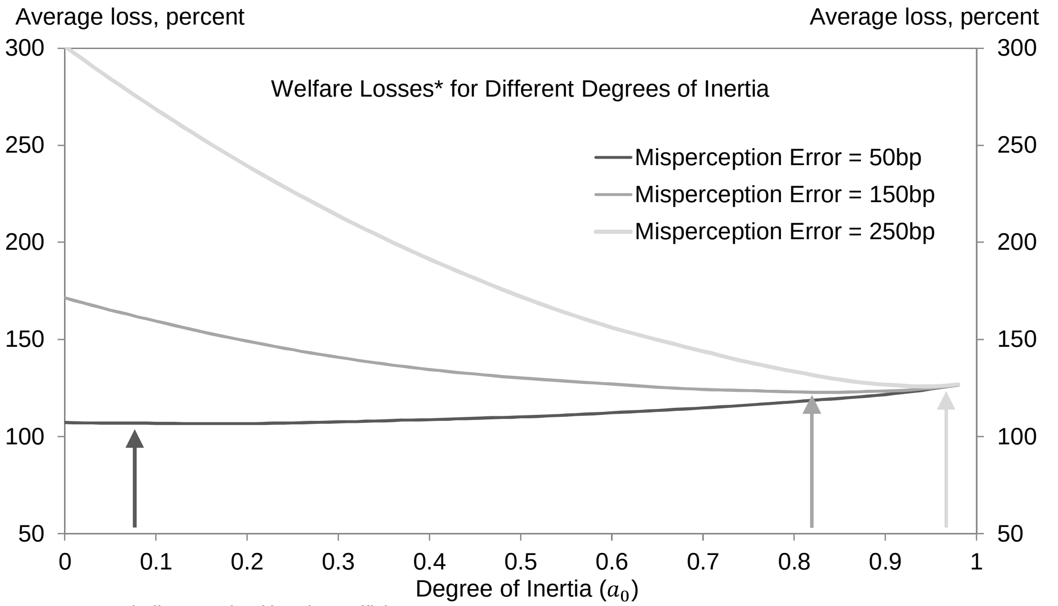
Note: Arrows indicate optimal inertia coefficients.
* Relative to no-misperception optimum.

Exhibit 6.6. Optimal inertia curves for different smoothing coefficients in loss function.



Note: Arrows indicate optimal inertia coefficients.
* Relative to no-misperception optimum.

Exhibit 6.7. Optimal inertia curves for different r^* errors with model-consistent expectations.



Note: Arrows indicate optimal inertia coefficients.
* Relative to no-misperception optimum.