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A PROCEDURE FOR GENERATING PARETO-EFFICIENT EGALITARIAN-EQUIVALENT ALLOCATIONS

BY VINCENT P. CRAWFORD[1]

This paper describes a simple, operational procedure that, under reasonable economic assumptions, always generates Pareto-efficient egalitarian-equivalent allocations (PEEEA) when agents know each other's preferences. The procedure constitutes a new, constructive proof of Pazner and Schmeidler's theorems on the existence of PEEEA, and shows that PEEEA, like fair and Pareto-efficient allocations, can be decentralized using less information than is required by the standard market procedure for decentralizing allocations that maximize a neoclassical, individualistic social welfare function.

1. INTRODUCTION

IN A RECENT PAPER, Pazner and Schmeidler **[7]** introduced an interesting new concept of equity, called "egalitarian-equivalence," which is ordinal and does not require interpersonal welfare comparisons. In a pure-trade economy, an allocation is *egalitarian-equivalent* if and only if its distribution of utilities could have been generated by equal division (ED) of some (not necessarily feasible) bundle of goods. Thus, egalitarian-equivalence of an allocation requires the existence of a fixed *egalitarian reference bundle* that is indifferent for each agent to his consumption bundle at the allocation.

Pazner and Schmeidler formulated the concept of egalitarian-equivalence to avoid a difficulty they perceived in fairness, a concept of equity that, like egalitarian equivalence, is ordinal and does not require interpersonal welfare comparisons. In what follows, an agent who would prefer another agent's bundle of goods to his own will be said to *envy* the other agent. An allocation at which no agent envies another will be called a *fair* allocation. In **[6]**, Pazner and Schmeidler showed that fairness may be inconsistent with Pareto-efficiency even in well-behaved economies with production. In **[8]**, Varian gave an example to demonstrate that fairness and Pareto-efficiency may be inconsistent in pure-trade economies if agents do not all have convex preferences.[2] Pazner and Schmeidler **[7]** proved that egalitarian-equivalence is consistent with Pareto-efficiency in both of the above situations, as well as in those well-behaved pure-trade economies where simultaneously fair and Pareto-efficient allocations exist.

It is well-known (see **[8]**, for example) that in pure-trade economies an equal-income competitive equilibrium (EICE) allocation (an allocation reached

[1] I have benefited from the comments of Walter P. Heller, R. Robert Russell, a referee, and, especially, Andrew Postlewaite, who stubbornly insisted on the importance of studying the auction mechanism discussed in this paper.

[2] Note that "fair" as used here is equivalent to Varian's **[8]** "equitable," and that his "fair" means "both equitable and Pareto-efficient." Varian has also shown that, if the economy's production technology is separable with respect to different agents' labors, it is possible to modify the concept of fairness to make it compatible with efficiency. Even with a separable technology, there are important problems with this approach, which are discussed in **[8]**.

when agents trade to a competitive equilibrium from equal initial endowments) is both fair and Pareto-efficient. That it is Pareto-efficient is, of course, one of the basic theorems of welfare economics; it is fair because all agents have the same budget set: an agent cannot envy another agent, because he could have bought the other's consumption bundle. Thus, when an EICE exists, dividing goods equally and setting up a competitive market is, at least when endowments are observable, an operational technique for generating a fair and Pareto-efficient allocation.

The availability of such a simple, usable procedure for decentralizing fair and Pareto-efficient allocations is, in those pure-trade economies where an EICE exists, an important advantage of the fairness approach over that of neoclassical welfare economics. Hurwicz's [**4**] demonstration that procedures which always generate Pareto-efficient, individually rational allocations when agents tell the truth about their preferences necessarily create incentives for agents to lie shows that any procedure based on the competitive mechanism may lose its optimality properties when agents are uncertain about each other's preferences, and can therefore get away with lying about them. If agents do know each other's preferences, it is, of course, true that, under standard assumptions, an allocation that maximizes a neoclassical, individualistic social welfare function can be decentralized as a competitive equilibrium for appropriately chosen initial endowments. The problem is that finding the right initial endowments is informationally virtually equivalent to computing the entire socially optimal allocation.[3] The competitive mechanism economizes greatly on the information a planner needs to find *some* Pareto-efficient allocation, but hardly at all on the information he needs to find the particular one that maximizes social welfare. Thus, the EICE procedure for decentralizing fair and Pareto-efficient allocations is operational in a sense in which the standard market procedure for decentralizing a social optimum is not.

In view of these observations and the fact that egalitarian-equivalence, unlike fairness, is consistent with Pareto-efficiency in a wide variety of environments, it would be of interest to find an operational procedure for generating Pareto-efficient egalitarian-equivalent allocations (PEEEA). The purpose of this paper is to exhibit such a procedure, and thus to show that egalitarian-equivalence shares the advantage of fairness just pointed out. The procedure also constitutes a constructive proof of Pazner and Schmeidler's [**7**] existence results.

The method for generating PEEEA I shall describe is obtained by a modification of a fair division device, of independent interest, that I have discussed elsewhere, in [**2**]. This device is, I have argued, an improvement upon the classical divide-and-choose procedure, which is discussed in [**1**]. In Section 2, I shall describe this fair division device and record some of its properties, in order to make the paper self-contained and put the results of [**2**] into a form that is more convenient for my present purpose. Section 3 describes a method for generating

[3] Conceivably, one could know that an allocation had the property that a competitive equilibrium starting from it would maximize social welfare, without also knowing the socially optimal allocation. But it is hard to imagine how, in the absence of divine intervention, one could obtain only that information.

PEEEA in pure-trade economies. Section 4 extends the method to allow construction of PEEEA in economies with production. Finally, in Section 5 I shall discuss possible further generalizations and summarize the results.

2. THE EQUAL-DIVISION DIVIDE-AND-CHOOSE METHOD

In [**2**], I presented a new fair division device, called the "equal-division divide-and-choose" method (EDDC), which was shown to have several desirable properties in a two-person pure-trade economy and some, but not all, of those properties in much more general environments. EDDC can be administered by the agents themselves, and is therefore likely to have administrative costs that are low relative to those of arbitration procedures that require the services of an arbitrator. In this section, I shall examine its properties in an n-person pure-trade economy in detail, recording several results that are needed for the analysis of Section 3.

Suppose that n agents, indexed $i = 1, \dots, n$, must agree on how to share a fixed bundle of perfectly divisible goods. In EDDC, one person, say, n, is designated "divider" in some equitable way, perhaps by the roll of a fair n-sided die. n then offers each of the other agents, $i = 1, \dots, n-1$, called "choosers," a choice between equal division (ED) and another allocation specified by n.

To study the properties of EDDC, I shall assume that agents have already agreed to use it, that their roles have already been determined, and that they do not anticipate further trade after EDDC. Each agent behaves noncooperatively, seeks only to obtain the most desirable bundle possible, cares only about his own consumption, and has preferences that can be represented by a continuous and strongly monotonic (but not necessarily quasiconcave) utility function. Each agent is assumed to know all other agents' preferences, which is not a completely natural assumption in all situations. But a study of the certainty case provides a necessary preliminary to a more general analysis and may serve as a reasonable description of some situations. A final assumption is included for expositional simplicity: whenever a chooser is indifferent between the allocations offered him, he can be counted on to choose as the divider would prefer him to. This assumption is innocuous, since a small adjustment in the divider's proposed allocation could induce the choosers to make the desired choice without perceptibly changing any agent's consumption bundle or welfare.

In the sequel, the following vector notation is used: if $a \equiv (a_1, \dots, a_r)$ and $b \equiv (b_1, \dots, b_r)$, $a \leqslant b$ means $a_i \leqslant b_i$, $i = 1, \dots, r$, and $a \leq b$ means $a \leqslant b$ but $a \neq b$; $\mathbf{0}$ and $\mathbf{1}$ denote a vector of zeros and a vector of ones, whose dimensionalities should be inferred from the context. Units are chosen so that the m-vector of goods to be allocated is $\mathbf{1}$. In EDDC, n offers each chooser a choice between ED and the allocation $z \equiv (z_1, \dots, z_n)$, where the m-vector z_i is agent i's consumption bundle; thus, agent i, $i = 1, \dots, n-1$, must choose between the bundles $(1/n)\mathbf{1}$ and z_i. Let U^i denote agent i's utility function, $i = 1, \dots, n$. The analysis of Section 3 depends on the properties of a generalization of EDDC, in which an arbitrary physically feasible *basis allocation* $\bar{z} \equiv (\bar{z}_1, \ldots, \bar{z}_n)$ plays the role of ED in EDDC. I shall call this generalization of EDDC "EDDC*."

In EDDC*, n offers each chooser a choice between the allocations z and $\bar{z}$; his optimal choice of z therefore solves the following problem:

$$\text{(2.A)} \qquad \max_{z \geqq 0} U^n(z_n) \quad \text{subject to} \quad \sum_{i=1}^{i=n} z_i \leqq \mathbf{1}$$

$$U^i(\bar{z}_i) \leqq U^i(z_i) \qquad (i = 1, \ldots, n-1).$$

The constraints of (2.A) ensure that z is a physically feasible allocation and that each chooser will voluntarily choose z over $\bar{z}$, subject only to the restriction that he breaks ties as n directs. I shall now record some of the properties of EDDC* allocations; the proofs are straightforward extensions of those in [**2**]. The following lemma provides an optimality condition that any solution of (2.A) must satisfy.

LEMMA 2.1: *Any solution of* (2.A) *must satisfy*

$$\text{(2.1)} \qquad U^i(\bar{z}_i) = U^i(z_i) \qquad (i = 1, \ldots, n-1).$$

In what follows, let $z^* \equiv (z_1^*, \ldots, z_n^*)$ denote both a solution of (2.A) and the corresponding EDDC* allocation. If an allocation is Pareto-superior or Pareto-indifferent to some other allocation, it will be said to be *individually rational* from the other allocation.

THEOREM 2.1: *EDDC* always generates a Pareto-efficient allocation.*

THEOREM 2.2: *EDDC* always generates an allocation that is individually rational from* $\bar{z}$.

Theorem 2.2 for $n = 2$ and $\bar{z} = (\frac{1}{2}\mathbf{1}, \frac{1}{2}\mathbf{1})$ can be used to show that EDDC always generates a fair allocation when there are only two agents and they have convex preferences; see [**2**, Theorem 3] for the details.

THEOREM 2.3: *In EDDC*, the role of divider is an advantage.*

In this section I have shown that EDDC* and EDDC have several desirable properties; [**2**] discusses some additional properties of these methods. Both always generate Pareto-efficient allocations that are individually rational from their respective basis allocations, $\bar{z}$ and ED. If the basis allocation is acceptable to agents as equitable in some sense, these devices therefore generate allocations that are satisfactory on efficiency and equity grounds, without requiring the intervention of an arbitrator. Indeed, EDDC can be shown to generate allocations that are fair when there are only two agents and they have convex preferences.

If $\bar{z}$ is taken as the status quo, EDDC* can be viewed as a way of making operational the dictum of classical welfare economics that allocational inefficiencies should be removed as long as those who gain compensate those who lose.

Theorem 2.1 ensures that all inefficiencies are removed and Theorem 2.2 guarantees that compensation is carried out. However, Lemma 2.1 and Theorem 2.3 show that the gains from removing inefficiencies are distributed very inequitably ex post: the divider gets all of them. Even though each agent has an equal ex ante chance of assuming the favored role, finding a way to remove this ex post asymmetry would be of interest. In Section 3, I shall exhibit a device based on EDDC, with all of EDDC's desirable properties, that treats agents symmetrically ex post as well as ex ante. An additional advantage is that this device always generates PEEEA.

3. A PROCEDURE FOR GENERATING PEEEA IN PURE-TRADE ECONOMIES

A natural way to try to remove the asymmetry from EDDC is by auctioning off the role of divider, instead of allocating it randomly. In this section I shall prove that, under the assumptions of Section 2, such a procedure, properly defined, always generates PEEEA in pure-trade economies. More precisely, let each agent bid some scalar multiple λ of a *numeraire bundle* $x \geqslant \mathbf{0}$, the highest bidder, say, n, winning the privilege of being divider in EDDC* with basis allocation

$$\bar{z} = \left(\left(\frac{1}{n}\mathbf{1} + \frac{\lambda}{n-1}x \right), \ldots, \left(\frac{1}{n}\mathbf{1} + \frac{\lambda}{n-1}x \right), \left(\frac{1}{n}\mathbf{1} - \lambda x \right) \right). \tag{3.1}$$

In words, n pays the choosers for the role of divider by adding the bundle $(\lambda/(n-1))x$ to each chooser's basis allocation bundle and subtracting a corresponding amount from his own. Of course, the divider's resulting basis allocation may have some negative components if λ is large, but EDDC* is still well defined in this case, since $\bar{z}_n$ does not appear in (2.A). (It is clear that no one will ever offer to pay so much for the role of divider that his problem becomes infeasible.) Each agent has a well defined reservation price for the role of divider, since he knows what level of utility he would attain as divider at any given price and, as is shown below, his utility as chooser for any given price is independent of which other agent is divider.

Normally, one would expect that in such a market for a single, indivisible privilege an infinity of different prices (those between the highest and second-highest of agents' reservation prices) would clear the market. In such cases, the final outcome would, in general, depend on the type of auction used, and agents' conflicting desires about price would be likely to lead to disputes about the choice of auction procedure. Surprisingly, in the procedure just described, once a numeraire has been agreed upon there can be no such disputes, because of the following result: Assume that the numeraire bundle is *desirable* in the sense that every agent would prefer some finite multiple of it to the bundle $\mathbf{1}$. (This assumption is implied, for example, by the strong monotonicity of the U^i if every component of x is strictly positive.) Then there exists a unique price that makes every agent indifferent between the roles of divider and chooser. This price will, of course, prevail in the market no matter how it is organized, so there can be no disputes about its organization.

THEOREM 3.1:[4] *In the auction described above, for any numeraire bundle there exists a unique price that makes every agent indifferent between the roles of divider and chooser.*

PROOF: First, I shall argue that, for each agent, there exists a unique price that makes him indifferent between the roles of divider and chooser. Let λ again denote the price of the role of divider. Equation (3.1) and Theorem 2.3 show that no agent would strictly prefer the role of chooser if $\lambda = 0$. Further, my assumption about the desirability of x clearly ensures that for any agent there exists a finite value of λ at which he would prefer the role of chooser. It is straightforward to show that each agent's utility as divider is a continuous function of λ. Equation (3.1), Lemma 2.1, and the continuity of the U^i imply that each agent's utility as chooser is a continuous function of λ, independent of which agent is divider.[5] Thus, by the Intermediate Value Theorem, there exists a finite, nonnegative value of λ for each agent that makes him indifferent between the roles of divider and chooser. The monotonicity of the U^i and the fact that making the feasible region of (2.A) smaller can never help the divider imply that these values of λ are unique.

Now, I shall prove that the values of λ just shown to exist for each agent are identical across agents. Let $\tilde{\lambda}$ denote the value of λ that makes n indifferent between the roles of divider and chooser and let $\hat{\lambda} \equiv \tilde{\lambda}/(n-1)$. By (3.1), Lemma 2.1, and the definition of $\hat{\lambda}$,

$$(3.2) \qquad U^n\left(\frac{1}{n}\mathbf{1} + \hat{\lambda}x\right) = \max_{z \geq \mathbf{0}} U^n(z_n) \quad \text{subject to} \quad \sum_{i=1}^{i=n} z_i \leq \mathbf{1},$$

$$U^i\left(\frac{1}{n}\mathbf{1} + \hat{\lambda}x\right) \leq U^i(z_i)$$

$$(i = 1, \ldots, n-1).$$

Theorem 2.1 implies that the allocation represented by the solution of the problem on the RHS of (3.2) is Pareto-efficient. By (3.2) and Lemma 2.1, this allocation gives each agent the same utility that the bundle $((1/n)\mathbf{1} + \hat{\lambda}x)$ would. Therefore, for any $j = 1, \ldots, n-1$,

$$(3.3) \qquad U^j\left(\frac{1}{n}\mathbf{1} + \hat{\lambda}x\right) = \max_{z \geq \mathbf{0}} U^j(z_j) \quad \text{subject to} \quad \sum_{i=1}^{i=n} z_i \leq \mathbf{1},$$

$$U^i\left(\frac{1}{n}\mathbf{1} + \hat{\lambda}x\right) \leq U^i(z_i)$$

$$(i = 1, \ldots, n; i \neq j),$$

since the problem on the RHS of (3.3) defines a Pareto-efficient allocation at

[4] The two-person version of this theorem was conjectured and independently proved by Andrew Postlewaite.

[5] The proof of Lemma 2.1 used the physical feasibility of $\bar{z}$ only to ensure that $\bar{z}_n \neq \mathbf{1}$. Since (3.1) and the fact that $\lambda x \geq \mathbf{0}$ also guarantee this, that the $\bar{z}$ specified in (3.1) may violate nonnegativity constraints for large values of λ does not invalidate the conclusion of Lemma 2.1.

which, by Lemma 2.1, every agent but j achieves the same utility he would get from the bundle $(1/n)\mathbf{1}+\hat{\lambda}x$. But (3.3) shows that $\tilde{\lambda}$ also makes all agents other than n indifferent between the roles of divider and chooser. *Q.E.D.*

I shall call the modified EDDC procedure just described, in which one agent pays the price $\tilde{\lambda}$ for the role of divider, "EDDCA," where the "A" stands for "auction." It is clear from the proof of Theorem 3.1 that this procedure generates the same utilities and has the same set of possible allocations no matter who assumes the role of divider. The next results establish some additional properties of EDDCA, which follow immediately from the arguments used to prove Theorem 3.1.

THEOREM 3.2: *EDDCA always generates a Pareto-efficient egalitarian-equivalent allocation with egalitarian reference bundle* $(1/n)\mathbf{1}+\hat{\lambda}x$.

THEOREM 3.3: *EDDCA always generates an allocation that is individually rational from ED.*

Theorem 3.3 can be used to show that, when there are only two agents and they have convex preferences, EDDCA always generates a fair allocation; see [**2**, Theorem 3] for the details. I have modified an example constructed by Andrew Postlewaite to obtain a pure-trade example with three agents, who have well-behaved preferences, in which the sets of fair and Pareto-efficient allocations and PEEEA are disjoint.[6] Thus, in view of Theorem 3.2, EDDCA cannot be shown to generate fair allocations when $n>2$ without making assumptions about preferences that are more restrictive than usual.

These results are illustrated in the Edgeworth box diagram of Figure 1, which depicts one of agent 1's indifference curves, I_1I_1, one of agent 2's, I_2I_2, and the EDDCA allocation when the numeraire bundle is x, A. Points other than A in Figure 1 are identified by agent 1's consumption bundles at them. Theorem 3.1 is illustrated by the fact that, as λ increases from 0, $\hat{\lambda}$ is the unique value at which the lens-shaped area that represents allocations better than, or indifferent to, $\frac{1}{2}\mathbf{1}+\lambda x$ for both agents first fails to have any interior points. (Note that when $n=2$, $\hat{\lambda}=\tilde{\lambda}$.) When $\lambda=\hat{\lambda}$, it is clear that the outcome of EDDCA is the same no matter who is divider. When $\lambda>\hat{\lambda}$, both agents prefer the role of chooser; when $\lambda<\hat{\lambda}$, both prefer the role of divider. Theorems 3.2 and 3.3 are also immediately clear from Figure 1.

In this section I have shown that, in pure-trade economies, EDDCA has all of EDDC's optimality properties and, in addition, always generates egalitarian-equivalent allocations and treats agents symmetrically. These advantages are gained at a sacrifice of some simplicity in the procedure. Indeed, it seems significantly more likely that the services of an arbitrator will be required in applications of EDDCA than in applications of EDDC.

[6] Pazner and Schmeidler [**7**] point out that fairness implies egalitarian-equivalence when $n=2$, but that this relationship no longer holds when $n>2$.

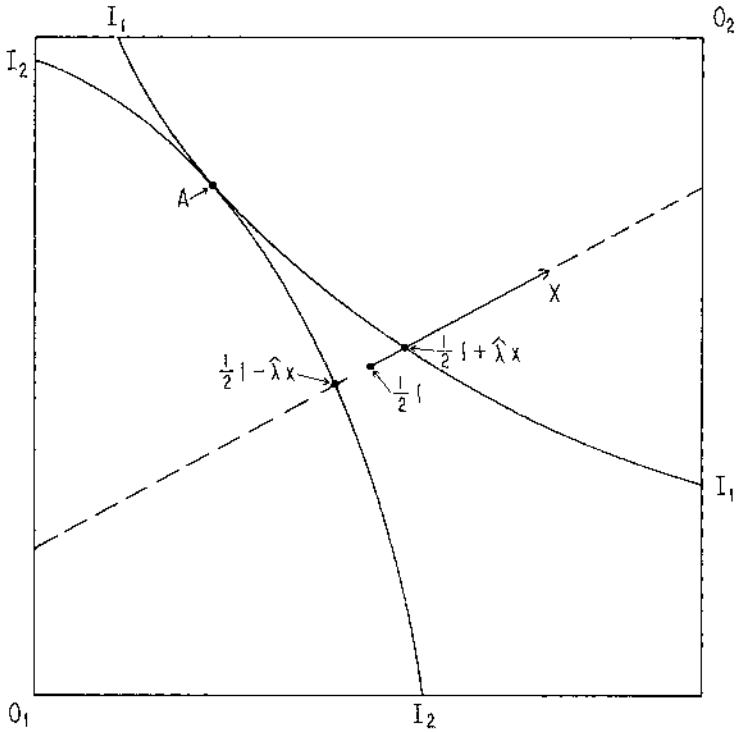

FIGURE 1.

An additional problem is that the outcome of EDDCA depends crucially on the choice, up to a scale factor, of the numeraire bundle, x, as the reader may convince himself by experimenting in Figure 1. Since EDDCA allocations are Pareto-efficient, agents will always have conflicts of interest about this choice. Since, like fairness, egalitarian-equivalence, even when coupled with Pareto-efficiency, does not single out a uniquely "best" allocation it should come as no surprise that the outcome of EDDCA depends on something. In view of Theorem 3.1, there is nothing it can depend on but the numeraire. In fact, a simple modification of EDDCA, in which $\mathbf{0}$ replaces $(1/n)\mathbf{1}$ in (3.1), extends its range to all possible PEEEA as the numeraire is varied.[7]

Still, there seems to be a reasonable chance of getting agents to agree on a choice of numeraire. In many applications, money is a natural choice; it is clearly reasonable to make my desirability assumption about money. When money does not seem appropriate, x might be made proportional to $\mathbf{1}$, the bundle of goods to be allocated; this choice also ensures that the desirability assumption is satisfied. In fact, this choice of numeraire in EDDCA leads to the "fair arbitration scheme" proposed by Pazner and Schmeidler in [7, Section III], and, as they point out, is the only choice that, for all convex preferences, yields an allocation that is also fair

[7] I am indebted to Andrew Postlewaite for this observation.

in two-person economies. But these choices are essentially arbitrary, as an agent who perceives that other numeraire bundles would lead to better outcomes for him is likely to point out. Beyond the fairness result just mentioned, the allocations they lead to have nothing to recommend them over those associated with other choices of numeraire but their salience. There is no apparent reason to believe that any particular EDDCA allocation has any special ethical significance beyond the fact that it is a PEEEA. Still, if agents can agree on a numeraire, EDDCA is an attractive, well defined way to arbitrate bargaining disputes.

4. EXTENSION TO ECONOMIES WITH PRODUCTION

Since Pazner and Schmeidler [7] have also shown that PEEEA exist in economies with production, it is natural to ask if a procedure like EDDCA can be constructed to generate them. In this section I shall exhibit such a procedure. Except as noted below, the assumptions and notation of this section are extended from earlier sections in the natural way.

Let $y \equiv (y_1, \ldots, y_n)$ denote an allocation, where y_i is the vector of agent i's consumptions of goods and supplies of the various types of labor in the economy. Let S denote the compact set of all allocations y that are technologically feasible for the economy, taking into account any initial endowments of goods that may exist. Following Pazner and Schmeidler [7], I define an *egalitarian-equivalent* allocation as one where every agent is indifferent between his allocation bundle and a fixed *egalitarian reference bundle*. This definition generalizes the one used for pure-trade economies, requiring that the allocation's distribution of utilities could have been generated by equal consumption of each good coupled with equal supply of each type of labor.[8] I shall also adopt a slightly strengthened version of Pazner and Schmeidler's [7] assumption of *utility-connectedness*: Let

$$(4.1) \qquad \underline{U}^i \equiv \min_{y \in S} U^i(y).$$

Then, for any $i = 1, \ldots, n$, if $y \equiv (y_1, \ldots, y_n) \in S$ satisfies $U^i(y_i) > \underline{U}^i$, for every $\varepsilon > 0$ there exists a $y' \equiv (y'_1, \ldots, y'_n) \in S$ such that $U^i(y'_i) > U^i(y_i) - \varepsilon$ and $U^j(y'_j) > U^j(y_j)$, $j = 1, \ldots, n$; $j \neq i$. In words, if an agent is above his minimum technologically feasible level of welfare, it is possible to increase the welfare of all other agents while reducing his welfare by as small an amount as desired.[9] This may be accomplished either by direct transfer of commodities, or indirectly through adjustments in the production plan. Any economy with reasonably well-behaved preferences and technology will satisfy this utility-connectedness assumption.

Consider an EDDC*-like procedure with basis allocation $\bar{y} \equiv (\bar{y}_1, \ldots, \bar{y}_n)$, $\bar{y} \in S$. If n is the divider, his optimal division solves a problem that is almost the

[8] Pazner and Schmeidler [7] consider some implications of an alternative definition of egalitarian-equivalence, in which the egalitarian reference bundle need not have equal labor supplies across agents, but the present definition seems more sensible and leads to exactly the same conclusions.

[9] Pazner and Schmeidler's [7] assumption differs in that it does not impose the limit on reductions in agent i's welfare.

same as (2.A):

$$\text{(4.A)} \qquad \max_{y \in S} U^n(y_n) \quad \text{subject to} \quad U^i(\bar{y}_i) \leq U^i(y_i) \qquad (i = 1, \ldots, n-1).$$

Under my assumption of utility-connectedness, it is straightforward to extend Lemma 2.1 and Theorems 2.1, 2.2, and 2.3 to the model of this section. Thus, solutions of (4.A) always satisfy

$$\text{(4.2)} \qquad U^i(\bar{y}_i) = U^i(y_i) \qquad (i = 1, \ldots, n-1),$$

and this procedure, like EDDC*, always generates Pareto-efficient allocations that are individually rational from $\bar{y}$ and favor n. The last three results are immediate extensions of Theorems 8, 9, and 10 in [**2**]. With these preliminaries taken care of, EDDCA can be extended to obtain a procedure for generating PEEEA in the production economy described above.

Suppose that agents have already agreed on a symmetrical and technologically feasible basis allocation $(y^e, y^e, \ldots, y^e)$ and a numeraire bundle $x \geq \mathbf{0}$ that is of the same dimensionality as y_i but has zeros for all components that correspond to labor supplies rather than consumptions of goods. (It is easy to find symmetrical technologically feasible basis allocations: take any production plan in which all agents have equal labor supplies and divide the output of consumption goods equally.) I assume, as in Section 3, that x is *desirable* in the sense that there exists some finite scalar λ such that each agent would prefer $y^e + \lambda x$ to any technologically feasible allocation bundle.

As in EDDCA, let each agent bid some scalar multiple λ of x, the highest bidder, say, n, winning the privilege of being divider in an EDDC*-like procedure with basis allocation

$$\text{(4.3)} \qquad \bar{y} = \left(\left(y^e + \frac{\lambda}{n-1} x \right), \ldots, \left(y^e + \frac{\lambda}{n-1} x \right), (y^e - \lambda x) \right).$$

The arguments of Section 3 can now be used to show that this procedure is well defined even though $y^e - \lambda x$ may violate nonnegativity constraints and that, under my assumption of utility-connectedness, the conclusions of Theorems 3.1, 3.2, and 3.3 are still valid. Thus, if $\tilde{\lambda}$ and $\hat{\lambda}$ are defined as in Section 3, the auction procedure described in this section always generates a PEEEA with egalitarian reference bundle $y^e + \hat{\lambda} x$, which is individually rational from y^e.

As before, agents will generally have conflicting interests in the choice of y^e and x that can only be resolved by arbitrary choices.[10] Still, if they can agree on some choice of basis allocation and numeraire bundle, this procedure treats agents symmetrically and always generates PEEEA in the wide class of environments in which they are known to exist, satisfying both the classical efficiency criterion and a reasonable equity condition.

[10] If anything, agreement on these choices will be harder to obtain when production is possible, since then there seems to be no sensible, salient choice of y^e.

5. FURTHER GENERALIZATIONS; CONCLUSIONS

The proofs of Section 4 indicate that EDDCA-like techniques could be used to generate PEEEA in environments much wider than the standard pure-trade and production economies studied here. Specifically, in any economy that satisfies a utility-connectedness assumption, has a symmetric basis allocation, and has a desirable numeraire bundle, an EDDCA-like procedure can be designed to generate PEEEA. These assumptions are satisfied in many (but not all) economies with externalities and public goods, for example, so Pazner and Schmeidler's existence results can be extended well beyond the standard cases studied here.

It would be of great interest to relax my assumption of perfect certainty about preferences. One problem that arises under uncertainty is that the conclusion of Lemma 2.1 no longer holds in general, which means that in EDDCA an agent's (expected) utility as chooser need no longer be independent of which other agent assumes the role of divider.[11] Thus, in formulating his reservation price, he will have to make some assumption about the probabilities of other agents becoming divider; since, ex post, only one agent actually assumes this role, it is not even clear that a "rational expectations" equilibrium exists in general in the market for roles. Certainly, even if one exists, it will not, in general, be unique as in the certainty case.

Further, Hurwicz's [4] demonstration that procedures which always generate Pareto-efficient, individually rational allocations when agents tell the truth necessarily create incentives for agents to lie about their preferences shows that EDDCA, like the competitive mechanism, may lose its optimality properties when agents are uncertain about each other's preferences. Designing a procedure whose Nash equilibria lead to PEEEA even when agents can get away with misrepresenting their preferences seems very difficult. Thus, the question posed in this paper and answered in the certainty case by EDDCA may not have an answer in the uncertainty case.[12] Still, investigating the consequences of uncertainty in these procedures is an important topic for future research.

In summary, this paper has exhibited a procedure that, if agents know each other's preferences, always generates PEEEA in a wide variety of economic environments; the procedure constitutes a new, constructive proof that PEEEA exist in such economies. Thus, egalitarian-equivalence, in addition to being compatible with Pareto-efficiency in a wider class of situations than fairness, also shares one of fairness' chief advantages as an approach to public choice: there exists a simple, operational procedure that decentralizes PEEEA, just as the

[11] Recall that in EDDC*, if any chooser strictly prefers $\bar{z}$ to the divider's proposed alternative, the final outcome is $\bar{z}$.

[12] After the final version of this paper was completed, I became aware of a result of Eric Maskin that confirms this conjecture. By Theorem 2 of his paper [5] it is impossible to construct a game form whose Nash equilibria, for all possible preference profiles, lead to outcomes in a social choice correspondence unless that correspondence satisfies his condition of *monotonicity*. Since it is easily verified that a social choice correspondence that always selects PEEEA is not, in general, monotonic, it is impossible to construct a procedure that always generates PEEEA when agents are uncertain about each other's preferences.

EICE decentralizes fair and Pareto-efficient allocations. This procedure is so simple that it might actually be used to generate equitable and efficient allocations in some real bargaining situations, particularly those involving only a few agents, who know each other well.

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