PART IV

NEW DIRECTIONS FOR POSITIVE ECONOMICS

CHAPTER 10

LOOK-UPS AS THE WINDOWS OF THE STRATEGIC SOUL

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Because human decisions are the result of cognitive processes, theories of human behavior rest at least implicitly on assumptions about cognition. Neuroeconomics reflects the belief that using evidence on neural correlates of cognition will lead us to better theories of decisions.

Gul and Pesendorfer (Chapter 1; henceforth GP) argue that, on the contrary, because economic theory was intended to explain only decisions, it should be tested only by observing decisions. They view neuroeconomics as a radical departure from economics in part because neural data concern involuntary, unconscious processes. Such processes are not decisions, so "our" theories cannot be about them.[1] Moreover, they argue, trying to extend our theories to explain neural data would require sacrificing important strengths of rational-choice analysis.

This chapter attempts to narrow the gap between these views by discussing some recent experiments that elicit subjects' initial responses to games with the goal of identifying the structure of their strategic thinking—subjects' attempts to predict others' decisions by taking their incentives into account.[2] Strategic thinking can, of course, be studied in experiments that elicit decisions alone, via designs in which different models of cognition imply different decisions, as in Stahl and Wilson [1994, 1995; henceforth "SW"], Nagel [1995], and Ho, Camerer, and Weigelt [1998; henceforth HCW]. But the experiments I discuss study strategic thinking more directly, by monitoring and analyzing subjects' searches for hidden but freely accessible payoff information, as in Camerer, Johnson, Rymon, and Sen [1993] and Johnson, Camerer, Sen, and Rymon [2002; henceforth collectively CJ, Costa-Gomes, Crawford, and Broseta [2001; henceforth CGCB], and Costa-Gomes and Crawford [2006, 2007; henceforth CGC]. My discussion draws extensively on CGC [2007], which reports and analyzes the information search data from CGC [2006].

CJ's, CGCB's, and CGC's analyses of search rest on explicit models of cognition and therefore raise some of the same issues that GP raise about neuroeconomics. But unlike neural correlates of cognition, search is a voluntary, conscious process. Rational-choice analysis can therefore be used to describe it, eliminating one source of resistance to studying cognition. Further, the clarity of the insights into behavior these analyses yield is a "proof of concept" that shows how much can be gained by expanding the domain of analysis beyond decisions. Although the analysis of search data sidesteps some important issues raised by studying neural data, I hope that considering it will bring us closer to agreement on how, and whether, to do neuroeconomics.

CJ's, CGCB's, and CGC's analyses suggest a concrete answer to GP's challenge: Why study cognition if our goal is only to understand and predict decisions? In CGC's decision data, for instance, most subjects deviate systematically from equilibrium, and their deviations are not well explained by noisy generalizations of equilibrium such as McKelvey and Palfrey's [1995] quantal response equilibrium ("QRE"). Following SW and CGCB, CGC described their behavior via a structural nonequilibrium model in which each subject's decisions are determined, in all games, by one of a small set of decision rules, or types (as they are called in this literature). The possible types were restricted to general principles of decision making, so that the theory's predictions would be potentially as portable to new games as equilibrium predictions. CGC showed that to the extent that subjects' deviations from equilibrium decisions can be distinguished from randomness, which is considerable, they are best explained by types that are rational and self-interested and that understand the game, but that base their beliefs on simplified models of others' decisions. In other words, subjects' deviations from equilibrium had mainly to do with how they model others' decisions, not with nonstandard preferences or irrationality.[3] Further, CGC's analysis of subjects' searches for hidden payoff information shows that extending the domain of the theory to include cognition and information search as well as conventional decisions allows more powerful tests and precise identification of subjects' types, sometimes directly revealing the algorithms subjects use to process payoff information into decisions and/or distinguishing intended decisions from errors.[4]

One could still choose to use subjects' decisions alone to model their behavior via revealed preference, as in GP's proposal. This requires a generalization of their proposal, because as stated, it refers only to individual decision problems.

In games, given the lack of a rational-choice model of nonequilibrium beliefs, revealed preference would need to be augmented by assumptions about players' beliefs, presumably by assuming that beliefs are in equilibrium.[5] However, as CGC's and previous experiments make clear, such an approach is unlikely to predict reliably beyond sample: in initial responses to new games, subjects' simplified models of others' decisions would yield different patterns of deviation from equilibrium. Only by coincidence would those patterns be well described by equilibrium (or QRE) with subjects' preferences as inferred from previous games. No empirically serious model of initial responses to games can ignore cognition, and relegating data other than conventional decisions to an inspirational role arbitrarily limits the use of a powerful tool for understanding and predicting decisions and may yield a biased view of behavior.

This critique highlights an important issue in judging GP's proposal. GP take the main problem of economics to be uncovering preferences, with rationality in the decision-theoretic sense (and in games, equilibrium) assumed. By contrast, CGC's and previous experiments on initial responses to games test (and mostly affirm) rationality, induce preferences (and leave little room for doubts about risk or social preferences), and focus instead on discovering the general principles that govern strategic thinking. Although many applications involve games whose players have enough clear precedents to justify equilibrium via learning, applications involving novel strategic situations, and where strategic thinking is the main source of uncertainty, are not uncommon. The costs and benefits of GP's proposal should be evaluated in the latter applications as well as in the cases involving individual decisions they use to develop and illustrate their proposal.

The rest of the chapter is organized as follows. I begin by reviewing CJ's and CGCB's designs and results, with the goal of introducing the key design and modeling issues in studying cognition via information search in the contexts in which they first emerged. I then review CGC's [2006] use of decision data to identify subjects' types and the evidence that the main source of their deviations from equilibrium is cognitive, not preference based. Next I introduce CGCB's and CGC's [2006, 2007] model of cognition and search and use CGC's search data to illustrate its use in interpreting search data. Following this, I address questions raised by CGC's [2006] analysis of decisions that seem likely to continue to resist analysis via decisions alone, but that search analysis might answer. Last, I outline a deeper explanation of the assumptions that underlie CGCB's and CGC's model of cognition and search, which views search strategies as rational decisions under plausible assumptions about the benefits and costs of search and constraints on working memory.[6] Throughout the chapter, I assume that subjects are rational, risk-neutral decision makers, but I allow "social" preferences that reflect altruism, spite, fairness, or reciprocity when they seem important, as indicated below.

Early Experiments That Studied Cognition in Games by Monitoring Information Search

In this section I review CJ's and CGCB's experimental designs and results. Their and CGC's experiments randomly and anonymously paired subjects to play series of different but related two-person games, with different partners each play and no feedback between plays. The goal was to suppress learning and repeated-game effects in order to elicit subjects' responses, game by game, to each game as if played in isolation, and so to reveal strategic thinking as clearly as possible.[7]

The structure of the games was publicly announced except for hidden, varying payoff parameters, to which subjects were given free access, game by game, one at a time, before making their decisions.[8] With low search costs, free access made the entire structure effectively public knowledge, allowing the results to be used to test theories of behavior in complete-information versions of the games.[9] Varying the payoff parameters makes it impossible for subjects to remember the current game's parameters from previous plays and so gives them incentives to search for the information their decision rules require. It also allows stronger separation of the decisions implied by equilibrium and leading alternative decision rules than in designs such as Nagel's or HCW's, in which subjects play the same game over and over again.

Camerer, Johnson, Rymon, and Sen's Alternating-Offers Bargaining Experiments

CJ [1993, 2002] pioneered the use of search for hidden payoff parameters to study cognition in games, eliciting subjects' initial responses to series of three-period alternating-offers bargaining games.[10] Previous experiments yielded large, systematic deviations from the subgame-perfect equilibrium offer and acceptance decisions when players have pecuniary preferences, such as those observed in ultimatum experiments. The deviations were attributed to cognitive limitations preventing subjects from doing the required backward induction, or believing that their partners would; to subjects having social preferences that modify their pecuniary payoffs; or both. Most researchers now agree that both factors are important, but in the early 1990s this was less clear.

CJ addressed the cognitive aspect of this question more directly by creating a design to study cognition via search and by deriving cognitive implications of alternative models of behavior and using them to analyze the search data. Within a publicly announced structure, they presented each bargaining game to subjects in extensive form as in figure 10.1, as a sequence of three pies and the associated

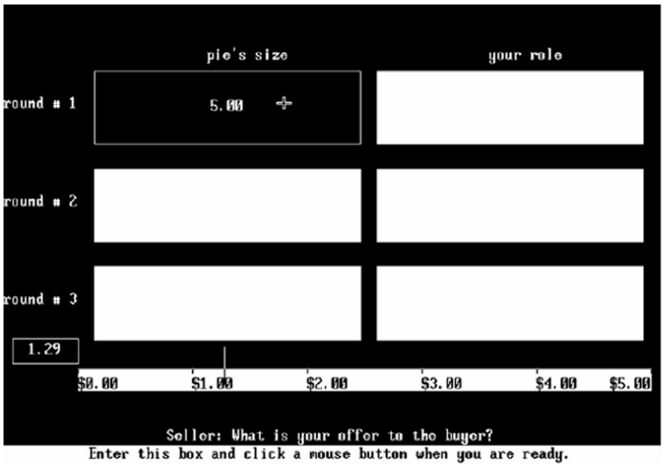

Figure 10.1. Display for Johnson, Camerer, Sen, and Rymon's [2002] alternating-offers bargaining experiments. From Johnson, Camerer, Sen, and Rymon [2002, figure 1].

offer and acceptance decisions. Discounting was simulated by shrinking the pies over time, from roughly $5.00 in round 1 to roughly $2.50 in round 2 and $1.25 in round 3, but the pies were varied slightly from game to game, to preserve subjects' incentives to search.

The pies were normally hidden in "boxes" as for rounds 2 and 3 in figure 10.1, but subjects were allowed to look them up as often as desired, one at a time. In figure 10.1 the subject has opened the box to look up the $5.00 round-1 pie.[11] Subjects' knowledge of the structure of the games and their free access to the pies allowed them to evaluate their own and their partners' pecuniary payoffs for any combination of offer and acceptance decisions.

If free access to the pies induces public knowledge of pecuniary payoffs, and if it is also public knowledge that subjects maximize their own expected pecuniary payoffs, then the results can be used to test theories of behavior in complete-information versions of the game, which has a unique subgame-perfect equilibrium whose offer and acceptance decisions are easily computed by backward induction. Even if players have privately observed social preferences, the incomplete-information version of the game has a generically unique sequential equilibrium whose strategies are easily computed by backward induction. In each case, the subgame-perfect or sequential equilibrium initial offer depends on both the second- and third-round pies, so the search requirements of equilibrium are mostly independent of preferences.[12] From now on I use "subgame-perfect equilibrium" to include pecuniary payoff maximization.

In CJ's baseline treatment, in which subjects were rewarded according to their payoffs playing the games against each other, subjects' decisions were far from the subgame-perfect equilibrium, replicating the results of previous studies and suggesting that requiring subjects to look up the pies did not significantly affect their decisions.

CJ took the analysis a step further by using a model of cognition and search to analyze the search data. They first noted that 10% of their baseline subjects never looked at the third-round pie and 19% never looked at the second-round pie. Thus, even if those subjects' decisions conform to equilibrium (given some specification of preferences, with or without a social component), they cannot possibly be making equilibrium decisions for the reasons the theory assumes. In a nonmagical world, their compliance with equilibrium cannot be expected to persist beyond sample.

This observation motivates a basic general restriction on how cognition drives search, which—anticipating CGCB's term for it—I call "occurrence" : If a subject's decision rule depends on a piece of hidden payoff information, then that piece must appear in her/his look-up sequence. Occurrence, as a cognitive restriction, goes against GP's proposal, but it is still uncontroversial enough to be widely accepted by theorists. In this case at least, the epistemic foundations of equilibrium have implications for the interpretation of decisions it is hard to justify ignoring.

If occurrence were the whole story, there would be little to gain from studying cognition via search. Because CJ's subjects who never looked at the second- or third-round pies tended to make decisions far from subgame-perfect equilibrium, there is little risk of misinterpreting them; even so, occurrence helps by ruling out explanations in which subjects' decisions are in sequential equilibrium for extreme distributions of social preferences. Inferences based on occurrence are sometimes useful in CGCB's and, as we will see, CGC's analyses as well, but the full power of monitoring search depends on analyzing the order, and perhaps the duration, of subjects' look-ups.

CJ's analysis of order and duration is based on the argument that in their design the backward induction that is the easiest way to compute sequential or subgame-perfect equilibrium decisions has a characteristic search pattern, in which subjects first look up the third-round pie, then the second-round pie (possibly rechecking the third), and so on, with most transitions from adjacent later to earlier round pies. Their argument rests on the empirical generalization that most subjects use the interface as a computational aid, making the comparisons or other operations on which their decisions are based via adjacent look-ups and relying on repeated look-ups rather than memory. This observation motivates another basic restriction, which—again anticipating CGCB's term—I call "adjacency" : the hidden parameters associated with the simplest of the operations on which a subject's decision rule depends will appear as adjacent look-ups in his look-up sequence.[13]

Adjacency, unlike occurrence, requires assumptions that not all theorists find compelling. It is theoretically possible for a subject to scan the pies in any order, memorize them, and then "go into his brain" to figure out what to do, in which case the order and duration of his look-ups will reveal nothing about cognition. (Here, brain imaging has a potential advantage over monitoring search because involuntary correlates of such a subject's thinking may still be observable.)

Fortunately, subjects' searches in designs such as CJ's, CGCB's, and CGC's exhibit strong regularities that make adjacency a reasonable working hypothesis. When challenged, CJ defended their adjacency-based characterization of backward-induction search by running a "robot" treatment with the same games as their baseline, in which subjects were told that they were playing against a computer that simulated a rational, self-interested player. This was followed after four periods by a "robot/trained subjects" treatment in which the same subjects received training in backward induction (but not search) and continued to play against robots as before. The latter subjects' search patterns were close to the backward-induction pattern [CJ 2002, figure 6]. Although the shift in search patterns was small prior to training, these results provide support for CJ's characterization, adjacency, and, of course, occurrence. As illustrated below, further (and sometimes stronger) support for adjacency is provided by CGCB's trained subjects, CGC's robot/trained subjects with high compliance with their assigned type's guesses, and CGC's baseline subjects with high compliance with their apparent rule's guesses (see tables 10.2 and 10.3 below for more details).

CJ's robot subjects' offer and acceptance decisions were shifted away from the baseline patterns toward subgame-perfect equilibrium, but were still far from it. Their robot/trained subjects' decisions were approximately in subgame-perfect equilibrium [CJ 2002, table II]. These shifts can be attributed to the robot treatment's "turning off" social preferences, assuming subjects don't think of experimenters or their funding agencies as "people" ; the robot treatment's eliminating strategic uncertainty; and/or cognition. CJ suggest that the deviations from equilibrium in the baseline are due to a combination of social preferences and cognition, with both important.

Returning to cognition and search, CJ's baseline subjects' searches were nearly the opposite of the searches of robot/trained subjects and CJ's characterization of backward induction search: baseline subjects spent 60–75% of the time looking up the first-round pie and only 20–30% looking up the second-round pie and 5–10% looking up the third-round pie, with most transitions forward, from earlier to later rounds. Importantly, subjects who looked up the second- and third-round pies more often, or had more backward transitions, also had a weak tendency to make, or accept, offers closer to the subgame-perfect equilibrium [CJ 2002, figures 4 and 5]. Thus, CJ's baseline subjects' deviations from backward induction search were correlated with their deviations from subgame-perfect equilibrium decisions, in the direction that an epistemic, procedural view of subjects' decision making would suggest. Although the correlation is weak, this result is an exciting first indication that subjects' search patterns might reveal something about their strategic thinking.

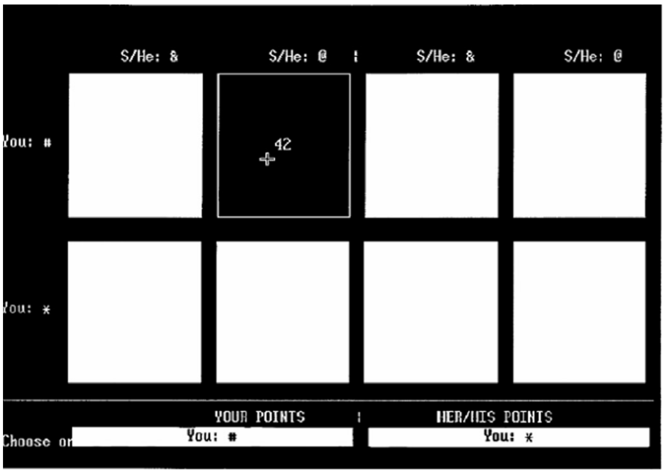

Figure 10.2. Display for a 2 × 2 Game in Costa-Gomes, Crawford, and Broseta's [2001] matrix-game experiments. From Costa-Gomes, Crawford, and Broseta [2001, figure 1].

Costa-Gomes, Crawford, and Broseta's Matrix-Game Experiments

CGCB adapted CJ's methods, building on SW's [1994, 1995] designs, to study cognition via search in a series of eighteen 2×2, 2×3, or 2×4 matrix games with unique pure-strategy equilibria, some of which can be identified by iterated dominance and some without pure-strategy dominance. The games were designed to turn off social preferences, and CGCB's results show little evidence of them. I therefore assume that CGCB's subjects maximized their own expected pecuniary payoffs.

Within a publicly announced structure, CGCB presented each game to subjects via MouseLab, as a matrix with players' payoffs spatially separated to ease cognition and clarify inferences from search. The payoffs were hidden, but subjects were allowed to look them up as often as desired. In the 2×2 game in figure 10.2, the subject, framed as the row player, has opened the box with his own payoff, 42, when he chooses decision # and his partner chooses @.[14] If free access induces public knowledge of the payoffs and it is public knowledge that subjects maximize their expectations, then the structure is public knowledge and the results can be used to test theories of behavior in complete-information versions of the games.

Although there are close connections between epistemic analyses of equilibrium decisions in extensive- and normal-form games, their cognitive foundations are very different. The different presentation of payoff information in CGCB's matrix games allows them to explore aspects of strategic thinking that do not come into play in CJ's bargaining games. Moreover, although CGCB's games have small strategy spaces, their sequence of 18 games creates a large space of possible decision histories, which allows their design to separate the implications of leading normal-form theories of decisions more strongly than in previous designs in which subjects play series of different matrix games with small strategy spaces, as in SW [1994, 1995], or in which they repeatedly play the same normal-form game with large strategy spaces, as in Nagel and HCW.

Finally, and most important here, the 8–16 hidden payoffs in CGCB's design create a large space of possible information searches, which allows the design to separate leading theories' implications for search as well as decisions. In CJ's design, a subject's searches can vary in only one important dimension: backward or forward in the pies. Measuring a subject's searches in this dimension can convey a limited amount of information about his strategic thinking—though this information can be quite revealing. In CGCB's games, by contrast, a subject's searches can vary in three important dimensions: up-down (or not) in his own payoffs, left-right (or not) in his partner's payoffs, and the frequency of transitions from his own to his partner's payoffs. With the subject framed as the row player in figure 10.2, it is clear that, assuming adjacency, the first of these dimensions is naturally associated with decision-theoretic rationality, the second with using others' incentives to predict their decisions, and the third with interpersonal payoff comparisons. It would be difficult to imagine an empirically successful theory of initial responses to this kind of game in which those three traits were not independently variable and important. Only a design with a search space as rich as CGCB's can separate the implications of alternative theories for both search and decisions strongly enough to identify their relationships.

In addition to a baseline treatment that paired subjects to play the 18 games with other subjects, CGCB conducted a trained subjects treatment, identical to the baseline except that each subject was trained and rewarded for identifying equilibrium decisions. This treatment confirms that subjects trained and motivated to find equilibrium guesses could do so, and provides data on equilibrium search behavior that are helpful in evaluating CGCB's model of cognition and search.

CGCB's games have unique equilibria that are easily identified by direct checking, best-response dynamics (which always converges in their games), or (in most of their games) iterated pure-strategy dominance. Yet, as in previous studies of initial responses to matrix games, CGCB found systematic patterns of deviation from equilibrium, with high equilibrium compliance in games solvable by one or two rounds of iterated dominance but much lower compliance in games solvable by three rounds or the circular logic of equilibrium without dominance [CGCB, table II]. These patterns are not well explained by noisy generalizations of equilibrium such as QRE. CGCB explained them via a structural nonequilibrium model of initial responses in the spirit of SW's, Nagel's, and HCW's models, in which each subject's decisions are determined, in all games, by one of a small set of types, which determines his decisions, with error, in each game. The possible types were restricted to general principles of decision making, so that the theory's predictions would be potentially as portable to new games as equilibrium predictions.

The leading types in CGCB's analysis include L1 (for level 1, as named by SW), called naive in CGCB and L1 here from now on, which best responds to a uniform random L0 "anchoring type" ; L2, which best responds to L1; equilibrium, which makes its equilibrium decision; D1 (dominance 1), which does one round of deletion of dominated decisions and then best responds to a uniform prior over the other's remaining decisions; D2, which does two rounds of iterated deletion and then best responds to a uniform prior over the other's remaining decisions; and sophisticated, which best responds to the probabilities of other's decisions, as estimated from subjects' observed frequencies, included to test whether subjects have prior understanding of others' decisions that transcends simple rules.[15] Because CGCB gave first priority to separating strategic from nonstrategic types, L1's decisions were perfectly confounded with those of a maximax type CGCB called optimistic. CGCB's econometric analysis of decisions alone estimated high frequencies of L1, L2, and D1. Because those types mimic equilibrium in simple games but deviate systematically in more complex games, this estimated type distribution allows the model to explain the aggregate relationship between complexity and equilibrium compliance.

Turning to CGCB's analysis of search, the main difficulty was imposing enough structure on the enormous spaces of possible decision and search histories to describe subjects' behavior in a comprehensible way. Although CJ identified a correlation (and a "right" direction for it) between subjects' decision and search deviations from subgame-perfect equilibrium in their alternating-offers bargaining games, their analysis does not show how to define or identify such a relationship in the higher dimensional spaces of possible decisions and searches created by CGCB's design.

CGCB addressed this issue by using the types as models of cognition and search as well as decisions. They took an explicitly procedural view of decision making, in which a subject's type and the associated cognitive process determine his search, and his type and search then determine his decision, game by game.[16] They characterized the link between cognition and search via the occurrence and adjacency restrictions described above, which generalize the ideas behind CJ's characterization of backward-induction search to a much wider class of games, patterns of hidden payoff information, and types. With these restrictions on cognition and search, the types provide a kind of basis for the spaces of possible decision and search histories, imposing enough structure to make it meaningful to ask whether subjects' decisions and searches are related in a coherent way.

Incorporating search into the econometric analysis yields a somewhat different view of subjects' deviations from equilibrium than previous analyses of decisions. It shifts CGCB's estimated type distribution toward L1 at the expense of optimistic and D1, leaving L1 and L2 as the only empirically important types. Part of this shift occurs because L1's searches, unlike L1's decisions, are clearly separated from optimistic's, and L1's search implications explain more of the variation in subjects' searches and decisions than optimistic's, which are too unrestrictive to be useful. Another part of the shift occurs because L1's search implications explain more of the variation in subjects' searches and decisions than D1's, which are much more restrictive than optimistic's but too weakly correlated with subjects' observed decisions. D1 loses frequency to L2, as well, even though their decisions are only weakly separated in CGCB's design, because L2's search implications explain more of the variation in subjects' searches and decisions. Thus, analyzing search not only yields more precise estimates of subjects' types, but also can correct distortions in type estimates based on decisions alone that stem from a design's failure to fully separate types.

These shifts illustrate an important principle. Because the number of experimental treatments and subjects that can be run is limited, data are scarce relative to the plausible theories of behavior, and trade-offs in discriminating among theories are inevitable. Gathering search (or other nondecision data) as well as decision data can make such trade-offs less stringent. Although gathering and analyzing nondecision data have their own costs, the optimal amount to gather is not always zero.

Overall, CGCB's analysis of decisions and search gives a strikingly simple view of behavior, with L1 and L2 making up 90% of the population. This type distribution and the clear relationships between subjects' cognition as revealed by search and their decisions support my claim that their deviations from equilibrium in these games are due mainly to how they think about others.

Costa-Gomes and Crawford's Two-Person Guessing Game Experiments

CGC [2006, 2007] adapted CGCB's methods to elicit subjects' initial responses to a series of 16 dominance-solvable two-person guessing games, cousins of Nagel's and HCW's *n*-person guessing games. In this section, I review CGC's design and their results for decisions, which provide even stronger evidence that the deviations from equilibrium in initial responses to games are due mainly to strategic thinking. In the following section, I review CGC's analysis of cognition and search.

CGC's Design

In CGC's games, newly designed for the purpose of studying cognition via decisions and search, two players make simultaneous guesses. Each player has his own lower and upper limit, both strictly positive, as in some of HCW's games, to ensure finite dominance solvability. Unlike in previous designs, however, players are not required to guess between their limits: to enhance the separation of types via search, guesses outside the limits are automatically adjusted up to the lower limit or down to the upper limit as necessary. Thus, the only thing about a guess that affects the outcome is the adjusted guess it leads to. Each player also has his own target, and (unlike in Nagel's and HCW's "winner-take-all" games) his payoff is higher, the closer his adjusted guess is to his target times his partner's adjusted guess.

In the most important departure from previous guessing designs, the targets and limits vary independently across players and games, with the targets either both less than one, both greater than one, or (unlike in previous designs) mixed.[17] The resulting games are asymmetric and dominance solvable in 3 to 52 rounds, with essentially unique equilibria determined (but not always directly) by players' lower limits when the product of the targets is less than one or their upper limits when the product is greater than one. In game 13 in figures 10.4–10.7, for instance, player *i* has limits 300 and 500 and target 0.7, and player *j* has limits 100 and 900 and target 1.5 [CGC, table 3]. The product of targets is $1.05 > 1$, player *i*'s equilibrium guess is at his upper limit 500, and player *j*'s equilibrium guess is at his best response to 500 of 750, below his upper limit.

From the point of view of studying decisions, CGC's design combines the main strengths of SW's and CGCB's designs, with subjects playing sequences of different but related games, and the main strengths of Nagel's and HCW's designs, games with very large strategy spaces. This combination greatly enhances the separation of equilibrium and other leading types' decisions.

CGC's games explore different aspects of strategic thinking than CJ's, CGCB's, or Nagel's and HCW's games. Of particular note is the subtle way in which the location of the equilibrium is determined by the product of players' targets, which adds greatly to the power of the design to distinguish equilibrium from boundedly rational strategic thinking. The only important difference between some of CGC's games is whether the product of targets is slightly greater or slightly less than one. Equilibrium responds very strongly to this difference, but low-level L*k*, or D*k*, types, whose guesses vary continuously with the targets, respond much less. Also noteworthy is the strong separation of L*k*'s, and D*k*−1's decisions, which are perfectly confounded in most of Nagel's and HCW's treatments and only weakly separated in their other treatments and in CGCB's design.

In addition to a baseline treatment that paired subjects to play the 16 games with other subjects, CGC conducted six different robot/trained subjects treatments, identical to the baseline except that each subject was trained and rewarded as a type: L1, L2, L3, D1, D2, or equilibrium. These treatments assess the types' cognitive demands, confirming, for instance, that subjects trained and motivated to make equilibrium guesses could do so; and provide data on the search behavior of subjects of known types that are helpful in evaluating the model of cognition and search.

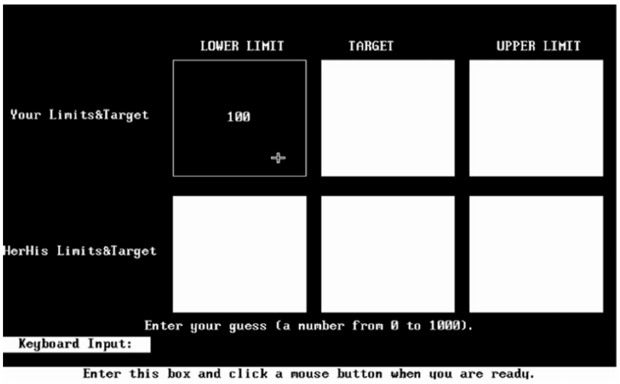

Figure 10.3. Display for Costa-Gomes and Crawford's [2006, 2007] two-person guessing games. From Costa-Gomes and Crawford [2006, figure 6].

In all treatments, within a publicly announced structure, CGC presented each game to subjects as an array of targets and limits, with those payoff parameters hidden but subjects allowed to look them up as often as desired, one at a time, using MouseLab's click option as in CGCB. In figure 10.3, the subject has opened the box to look up his own ("Your") lower limit, 100.

CGC's Analysis of Decisions

The strong separation of types' implications for guesses [CGC 2006, figure 5] and the clarity of CGC's baseline subjects' responses allow many of their types to be confidently identified from guesses alone. Of 88 subjects, 43 have clear "fingerprints" in that they made guesses that complied exactly (within 0.5) with one type's guesses in 7–16 of the games (20 L1, 12 L2, 3 L3, and 8 equilibrium).[18] figure 10.4 [CGC 2006, figure 2] shows the fingerprints of the 12 whose apparent types were L2. Of their 192 ($= 12 \times 16$) guesses, 138 (72%) were exact, which means they tracked the complex pattern of the games' L2 guesses with a remarkable degree of accuracy. I stress that these baseline subjects, unlike the robot/trained subjects, were taught nothing about strategic thinking: The models of others' guesses implicit in their apparent types were self-generated.

Given how strongly CGC's design separates types' guesses, and that guesses could take 200–800 different rounded values, these 43 subjects' compliance is far higher than could occur by chance. Further, because the types specify precise, well-separated guess sequences in a very large space of possibilities, their compliance rules out alternative interpretations of their guesses.[19] In particular, because the types build in risk-neutral, self-interested rationality and perfect models of the game, the deviations from equilibrium of the 35 whose apparent types are L1, L2, or L3 can be attributed to nonequilibrium beliefs, not irrationality, risk aversion, altruism, spite, or confusion.

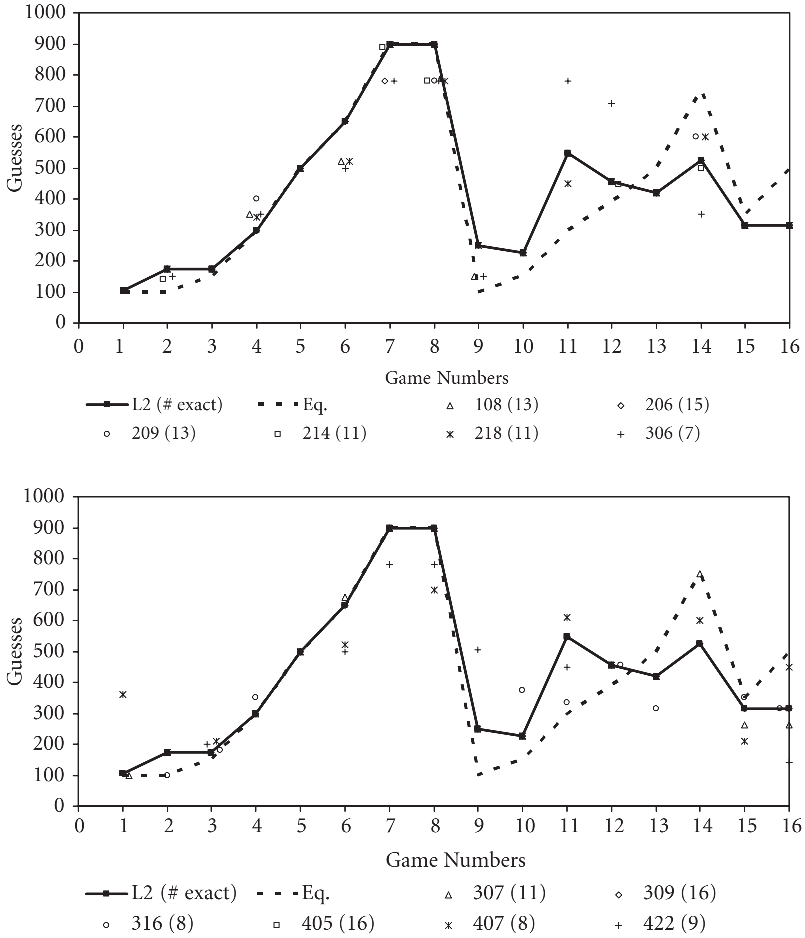

Figure 10.4. "Fingerprints" of 12 apparent level-2 (L2) baseline subjects. Only deviations from L2's guesses are shown 138 (72%) of these subjects' 192 guesses were exact L2 guesses. From Costa-Gomes and Crawford [2006, figure 2].

CGC's other 45 subjects' types are less apparent from their guesses, but L1, L2, and hybrids of L3 and/or equilibrium are still the only types that show up in econometric estimates.[20] The fact that most subjects follow low-level L*k* types, which mimic equilibrium in games that are dominance solvable in small numbers of rounds but deviate systematically in some more complex games also explains the inverse relationship between strategic complexity and equilibrium compliance observed in CGCB and previous experiments [CGCB, table II].

CGC's results for decisions provide very strong evidence that subjects' deviations from equilibrium in initial responses to games are due mainly to nonequilibrium strategic thinking, not preferences or irrationality. As noted in the introduction to this chapter, one could still use subjects' guesses alone to model their behavior via revealed preference, but such a model would misattribute the cause of the deviations and so would predict well beyond sample only by coincidence.

Costa-Gomes and Crawford's Analysis of Cognition and Search

CGC's [2006, section II.E; 2007] model of cognition and search refines CGCB's model, adapting their occurrence and adjacency restrictions to give a tractable characterization of each type's search requirements. With regard to search, CGC's design combines the strengths of CJ's presentation of games as functions of a small number of hidden parameters within an intuitive common structure, which allows subjects to focus on predicting others' responses without getting lost in the details of the structure; and CGCB's high-dimensional search spaces, which make search more informative and allow greater separation via search. CGC's design strongly and independently separates the implications of leading types for search and decisions, which makes it easier to identify relationships between them and multiplies the power of the design. Finally, it makes each type's implications search independent of the game, which simplifies the analysis.[21]

This section begins with a discussion of the issues that arise in specifying a model of cognition and search. It then presents CGC's leading types' search requirements and illustrates how they are derived. Finally, it presents sample search data for some of CGC's robot/trained and baseline subjects. As these data will be used to show, CGC's design and characterization of types' search implications make it possible to read the algorithms that a large minority of subjects used to choose their guesses directly from their search sequences. Other subjects' cognition is not apparent from their searches, but CGC's [2006] measures of their compliance with leading types' search implications have considerable discriminatory power in the econometric analysis, often allowing those subjects' types to be reliably estimated from searches alone, without regard to guesses.

Specification Issues

Studying cognition via search requires a model of how cognition determines subjects' look-up sequences. Previous articles have taken quite different positions on this issue. CJ's analysis gave roughly equal weight to look-up durations and total numbers of look-ups ("acquisitions") of each pie and to the numbers of transitions between look-ups of adjacent pies. Rubinstein's [2007] analysis considered only durations. Gabaix et al. [2006] focused on total numbers of look-ups rather than durations, but also considered some aspects of the order of look-ups. CGCB's and CGC's analyses focused instead on which look-ups subjects make, in the sense of occurrence, and on the order of look-ups in the sense of adjacency, relegating durations to a secondary role.

On another dimension, CJ's and Rubinstein's analyses and most of Gabaix et al.'s aggregated search data across subjects and over time, while CGCB and CGC took the position that cognition and search are so heterogeneous that it is essential to study them at the individual level.

CGCB's and CGC's focus on occurrence and adjacency follows naturally from a procedural view of decision making and the empirical tendency, now confirmed by a large body of MouseLab data, of most subjects to perform the operations on hidden parameters on which their decisions are based via adjacent look-ups, relying on repeated look-ups rather than memory. In this view—perhaps too extreme—duration is unimportant because the information content of a look-up is independent of its length as long as it suffices for cognition; look-ups too short for comprehension (< 0.18 sec) were filtered out in the analyses discussed here. Although duration might still be correlated with time spent thinking about a particular parameter, which might be important in a more refined model of cognition, search, and decisions, a procedural view does not suggest such a correlation, and CGCB's and CGC's subjects sometimes left boxes open for long periods while staring out the window, and so on which would weaken any such correlations.[23] Total numbers of look-ups are important but are captured indirectly through CGC's notion of search compliance.

CGC's Model of Cognition and Search

In CGC's model of cognition and search, each leading type implies a generically unique, pure adjusted guess in each game, which maximizes its expected payoff given the beliefs regarding others' guesses implicit in the type. (The leading types all specify best responses to some beliefs.) Each type is thereby naturally associated with algorithms that process hidden payoff information into decisions, which CGC used as models of cognition. Given the need to go beyond occurrence and the lack of an accepted theory of cognition and search, the goal was to add enough restrictions to extract the signal from subjects' search sequences but not so many that they distort its meaning. CGC derived types' minimal search implications under conservative assumptions, based on occurrence and adjacency, about how cognition determines search [CGC 2006, section I.B].

The leading role in these derivations is played by a type's ideal guesses, those that would be optimal given the type's beliefs, ignoring its limits. Given the quasi concavity of CGC's payoff functions, a subject can enter his ideal guess and know that his adjusted guess will be optimal without checking his own limits. Thus, a type's ideal guess not only determines its adjusted guess and the resulting outcome but also determines the type's minimal search implications.

Table 10.1. Types' Ideal Guesses and Minimal Search Sequences [Costa-Gomes and Crawford, 2006].

Type	Ideal	Guess
L1	$p^i[a^j + b^j]/2$	$\{[a^j, b^j], p^i\} \equiv \{[4,\ 6], 2\}$
L2	$p^i R(a^j, b^j; p^j[a^i + b^i]/2)$	$\{([a^i, b^i], p^j), a^j, b^j, p^i\} \equiv \{([1,\ 3], 5), 4,\ 6,\ 2\}$
L3	$p^i R(a^j, b^j; p^j R(a^i, b^i; p^i[a^j + b^j]/2))$	$\{([a^j, b^j], p^i), a^i, b^i, p^j\} \equiv \{([4,\ 6], 2), 1,\ 3,\ 5\}$
D1	$p^i(\max\{a^j, p^j a^i\} + \min\{p^j b^i, b^j\})/2$	$\{(a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^i\} \equiv \{(4, [5, 1]), (6, [5,\ 3]), 2\}$
D2	$p^i[\max\{\max\{a^j, p^j a^i\}, p^j \max\{a^i, p^i a^j\}\} + \min\{p^j \min\{p^i b^j, b^i\}, \min\{p^j b^i, b^j\}\}]/2$	$\{(a^i, [p^i, a^j]), (b^i, [p,^i\ b^j]), (a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^j, p^i\} \equiv \{(1, [2,\ 4]), (3, [2,\ 6]), (4, [5,\ 1]), (6, [5,\ 3]), 5,\ 2\}$
Equilibrium	$p^i a^j$ if $p^i p^j < 1$ or $p^i b^j$ if $p^i p^j > 1$	$\{[p^i, p^j], a^j\} \equiv \{[2, 5], 4\}$ if $p^i p^j < 1$ or $\{[p^i, p^j], b^j\} \equiv \{[2, 5], 6\}$ if $p^i p^j > 1$
Sophisticated	(No closed-form expression, but CGC took *sophisticated*'s search implications to be the same as D2's]	$\{(a^i, [p^i, a^j]), (b^i, [p^i, b^j]), (a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^j, p^i\} \equiv \{(1, [2,4]), (3, [2,6]), (4, [5,1]), (6, [5,3]), 5, 2\}$

Note: $R(a, b; x) \equiv \min\{b, \max\{a, x\}\} \equiv \max\{a, \min\{b, x\}\}$ denotes x's adjusted guess with limits a and b.

The left-hand side of table 10.1 [CGC 2006, table 4] lists the formulas for the leading types' ideal guesses in CGC's games, which are easily derived as in CGC [2006, section I.B], using CGC's notation for the limits and targets, a^i for the player's own lower limit, b^i for the player's own upper limit, and p^i for the player's own lower target, with analogous notation using superscript j for the player's partner's limits and target. The right-hand side of table 10.1 lists the leading types' minimal search implications expressed as sequences of parameter look-ups, first in CGC's notation and then in terms of the associated box numbers (1 for a^i, 2 for p^i, 3 for b^i, 4 for a^j, 5 for p^j, 6 for b^j) in which MouseLab records subjects' look-up sequences in our design. Table 10.1 shows look-ups in the order that seems most natural, but that order is not required in the analysis.[24]

The search implications are derived as follows. Evaluating a formula for a type's ideal guess requires a series of arithmetic operations, some of which—the innermost operations, whose parameters are in square brackets in the right-hand side of table 10.1, such as $[a^j, b^j]$ for L1—are basic in that they logically precede other operations. Like CGCB, CGC assumed that subjects perform basic operations via adjacent look-ups, remembering their results, and otherwise relying on repeated look-ups rather than memory. Basic operations are then represented by adjacent look-ups that can appear in either order but cannot be separated by other look-ups. The look-ups of other operations can appear in any order and are (conservatively) allowed to be separated. In table 10.1 such operations are represented by look-ups within braces or parentheses.[25]

An L1 player i, for instance, best responds to the belief that player j's guess is uniformly distributed between his limits. This yields a guess for j that is never adjusted, and that averages $[a^j + b^j]/2$. CGC [2006, section I.B] shows via a certainty equivalence property of CGC's games (observation 2) that L1's ideal guess is $p^i[a^j + b^j]/2$, which will be automatically adjusted, if necessary, to $R(a^i, b^i; p^i[a^j + b^j]/2) \equiv min\{b^i, max\{a^i, p^i[a^j + b^j]/2\}\}$. The only basic operation is $[a^j + b^j]$. An L1 player i therefore has minimal look-up sequence: $\{[a^j, b^j]$ (to compute j's average guess), p^i (to identify i's ideal guess)$= \{[4, 6], 2\}$, of which $[4, 6]$ cannot be separated.

An L2 player i best responds to the belief that player j is L1, taking the adjustment of j's guess into account. An L1 player j's adjusted guess is $R(a^j, b^j; p^j[a^i + b^i]/2)$, so an L2 player i's ideal guess is $p^i R(a^j, b^j; p^j[a^i + b^i]/2)$, which will be automatically adjusted to $R(a^i, b^i; p^i R(a^j, b^j; p^j[a^i + b^i]/2))$. An L2 player i therefore has look-up sequence $\{([a^i, b^i], p^j)$ (to predict j's L1 ideal guess), a^j, b^j (to predict j's L1 adjusted guess), p^i (to identify i's ideal guess)$\} = \{([1, 3], 5), 4, 6, 2\}$.[26] This illustrates the fact that CGC's design separates the search implications of different types as strongly as it separates their implications for guesses [CGC 2006, figure 5].

In CGC's [2006] econometric analysis of search, not discussed here, search compliance for a given subject, type, and game is measured by the density of the type's complete minimal search sequence in the subject's look-up sequence for the game, allowing for the heterogeneity of search behavior.[27] CGC's measure is a significant advance on CGCB's measure, which is based on the percentages of a type's occurrence and adjacency requirements satisfied by the entire sequence.

Sample Search Data

Table 10.2 gives a sample of the information search data for CGC's robot/trained subjects, and table 10.3 gives an analogous sample for baseline subjects of various assigned or apparent types (see table 10.1 for the search implication for each type). Table 10.4 shows how the numbers in tables 10.2 and 10.3 are used to represent different MouseLab boxes. In each case, the subjects were chosen for high exact compliance with their types' guesses, not for compliance with any theory of search; subjects' frequencies of exact guesses are in parentheses after their types. Only the orders of look-ups are shown, and only from the first two or three games, but those games are representative.

Recalling that the theory allows any order of look-ups grouped within brackets, braces, or parentheses, the searches of high-guess-compliance robot/trained or baseline subjects conform closely to CGC's theory, with a subject's assigned or apparent type's minimal sequence unusually dense in his observed sequence.[28] The only exception is the equilibrium subjects, who search far longer and in more

Table 10.2. Selected Robot/Trained Subjects' Information Searches.

Subject	Type/Alt[a]	Game 1[b]	Game 2[b]
904	L1 (16)	1234564623	1234564321
1716	L1 (16)	14646213464623	46246213
1807	L1 (16)	462513	46213225
1607	L2 (16)	1354621313	1354613546213
1811	L2 (16)	1344465213*46	13465312564231356252
2008	L2 (16)	1113131313135423	131313566622333
1001	L3 (16)	46213521364*24623152	4621356425622231462562*62
1412	L3 (16)	1462315646231	462462546231546231
805	D1 (16)	1543564232132642	51453561536423
1601	D1 (16)	25451436231	5146536213
804	D1 (3)/L2 (16)	1543465213	5151353654623
1110	D2 (14)	1354642646*313	135134642163451463211136 414262135362*146546
1202	D2 (15)	246466135464641321342462 4226461246255*1224654646	123645132462426262241356 462*135242424661356462
704	DEq (16)	123456363256565365626365 6526514522626526	123456525123652625635256 262365456
1205	Eq (16)	1234564246525625256352*465	123456244565565263212554 14666265425144526*31
1408	Eq (15)	12312345644563213211	1234564561236435241
2002	Eq (16)	142536125365253616361454 61345121345263	14362536142514252362556563

[a] Shows the assigned type of each subject and, in the case of subject 804, an alternative assignment, as well. The subjects' frequencies of making their assigned types' (and, where relevant, alternatives types) exact guesses are in parentheses after the assigned type.
[b] An asterisk in a subject's look-up sequence means that the subject entered a guess without immediately confirming it.

complex patterns than CGC's theory suggests, perhaps because its minimal equilibrium search requirements allow more luck than these subjects enjoyed.[29] Baseline L1, L2, and perhaps L3 and equilibrium subjects' searches are very close to those of their robot/trained counterparts, suggesting that (unlike in CJ) training had little effect on their search behavior.[30] Perhaps equilibrium search in normal-form games is less unnatural than backward-induction search in CJ's extensive-form games. For the simpler types L1, L2, and perhaps L3, the algorithms that subjects use to identify their types' guesses can be directly read from their searches.

CGC's [2006, section II.E, table 7] econometric analysis shows that such inferences are usually consistent with estimates based on guesses alone, and that search compliance as measured here is also useful in identifying the types of subjects whose types are not apparent from their searches. For some subjects, econometric estimates based on guesses and search together resolve tensions between guesses-only and search-only estimates in favor of a type other than the guesses-only estimate.

Table 10.3. Selected Baseline Subjects' Information Searches.

Subject	Type/Alt[a]	Game 1[b]	Game 2[b]	Game 3[b]
101	L1 (15)	146246213	46213	462*46
118	L1 (15)	24613462624132*135	2462622131	246242466413*426
413	L1 (14)	1234565456123463*	12356462213*	264231
108	L2 (13)	135642	1356423	1356453
206	L2 (15)	533146213	53146231	5351642231
309	L2 (16)	1352	1352631526*2*3	135263
405	L2 (16)	144652313312546232 12512	1324562531564565 4546312315656262	3124565231*123654 55233**513
210	L3 (9) Eq (9) D2(8)	123456123456213456 254213654	1234564655622316 54456*2	1234556456123
302	L3 (7) Eq (7)	221135465645213213 45456*541	2135465662135454 6321*26654123	265413232145563214 563214523*654123
318	L1 (7) D1 (5)	13245646525213242* 1462	132465132*462	1346521323*4
417	Eq (8) L3 (7) L2 (5)	252531464656446531 6412524621213	25523662*3652435 63	5213636415265263* 652
404	Eq (9) L2 (6)	462135464655645515 21354*135462426256 356234131354645	46246135252426131 5463562	462135215634*52
202	Eq (8) D2 (7) L3 (7)	123456254613621342 *525	1234564456132554 6251356523	1234561235623
310	Eq (11)	123126544121565421 254362*21545 4*	1235462163262314 56*62	123655463213
315	Eq (11)	213465624163564121 325466	1346521246536561 213	132465544163*3625

[a] Shows the assigned type of each subject and, in some cases, an alternative assignment, as well. The subjects' frequencies of making their assigned types' (and, where relevant, alternatives types) exact guesses are in parentheses after the assigned type.
[b] An asterisk in a subject's look-up sequence means that the subject entered a guess without immediately confirming it.

Table 10.4. MouseLab Box Numbers.

Person	a	p	b
You (i)	1	2	3
S/he (j)	4	5	6

Those estimates confirm the presence of significant numbers of subjects of types L1, L2, equilibrium, or hybrids of L3 and/or equilibrium in the population, and the absence of significant numbers of subjects of other types. Once again, subjects' deviations from equilibrium can be attributed mostly to nonequilibrium strategic thinking, not preferences or irrationality.

For some subjects, search is an important check on type inferences based on guesses. Baseline subject 309, whose 16 exact L2 guesses seem overwhelming evidence that his type is L2, violated L2 occurrence by missing one of its required look-ups in each of games 1–5 (table 10.3 shows his look-ups for games 1–3). Just as for CJ's subjects who never looked at the second- or third-round pie, in games 1–5 this subject could not have been making L2 guesses for the reason the theory assumes, and his compliance could not be expected to persist beyond sample. Fortunately, 309 had a Eureka! moment after game 5, and from then on complied almost perfectly with L2's search requirements as well as its guess requirements.[31]

Further Questions Search Analysis Might Answer

To illustrate some of the further possibilities for search analysis, this section discusses two questions raised by CGC's [2006] analysis of decisions that resist analysis via decisions alone. These questions are addressed in CGC [2007].

What Are CGC's Baseline Apparent Equilibrium Subjects Really Doing?

Figure 10.5 [CGC 2006, figure 4] graphs the guesses of CGC's eight baseline subjects with seven or more exact equilibrium guesses. The 16 games are ordered by strategic structure as in CGC [2006, table 3] (not in the randomized order in which subjects played them), with the eight games with mixed targets (one greater and one less than one) in the right half of the figure. Of these subjects' 128 guesses in the 16 games, 69 (54%) were exact equilibrium guesses. In CGC's [2006] likelihood-based econometrics, given their a priori specification of possible types and the large strategy spaces of CGC's games, this is overwhelming evidence that their types are equilibrium. But as figure 10.5 makes clear, their equilibrium compliance was far higher for games without mixed targets (55 out of 64 possible exact equilibrium guesses, or 86%) than for games with mixed targets (14 out of 64, or 22%). Thus, it is (even nonparametrically) clear that these subjects, despite equilibrium compliance that is off the scale by normal standards, are actually following a rule that only mimics equilibrium, and only in games without mixed targets.

The puzzle is deepened by noting that all the ways game theorists teach people to identify equilibria (best-response dynamics, equilibrium checking, and iterated dominance) work equally well with and without mixed targets. Further, CGC's equilibrium robot/trained subjects who were taught these three ways to identify their equilibrium guesses have roughly the same equilibrium compliance with and without mixed targets [figure 10.6; CGC 2007]. Thus, whatever the baseline apparent

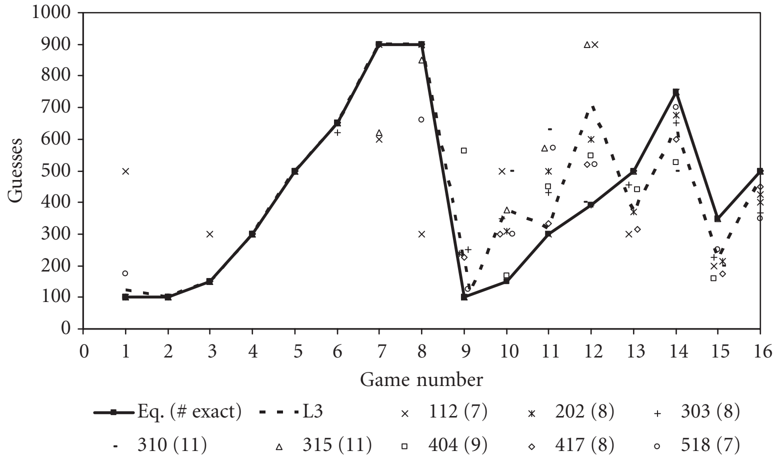

Figure 10.5. "Fingerprints" of eight apparent equilibrium baseline subjects. Only deviations from equilibrium's guesses are shown; 69 (54%) of these subjects' 128 guesses were exact equilibrium guesses. From Costa-Gomes and Crawford [2006, figure 4].

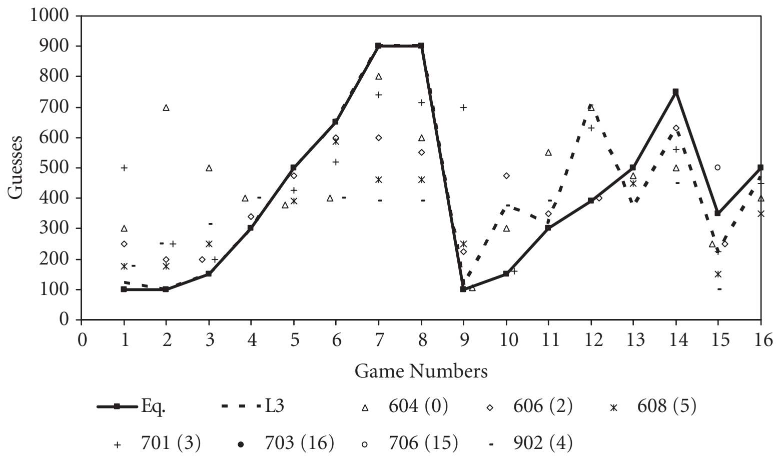

Figure 10.6. "Fingerprints" of 10 UCSD equilibrium robot/trained subjects. Only deviations from equilibrium guesses are shown; three subjects (603, 704, 705) had 16 exact guesses, and 92 (58%) of these subjects' 160 guesses were exact equilibrium guesses.

equilibrium subjects were doing, it is not one of the first things a game theorist would think of. (Subjects' debriefing questionnaires did not reveal what it was.) Nonetheless, the rule or rules they follow have a decidedly nonrandom structure: all 44 of those subjects' deviations from equilibrium (the solid line in figure 10.5) when it is separated from L3 (dotted line), with or without mixed targets, are in the direction of (and sometimes beyond) their L3 guesses, though this could reflect the fact that in CGC's games, L3 guesses are always less extreme than equilibrium guesses.

CGC's [2007] analysis tries to resolve the puzzle by using the search data to answer the following questions:

1. How do the baseline apparent equilibrium subjects find their equilibrium guesses in the games without mixed targets: best-response dynamics, equilibrium checking, iterated dominance, or something else that doesn't "work" with mixed targets? Refining CGC's [2006] characterization of equilibrium search to separate the three methods and redoing the estimation with the refined compliance measures, separately for games with and without mixed targets, should be revealing. The absence of baseline Dk subjects suggests that iterated dominance, even finitely truncated, is unlikely. Best-response dynamics, perhaps truncated after one or two rounds, seems more likely.
2. How do the baseline apparent equilibrium subjects' search patterns differ in games with and without mixed targets? How do the differences compare to the differences for baseline L1, L2, or L3 subjects? CGC's 20 apparent baseline L1 subjects' compliance with L1 guesses is almost the same with and without mixed targets [CGC, 2006, figure 1], which is unsurprising because whether or not the targets are mixed is irrelevant to subjects who do not try to model others' responses to incentives. But the 12 apparent L2 [figure 10.4; CGC, 2006, figure 2] and 3 apparent L3 [CGC, 2006, figure 3] subjects' compliance with their types' guesses is much lower with than without mixed targets. This is curious, because for L2 and L3, unlike for equilibrium, games with mixed targets require no deeper understanding.
3. How do equilibrium robot/trained subjects with high compliance find their equilibrium guesses even in games with mixed targets? How do their searches in those games differ from baseline apparent equilibrium subjects' searches? CGC strove to make equilibrium robot/trained subjects' training as neutral as possible, but something must come first, and they were taught equilibrium checking first, then best-response dynamics, then iterated dominance. To the extent that these subjects used one of those methods, it explains why they have equal compliance with and without mixed targets. But if some of them used something else that deviates from equilibrium mainly in games with mixed targets, it might provide important clues to what the baseline equilibrium subjects did.

Why Are L*k* the Only Types Other Than Equilibrium with Nonnegligible Frequencies?

CGC's [2006] analysis of decisions and search estimated significant numbers of subjects of types L1, L2, equilibrium, or hybrids of L3 and/or equilibrium, and nothing else that does better than a random model of guesses for more than one subject.

Why do these types predominate, out of the enormous number of possibilities? Why, for instance, are there no significant numbers of Dk types, which are closer to what game theorists teach?

CGC's [2007] analysis tries to answer this question by using search and other methods to look more deeply into the following phenomena:

1. Most robot/trained subjects could reliably identify their type's guesses, even for types as difficult as equilibrium or D2. Individual subjects' exact compliance with their type's guesses was usually bimodal within type, on very high and very low. Even so, there are several signs of differences in difficulty across types.
2. None of CGC's 70 robot/trained Lk subjects ever failed their type's understanding test, while 1 of 31 failed the D1 test, 1 of 20 failed the D2 test, and 7 of 36 failed the equilibrium test.
3. For those who passed the test, compliance was highest for Lk types, then equilibrium, then Dk types. This suggests that Dk is harder than equilibrium, but more analysis is needed to tell if this was an artifact of the more stringent screening of the equilibrium test.
4. Within the Lk and Dk type hierarchies, compliance was higher for lower k as one would expect, except that L1 compliance was lower than L2 or L3 compliance. This may be because L1 best responds to a random L0 robot, which some subjects think they can outguess, but L2 and L3 best respond to a deterministic L1 or L2 robot, which doesn't invite gambling.
5. Remarkably, 7 of our 19 robot/trained D1 subjects who passed the D1 understanding test, in which L2 answers are wrong, then "morphed" into L2s when making their guesses, significantly reducing their earnings (figure 10.7 and subject 804 in table 10.2; recall that L2 and D1 are cousins, both making 2-rationalizable guesses). This kind of morphing is the only kind that occurred, which seems compelling evidence that Dk types are unnatural. But a comparison of Lk's and D$k-1$'s search and storage requirements may have something to add.

A Rational-Choice Model of Optimal Search for Hidden Payoff Information

This section outlines a simple rational-choice analysis in support of the occurrence and adjacency assumptions that underlie CGCB's and CGC's models of cognition and search. The analysis is general in that it takes as given the formula that

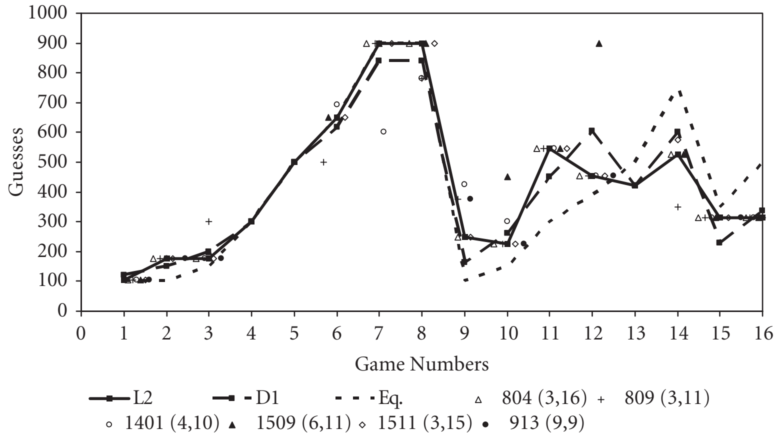

Figure 10.7. "Fingerprints" of six York robot/trained subjects who "morphed" from dominance-1 (D1) to level-2 (L2). Only deviations from D1's guesses are shown; 28 (29%) of these subjects' 96 guesses were exact D1 guesses, and 72 (75%) were exact L2 guesses.

relates a type's decision to the hidden parameters. It views search for hidden payoff information as just another kind of rational decision, deriving subjects' demand for it from the benefits of making better decisions under plausible assumptions about the benefits and costs of search and storing numbers in working memory.

The model rests on two assumptions about cognition and search:

1. The costs of look-ups are small. There is a great deal of evidence that subjects in experiments with hidden but freely accessible payoff parameters perceive the cost of looking them up as negligible, scarcely larger than the cost of reading them in a printed table. Having to look things up has small effects on their decisions (as shown in CGCB's and CGC's [2006] open boxes control treatments); subjects usually make many more look-ups than efficient search requires, and they usually make some motivated purely by curiosity.
2. There is a flow cost of keeping numbers in working memory, which starts small for the first number but even then is larger than the cost of a look-up, and which increases with the number of stored numbers. Total memory cost is the time integral of the flow cost and is therefore proportional, other things equal, to total storage time, and increasing in the number of stored numbers. (If working memory were free, nothing would prevent the scanning and memorization referred to in my discussions of CJ and CGCB, but this is plainly unrealistic.)

Occurrence follows immediately from assumption 1. A rational player looks up all costlessly available information that might affect his beliefs. When, as in these designs, information comes in discrete quanta with nonnegligible effects on beliefs and the optimal decision, this conclusion extends to information available at low cost.[32]

Given occurrence, adjacency [in CGC's sense that the basic (innermost) operations in square brackets in the right-hand side of Table 10.1 are represented by adjacent look-ups] follows from assumption 2. Under this assumption, a player minimizes the total memory plus look-up cost by processing the basic operations needed to evaluate the expression for his ideal guess before other operations with whose results they are to be combined, storing the results (meanwhile "forgetting" the parameters they combine), and then combining them. Basic operations take precedence over other operations because "distributing" them increases memory cost.[33] For example, in evaluating the expression $p^i[a^j + b^j]/2$ for L1's ideal guess, processing $[a^j + b^j]$ first, storing the result, and then combining it with p^i yield the following sequence of numbers of numbers in working memory: 1, 2, 1, 2, 1. The distributed alternative of processing $p^i\ a^j$, storing the result, and then processing p^i b^j and combining it with $p^i\ a^j$ yields the sequence 1, 2, 1, 2, 3, 2, 1, which dominates the first sequence. The first method also saves the cost of looking up p^i a second time, but this is much less important.

Although occurrence and adjacency are only necessary conditions for optimal search, I stop with them because they have considerable empirical support, they make the main patterns of subjects' search behavior in CJ's extensive-form and CGCB's and CGC's normal-form games intelligible, and they seem more transparent than other conditions for optimality and thus more likely to be descriptive of subjects' search behavior.

I close by noting that although this model supports CJ's, CGCB's, and CGC's use of occurrence and adjacency, it says nothing directly about how to measure search compliance in an econometric analysis. CGC's use of the density of a type's minimal search sequence in the part of the observed sequence where the subject tends to make his relevant look-ups (his search "style," in CGC's terminology) is a judgment call, which seems to be well supported by inspectingthe data.

Conclusion

CJ's, CGCB's, and CGC's analyses of cognition in games via monitoring subjects' searches for hidden but freely accessible payoff information bridge part of the gap between neuroeconomics and conventional economics because they rest on explicit models of cognition, but search, unlike neural correlates of cognition, can be viewed as a rational choice. This chapter has used those analyses to make two points about the potential uses of neural data in economics.

First, standard assumptions of rational choice and equilibrium have yielded successful explanations of many phenomena, which as GP note can usefully be tested via revealed preference analysis of decision data. But there are other, equally important phenomena that appear to stem from failures of the implicit assumptions about cognition that underlie standard analyses, for which tests that don't take cognition explicitly into account are likely to be biased and misleading.

Second, with unbounded capacity for experimentation, it might be possible to discover all we need to know about behavior by observing decisions alone. But this is an arbitrary constraint, and CJ's, CGCB's, and CGC's analyses show that expanding the domain of analysis beyond decisions can yield a clearer view of behavior than is practically achievable by observing only decisions.

NOTES

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1. GP do allow an "inspirational" role for data other than decisions, but they exclude such data from theory testing.

2. Why study strategic thinking when with enough experience in a stationary environment, even amoebas—or human reinforcement learners, who need not even know that they are playing a game—usually converge to equilibrium? Many applications of game theory involve situations with no clear precedents. (Should you sell U.S. airline stocks when the market reopens after 9/11, or buy them on the anticipation that others will overreact?) Comparative statics and design questions inherently involve new games with new equilibria, which players cannot reach by copying behavior from analogous games. In such situations, subjects' initial responses are often plainly "strategic" but nonetheless deviate from equilibrium. Even in settings in which players can be expected to converge to equilibrium, the structure of strategic thinking can influence the rate of convergence and equilibrium selection.

3. This conclusion is consistent with SW's, Nagel's, HCW's, and CGCB's results, but their evidence is less clear.

4. CGCB's and CJ's analyses make this point in different ways. Camerer (chapter 2), Caplin (chapter 15), and Schotter (chapter 3) argue cogently, in complementary ways, for the use of nonchoice data and outline frameworks to guide their use in analyses. Köszegi and Rabin (chapter 8) discuss using decision data to distinguish intended decisions from errors.

5. Or, if the rules and possible preferences allow multiple equilibria, the equilibrium identified by some agreed-upon selection principle. Although GP's chapter focuses entirely

on individual decisions, private communications suggest that they accept the need for extending their proposal to games by assuming equilibrium.

6. The proposed explanation differs greatly from classical search theory in purpose, but only slightly in methods.

7. "Eureka!" learning remains possible, but it can be tested for and seems to be rare. Initial responses yield insights into cognition that also help us think about how to model learning from experience, but that is another story.

8. Access was via a MouseLab interface that automatically records a sequence of parameter opening and closing times, which makes it possible to test models of the order and/or duration of parameter look-ups. Subjects were not allowed to write, and the frequencies with which they looked up the parameters made clear that they did not memorize them. Subjects were taught the mechanics of looking up parameters and entering decisions, but not information-search strategies. MouseLab is an automated way to track search as in eye-movement studies of individual decisions (Payne, Bettman, and Johnson [1993]; www.cebiz.org/mouselab.htm). Wang, Spezio, and Camerer [2006] illustrate the use of a modern, more powerful eye-tracking method.

9. A partial exception is that CJ's experiments evoked nonpecuniary social preferences like those in ultimatum experiments, and these and subjects' risk aversion are uncontrolled and privately known. Privately known social preferences are easily accommodated in the analysis of CJ's results, and risk aversion was probably insignificant.

10. CJ's [1993, 2002] designs differed in some ways, for example framing in losses versus in gains, that are not important for my purposes and are not discussed here. At roughly the same time in the early 1990s, Camerer and Johnsan [2004] did a MouseLab study of forward induction in extensive-form games. Algaze (Croson) [1990] reported a very brief study of search for hidden payoff information in matrix games. Neither of the latter papers is discussed here.

11. CJ used a "rollover" option in MouseLab, in which subjects could open the box that concealed a pie by moving the cursor into it, revealing the pie for as long as the cursor was in the box. Subjects could also use the interface to look up their roles in each round, but these were known, and those look-ups were not reported or analyzed.

12. Only "mostly" because with only pecuniary preferences, the first-round pie, as long as it is large enough, does not affect the equilibrium initial offer. With social preferences, the first-round pie may be relevant because it may influence the responder's acceptance decision.

13. This informal definition, like the one for occurrence, is intentionally vague regarding how often look-ups or operations appear to accommodate variations in CJ's, CGCB's, and CGC's use of occurrence and adjacency. The notions are made more precise in CGCB's analysis and, as explained below, CGC's. Note that both are general restrictions on how cognition drives search, which can be applied across a variety of games and decision rules.

14. Instead of the rollover option CJ used, CGCB used a "click" option, in which subjects could open a box by moving the cursor into it and left-clicking the mouse. Before he could continue, a subject had to close the box by right-clicking, which could be done from anywhere in the display.

15. Lk's and D$k-1$'s decisions both survive k rounds of iterated elimination of dominated decisions and so in two-person games are k-rationalizable [Bernheim 1984].

Although D$k - 1$ types are closer to how theorists analyze games, Lk types seem more natural and predominate in applications.

16. Because a type's search implications depend not only on what decisions it specifies, but also on why, something like a types-based model seems necessary here. In CJ [1993], types are implicit in the discussion and limited to two, which might be called "subgame-perfect equilibrium" and "other." CJ [2002] adapted CGCB's analysis by defining extensive-form "types" modeled after CGCB's and SW's normal-form types, using them to construct a more structured data analysis than CJ's [1993].

17. In previous designs, the targets and limits were the same for both players and varied only across treatments.

18. Eleven of these subjects were from an "open boxes" treatment, not discussed here, identical to the baseline but with the parameters continually visible. The results of this treatment (and analogous treatments in CJ and CGCB) confirm that making subjects look up the parameters does not significantly affect their decisions, so the data can be pooled with baseline decision data, as here. CGC's open boxes subjects have numbers that begin with a 5.

19. By contrast, in SW's or CGCB's matrix-game designs, even a perfect fit does not distinguish a subject's best-fitting type from nearby omitted types; and in Nagel's and HCW's guessing-game designs, with large strategy spaces but with each subject playing only one game repeatedly, the ambiguity is worse.

20. Nagel's results are often viewed as evidence that subjects perform finitely iterated dominance, as in D$k - 1$. But Lk's and D$k - 1$'s decisions are perfectly confounded in Nagel's main treatments and weakly separated in Nagel's and HCW's other treatments and in CGCB's design. CGC's clear separation of Lk from D$k - 1$ allows them to conclude that Dk types don't exist in significant numbers, at least in this setting, and thus that subjects respect low levels of iterated dominance as a by-product of following Lk types, not because they explicitly perform it. Sophisticated, which is clearly separated from equilibrium, also doesn't exist in significant numbers. CGC's [2006, section II.D] specification test rules out significant numbers of other types omitted from the specification.

21. By contrast, the lack of a simple common structure in CGCB's design makes rules' search implications vary from game to game in ways so complex you need a "codebook" to identify them.

22. CGCB and CGC made no claim that durations are irrelevant, just that durations don't deserve priority. CGCB [table IV] present some results on durations under the heading of "gaze times."

23. Wang, Spezio, and Camerer's [2006] eye-tracking methods have an advantage in avoiding this ambiguity.

24. In CGC's design, unlike in CGCB's, equilibrium's minimal search implications are simpler than any boundedly rational type's implications. This makes it harder to explain deviations from equilibrium by cognitive complexity. But we will see that high-compliance equilibrium robot/trained subjects search more than high-compliance robot/trained subjects of other types, so CGC's equilibrium search implications may not reflect its complexity.

25. L1's search implications illustrate an important advantage of the automatic adjustment feature of CGC's design. L1's ideal guess depends on its own target but only its partner's limits, while L2's and D1's depend on both players' targets and limits and

equilibrium's depends on both players' targets and a combination of its own and its partner's lower or upper limits. In other designs, such as CGCB's, L1's decisions almost inevitably depend only on its own payoff parameters, and more sophisticated types' decisions depend on both own and other's parameters. Thus, the automatic adjustment feature allows CGC to separate solipsism from the strategic naivete of L1. CGC's data give no evidence of solipsism, but a great deal of evidence of naivete. CGC's data also show that most subjects understood and relied upon automatic adjustment, which was carefully explained to them.

26. With automatic adjustment, an L2 player i does not need to know his own limits to play the game or think about the effects of his own guess being adjusted, only to predict j's L1 guess. By contrast, an L1 player i doesn't need to know his own limits, only j's. Because the possible values of the limits are not public knowledge, an L2 player i cannot infer that adjustment of player j's ideal guess can occur only at his upper (lower) limit when $p^j > 1$ ($p^j < 1$). An L2 subject who incorrectly infers this may omit $a^j = 4$ ($b^j = 6$) when $p^j > 1$ ($p^j < 1$).

27. As is evident from Tables 10.2 and 10.3, subjects' look-up sequences vary widely in what CGC called "style" : Most robot/trained and baseline subjects with high exact compliance consistently look first at their type's minimal search sequence and then continue looking, apparently randomly, or stop and enter their guess (for example L2 robot/trained subject 910, L3 subject 1008, and D1 subject 1501 in Table 10.2; and L2 baseline subjects 108 and 206 in Table 10.3). But some such subjects look randomly first and turn to the relevant sequence at the end (L1 robot/trained subject 904). CGC's [2006, Section II.E] econometric analysis uses a binary nuisance parameter to distinguish these "early" and "late" styles and filter them out to obtain a better measure of search compliance.

28. CGC's specification analysis turned up only one clear violation of their proposed characterization of types' search implications, which is instructive. Baseline subject 415 (not shown in table 10.3), whose apparent type was L1 with nine exact guesses, had zero L1 search compliance in 9 of the 16 games because he had no adjacent $[a^j, b^j]$ pairs. His look-up sequences, however, were rich in (a^j, p^i, b^j) and (b^j, p^i, a^j) triples, in those orders, but not in such triples with other superscripts. This strongly suggests that 415 was an L1 who happened to be more comfortable with three numbers in working memory than CGC's characterization of search assumes, or than their other L1 subjects were. But because this violated CGC's assumptions on search, this subject was "officially" estimated to be D1.

29. One of the methods CGC allow for identifying equilibrium guesses is equilibrium checking, which has the least search requirements among all methods. Equilibrium checking can identify the equilibrium guess very quickly if the player has the luck to check the equilibrium first [CGC 2006, appendix H; CGC 2007]. Allowing this is unavoidable without risking incorrectly concluding that a subject has violated equilibrium's search implications.

30. CGC's baseline subjects with high compliance for some type are like robot/untrained subjects, which do not usually exist because one cannot tell robot subjects how they will be paid without training them in how the robot works. These "naturally occurring" baseline robot subjects provide an unusual opportunity to separate the effects of training and strategic uncertainty, by comparing their behavior with robot/trained subjects' behavior.

31. Subject 309 omitted look-ups 4 and 6 (his partner's lower and upper limits) in game 1 and look-up 4 in games 2–5. This suggests that he did not yet understand the need to check his partner's lower limit to be sure of his L2 guess even when his own target, or the product of targets, was greater than 1. However, he omitted look-up 4 even in game 4 where both targets were less than 1, showing that his error was probably more complex. That these omissions did not lead to non-L2 guesses in games 1–5 is an accident of our design with no greater significance.

32. Note that because MouseLab allows a subject to enter a tentative guess without confirming it (the asterisks in the data in tables 10.2 and 10.3), thereby saving storage cost, the variations in search style described in note 24 are consistent with optimality when look-up costs are negligible even if storage costs are not.

33. This effect is related to the reason that backward induction is the most efficient way to solve a finite-horizon dynamic programming problem such as those that subjects faced in CJ's design: other ways are feasible, but wasteful of storage and computational capacity (though the latter is assumed to be freely available here).

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