STRATEGIC THINKING

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Abstract: Most applications of game theory assume equilibrium, justified by presuming that learning will have converged to one; or in settings where that is implausible, that equilibrium approximates people's strategic thinking without learning. Yet recent experimental work suggests that initial responses to many kinds of games deviate systematically from equilibrium, and that certain nonequilibrium models can then out-predict equilibrium models of thinking. Even when learning converges to equilibrium, such nonequilibrium models of initial responses allow better prediction of history-dependent limiting outcomes. This paper reviews recent theoretical and empirical work on nonequilibrium models of strategic thinking and illustrates their applications in economics.

Keywords: behavioral game theory, experimental game theory, strategic thinking, Nash equilibrium, quantal response equilibrium, level-*k* models, cognitive hierarchy models, coordination, salience, strategic communication
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1. *Introduction*

Strategic thinking pervades human interaction. As soon as children develop enough “theory of mind” to model other people as independent decision makers, they must be taught to look both ways before crossing one-way streets—suggesting that they instinctively assume rationality in predicting others’ decisions.[2] Our adult attempts to predict other people’s responses to incentives are shaped by similar, though usually more subtle, rationality-based inferences.

The canonical model of strategic thinking is the game-theoretic notion of Nash equilibrium, defined as a combination of strategies, one for each player, such that each player’s strategy maximizes his expected payoff, given the others’ strategies. Although equilibrium can be defined and applied without reference to its interpretation, it is best thought of as an “equilibrium in beliefs,” in which players who are rational in the decision-theoretic sense have beliefs about each other’s strategies that are correct, given the rational strategy choices they imply.

In games, rationality alone seldom restricts behavior enough to be useful. Equilibrium therefore augments rationality with the “rational-expectations” assumption that players’ beliefs are correct.[3] The structure this adds often gives a precise and plausible account of strategic behavior; and the generality, simplicity, and tractability of equilibrium analysis have made it the method of choice in strategic applications (Roger B. Myerson 1999).

Although equilibrium is almost universally assumed in applications, it is better justified in some than others. When players have abundant prior experience with perfectly analogous games, both theory and experimental results suggest that under mild assumptions about cognition, learning has a strong tendency to converge to equilibrium (e.g. Drew Fudenberg and David K. Levine 1998; Camerer 2003, Chapter 6; Camerer and Ho 1999).[4]

However, in many settings players’ interactions have only imperfect precedents, or none at all. If assuming equilibrium is justified in such settings, it must be via strategic thinking rather than learning. The literature on epistemic game theory (e.g. Adam Brandenburger 1992) gives conditions under which reasoning based on iterated knowledge of rationality and beliefs focuses

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[2] In this case their reliance on rationality is excessive, which is why adults have something to teach them. This example originally appeared in Camerer (2003, Chapter 1), courtesy of one of the authors.

[3] Even common knowledge of rationality implies only that players’ strategies are rationalizable (Bernheim 1984 and David Pearce 1984), which leaves behavior completely unrestricted in many games of interest to economists. Because many interesting games have multiple equilibria, equilibrium is often further augmented by refinements, as in Eric Van Damme (1987) or John C. Harsanyi and Reinhard Selten (1987), with the goal of deriving unique predictions.

[4] In reinforcement learning, for example, players need not even know that they are playing a game. Our statement omits some qualifications that are important only for extensive-form games.

players' beliefs on a particular equilibrium, even in their initial responses to a game. But in many games such reasoning is complex, making the thinking justification for equilibrium behaviorally far less plausible than the learning justification is when players have abundant prior experience.

This makes the desirability of improving upon equilibrium models of initial responses clear in applications involving novel games.[5] But better models of initial responses may help even in applications where it is reasonable to assume that learning has already converged to equilibrium. In many such applications, an equilibrium is selected from multiple possibilities via history-dependent learning dynamics, whose limiting outcome is influenced by players' initial responses (Crawford 1995; John Van Huyck, Joseph Cook, and Raymond Battalio 1997). And in other applications, such as the FCC spectrum auction (R. Preston McAfee and John McMillan 1996), initial responses are important for their own sake. In either case, better models of strategic thinking allow more useful predictions than those based on equilibrium analysis alone.

Even those who grant the potential desirability of improving upon equilibrium models of initial responses may doubt its feasibility. How can any model systematically out-predict a rational-expectations notion such as equilibrium? And how can one identify simple models that allow such improvements among the huge number of logically possible nonequilibrium models? We suspect that when equilibrium is assumed despite weak justification, or when the scope of its learning justification is overestimated, it is because analysts hope equilibrium will still be correct on average, or fear that without equilibrium there can be no basis for analysis.

Yet there is now a large body of experimental research that studies strategic thinking by eliciting initial responses to games, which suggests that in many applications in which equilibrium is assumed, neither the hope nor the fear is justified. This paper reviews the theoretical and experimental/empirical research on strategic thinking and explores its implications for modeling strategic behavior.[6]

[5] An analysis of strategic thinking may also eventually add to our understanding of learning from imperfect analogies as well, but that possibility will not be discussed here. Such analysis can also yield insights into cognition that elucidate the structure of learning rules, where assumptions about cognition determine which analogies between current and previous games players recognize and sharply distinguish reinforcement from beliefs-based and more sophisticated rules.

[6] See also the important contribution of Camerer, Ho, and Juin Kuan Chong (2004) and other contributions summarized in Camerer (2003, Chapter 5) and Crawford (1997, Sections 4 and 5). Although most empirical work in economics has relied on field data, laboratory experiments have played the leading role in empirical work on strategic behavior. Because behavior in games is notoriously sensitive to the details of the environment, strategic models carry a heavy informational burden, which is often compounded in the field by an inability to observe all relevant variables. Important advances in experimental methods over the past few decades allow a control that often gives experiments a decisive advantage in identifying the relationship between behavior and the environment. We discuss clear evidence from field data below whenever possible, but the bulk of our discussion of necessity concerns experimental data.

The experimental research shows with progressively increasing clarity that people's responses to novel games often deviate systematically from equilibrium. The results also show that the deviations have a large structural component that can be modeled in a simple way: Thinking systematically avoids the fixed-point or indefinitely iterated dominance reasoning that equilibrium sometimes requires, in favor of rules of thumb that anchor beliefs in an instinctive reaction to the game and then adjust them via a small number of iterated best responses.[7]

These rules of thumb—called "types" in this context (no relation to players' private-information variables)—are cognitively simple, have strong intuitive appeal, and correspond closely to clear informal descriptions of strategic thinking. Although people's thinking is typically heterogeneous, their types are drawn from a population distribution concentrated on one to three best-response iterations. The results identify a class of "level-*k*" or "cognitive hierarchy" ("CH") models that share the generality and much of the simplicity and tractability of equilibrium analysis, but which can in many settings systematically out-predict equilibrium.[8]

Although level-*k*/CH models are alternatives to equilibrium analysis, they generalize equilibrium rather than replacing it. Level-*k* types are rational in the sense of best-responding to some beliefs; they depart from equilibrium only in that their beliefs are derived from simplified, nonequilibrium models of other players.[9] In sufficiently simple games, the low-level types that describe most subjects' behavior mimic equilibrium strategy choices, even though they deviate from equilibrium thinking. But in more complex games, some or all such types may deviate systematically from equilibrium choices. Importantly, the models not only predict that such deviations will sometimes occur: They also identify which settings evoke deviations; what forms they take; and, given the population type frequencies, with what frequencies they occur.

We stress that while level-*k*/CH models appear to predict a sizeable fraction of the deviations from equilibrium in many settings, they stop well short of predicting all deviations in all settings.

[7] As Selten (1998) put it, "Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties…. Boundedly rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found." We stress that there is no implication that learning cannot make people converge to something that an analyst would need fixed-point or multiple-round reasoning to characterize; just that such reasoning does not *directly* describe most people's thinking. Costa-Gomes, Crawford, and Bruno Broseta (2001, Table II) show that reliance on iterated dominance seldom goes beyond three rounds.

[8] In applications the behavioral parameters that describe this distribution are usually estimated from the data or calibrated using previous estimates. Although estimates vary somewhat across settings and populations, in most applications a stable distribution that puts significant probability only on the lowest levels captures the main deviations from equilibrium. We illustrate this below by using a representative constant calibration whenever possible.

[9] Type level-*k* (though not its CH counterpart beyond $k = 1$) respects *k*-rationalizability, the condition that corresponds in two-person games to the result of *k* rounds or iterated deletion of dominated strategies (Bernheim 1984).

Even so, we view it as encouraging that models as simple and tractable as these are can predict something as elusive as deviations from equilibrium. Moreover, the experimental results also suggest that the strategic thinking-related deviations the models do *not* predict may have little structure that can be predicted by models of comparable generality. Thus, level-*k*/CH models generalize equilibrium analysis in a way that is likely to be useful in settings where deviations from equilibrium are important, while ignoring little that cannot reasonably be modeled as errors. We therefore believe there is a strong case for adding such models to the analyst's toolkit.

Our discussion will illustrate several ways in which a level-*k*/CH analysis can improve upon an equilibrium analysis. In settings where the types that best describe most subjects' behavior mimic equilibrium choices, a level-*k*/CH analysis can establish the robustness of equilibrium predictions to deviations from their strong behavioral assumptions.[10] In settings where it is implausible to assume equilibrium, a level-*k*/CH analysis can challenge equilibrium predictions and resolve empirical puzzles by explaining the deviations from equilibrium some games evoke. Level-*k*/CH models also give a more plausible view of coordination than a traditional analysis of equilibrium selection. And finally, such models elucidate the effects of strategic communication in both "outguessing" games, where deception is an important factor that is ruled out by assuming equilibrium; and coordination games, where reassurance and symmetry-breaking are potentially important but have unrealistically limited scope when equilibrium is assumed.

The rest of this paper is organized as follows. To keep the discussion manageable, except as noted we assume that players have accurate models of the games they play and that, except for errors, their strategies are rational responses to some beliefs about others' strategies.[11] We also focus on normal-form games, including extensive-form games only to study communication.

Section 2 reviews the leading alternative models of strategic thinking, their cognitive requirements, and how they are implemented in applications. We start with equilibrium and continue with finitely iterated dominance and *k*-rationalizability (Bernheim 1984 and Pearce 1984); quantal response equilibrium ("QRE"; Richard S. McKelvey and Thomas R. Palfrey 1995); level-*k* (Rosemarie Nagel 1995, Dale O. Stahl and Paul Wilson 1994, 1995, Costa-Gomes, Crawford, and Broseta 2001, Costa-Gomes and Crawford 2006) and CH (Camerer, Ho,

[10] The robustness then resembles that established by a rationalizability-based analysis; but as we will see, a level-*k* /CH analysis adds useful structure, and its results may well deviate from equilibrium-based conclusions.

[11] Although people do sometimes misperceive the games they are playing or deviate from decision-theoretic rationality in the sense of best responses to some beliefs, and such misperceptions or deviations might interact with strategic thinking, those factors are conceptually distinct from our focus of thinking about others' responses to incentives.

and Chong 2004) models; and noisy introspection models ("NI"; Jacob K. Goeree and Charles A. Holt 2004).

The remaining sections interweave experimental evidence with strategic and economic applications, ordering topics by strategic rather than economic issues.[12]

Throughout the paper, the experimental evidence is linked to informal evidence from "folk game theory", which illustrates the need for nonequilibrium models of strategic thinking, the issues that a successful model must address, and the range of potential applications.

Our term is meant to suggest an analogy with folk physics, untrained people's intuitive beliefs about the laws of physics. Why study folk instead of "real" game theory? Folk physics is an imperfect reflection of real physics, but it yields considerable insight into human cognition. Folk game theory is an imperfect reflection of traditional game theory, but unlike folk physics it is a direct reflection of its observable counterpart, namely the part of behavioral game theory that concerns strategic thinking. Moreover, we shall argue that folk game theory provides powerful additional evidence for the lessons from experimental evidence about strategic thinking.[13]

Section 3 introduces the experimental evidence on strategic thinking, starting with guessing games in the style of Keynes' (1936, Chapter 12) beauty-contest example and other guessing and normal-form games with complete information (Nagel 1995; Stahl and Wilson 1994, 1995; Ho, Camerer, and Keith Weigelt 1998; Costa-Gomes, Crawford, and Broseta 2001; Antoni Bosch-Domènech et al. 2002; Costa-Gomes and Crawford 2006; and Costa-Gomes and Georg Weizsäcker 2008). This evidence generally favors level-*k*/CH models over the alternatives.

Section 4 illustrates the workings of level-*k*/CH and alternative models in more detail, considering complete-information outguessing games with unique mixed-strategy equilibria such as perturbed Matching Pennies, in which the key issue is how to respond to payoff asymmetries. In such games level-*k*/CH models' predictions "quasi-purify" something roughly like a mixed-strategy equilibrium via the predictable heterogeneity of players' strategic thinking, while avoiding some implausible comparative statics implications of equilibrium.

[12] Level-*k*/CH aficionados will see that the topics are grouped by the principles by which the anchoring *L0* type is specified, and ordered to facilitate explaining to non-aficionados how the models work and their economic implications.

[13] Michael Suk-Young Chwe (2010) gives a fascinating complementary discussion of folk game theory. We note that there may be selection effects, in that level-*k* reasoning may be easier to express in aphorisms than other kinds of strategic thinking. It may also be that people who have studied level-*k* models as much as we have are more likely to notice them in other people's writings. Even so, we think the extent to which folk game theory supports level-*k* models is surprising and informative.

Although most laboratory evidence on strategic thinking comes from designs that induce symmetric information, most field evidence comes from settings with clear informational asymmetries. Section 5 extends level-*k*/CH models to allow asymmetric information and to use the models to interpret the data from experiments and field settings. It first discusses evidence from experiments on leading examples of games with informational asymmetries: zero-sum betting (Camerer, Ho, and Chong 2004 and Isabelle Brocas, Juan D. Carrillo, Camerer, and Stephanie W. Wang 2010) and auctions with private information (Crawford and Iriberri 2007a). It then discusses the use of such models to analyze field data (Robert Östling, Joseph Tao-Yi Wang, Eileen Chou, and Camerer Forthcoming, and Alexander Brown, Camerer, and Dan Lovallo 2010). Section 5 concludes by discussing the design of revenue-maximizing auctions with level-*k* bidders (Crawford, Tamar Kugler, Zvika Neeman, and Ady Pauzner 2009).

Section 6 uses level-*k*/CH models to analyze coordination via symmetry-breaking. Following Camerer, Ho, and Chong (2004, Section III.C), complete-information versions of the models are used to explain the remarkable experimental results of Amnon Rapoport et al. (1998), Daniel Kahneman (1998), and Rapoport and Darryl A. Seale (2002), in which subjects playing Battle-of-the-Sexes or *n*–person market entry games achieve systematically better ex post coordination than in the symmetric mixed-strategy equilibrium benchmark. Section 6 then reviews recent work that uses incomplete-information CH models to analyze field evidence from market entry games (Avi Goldfarb and Botao Yang 2009; Goldfarb and Mo Xiao 2011).

Section 7 uses level-*k*/CH models to study coordination via assurance in Stag Hunt games with structures like Douglas W. Diamond and Philip H. Dybvig's (1983) model of bank runs. The workhorse model of behavior in bank runs games has been "global games" (Hans Carlsson and Eric van Damme 1993, Carlsson and Mattias Ganslandt 1998, Stephen Morris and Hyun Song Shin 1998, and David M. Frankel, Morris, and Ady Pauzner 2003), which achieves unique equilibrium selection without refinements via noncooperative arguments, which selection in simple bank-runs games coincides with risk-dominance. We argue in Section 7 that the global games approach viewed as a model of initial responses—as it must be to describe bank runs—has questionable behavioral foundations.

We then reconsider the global games approach from the viewpoint of level-*k*/CH models, with the goal of evaluating its robustness to failures of the equilibrium assumption or its model of equilibrium selection. The level-*k*/CH approach yields a quite different view of the process of coordination than the global-games approach, whose conclusions are nonetheless strikingly similar in the simplest bank-runs games. However, in more complex games equilibrium selection may not follow predictions based on risk-dominance or global games; and a level-*k*/CH analysis highlights an issue that is not considered in the global-games literature, how players model the correlation of others' strategy choices, on which the evidence challenges the standard view.

For settings involving market entry games, coordination games, or auctions in which equilibrium seems implausible, recent work has proposed "incomplete" models based on *k*-rationalizability (Andres Aradillas-Lopez and Tamer 2008; Federico Ciliberto and Tamer 2009). Other work has used the flexibility of incomplete models for settings where some equilibrium seems likely to emerge but equilibrium selection seems difficult to predict (Timothy F. Bresnahan and Peter C. Reiss 1991; Steve Berry and Elie Tamer 2006; Federico Echenique and Ivana Komunjer 2009). Although incomplete models have been extremely useful in econometric analyses of individual decisions (Charles F. Manski 2007), in strategic settings their prediction ambiguity often "multiplies up" to the point where rationalizability or equilibrium unaugmented by a model of selection yields very weak identification. This problem is illustrated by Aradillas-Lopez and Tamer's (2008) analyses of market entry and auction games. Section 8 considers the benefits for identification and estimation of using level-*k*/CH models as structural alternatives to such incomplete models, as in Costa-Gomes, Crawford, and Iriberri's (2009) analysis of coordination games and Gillen's (2010) analysis of auctions.

Sections 9 and 10 consider the new issues raised by games played on non-neutral salience landscapes. Section 9 discusses complete-information hide-and-seek and outguessing games where the key issue is how to respond to salience (Ariel Rubinstein 1999 and Crawford and Iriberri 2007b). Section 10 considers complete-information coordination games à la Thomas C. Schelling (1960), with asymmetric tensions between Schelling salience and the inherent salience of higher own payoffs (Crawford, Uri Gneezy, and Yuval Rottenstreich 2008).

Sections 11 to 13 consider models of strategic communication. Section 11 considers deception via communication of intentions in outguessing games (Crawford 2003). Section 12 studies communication of intentions in coordination games (Crawford 2007 and Tore Ellingsen and Robert Östling 2010). Section 13 studies communication of private information in sender-receiver games (Crawford 2003 and Joseph T.-Y. Wang, Michael Spezio, and Camerer 2010).

Section 14 is the conclusion.

2. *Alternative Models of Strategic Thinking*

Until recently the choices for modeling nonequilibrium initial responses to games were quite limited, but now there are several alternatives. This section sets the stage by reviewing them, their cognitive requirements, and how they are implemented in applications. We start with equilibrium and continue with finitely iterated (strict) dominance and *k*-rationalizability, quantal response equilibrium, level-*k* and cognitive hierarchy, and noisy introspection models.

2.1. *Equilibrium plus Noise*

Any notion that is to be taken to data must allow for errors in some way. The most obvious choice, "equilibrium plus noise", adds errors with a specified distribution with zero mean and estimated precision to equilibrium predictions. The distribution is often allowed to be sensitive to the payoff costs of deviations, as with logit errors; but in equilibrium plus noise, unlike in the QRE models discussed in Section 2.3, the payoff costs of a player's deviations from equilibrium are evaluated assuming that other players play their equilibrium strategies without errors.

In judging theories of strategic thinking, cognitive requirements are relevant because if a theory is not consistent with a player's thinking, it will predict behavior accurately in general only by chance. In particular, no "as if" thinking justification for equilibrium is plausible because if a player's thinking does not accurately model others' strategy choices, then except in games where the rules he follows mimic equilibrium, he will deviate systematically. Depending on the game, an equilibrium player can find his equilibrium decision via one of several methods, which sometimes require fixed-point or indefinitely iterated dominance reasoning. The more complex this reasoning becomes, the less behaviorally plausible it is as a model of thinking.

In many applications equilibrium plus noise fits experimental results well. But even in games with unique equilibria, subjects' initial responses often deviate systematically from equilibrium, in ways that are sensitive not only to a given subject's out-of-equilibrium payoffs when others play their equilibrium strategies but also to his out-of-equilibrium payoffs when others do not play their equilibrium strategies. Further, in games with multiple equilibria equilibrium plus noise is incomplete in that it does not specify a unique prediction conditional on the values of its behavioral parameters (in this case, the error precision). Such multiplicity has been dealt with by estimating an unrestricted probability distribution over equilibria (Bresnahan and Reiss 1991), but such a model may overfit the data (Costa-Gomes, Crawford, and Iriberri 2009). With multiple equilibria, to put equilibrium plus noise on an equal footing with the other models considered here, which except for *k*-rationalizability are complete, it is natural to add a coordination refinement such as Harsanyi and Selten's (1987) risk- or payoff-dominance.

2.2. *Finitely Iterated Strict Dominance and* k*-Rationalizability*

A common, plausible reaction to the behavioral implausibility of the thinking justification for equilibrium is to maintain some or all of equilibrium's reliance on players' rationality and iterated mutual knowledge of rationality while relaxing its strong rational-expectations assumption that players' beliefs are correct. This leads to set-valued restrictions on individual players' strategies derived from iterated deletion of strictly dominated strategies and the associated notions of rationalizability and *k*-rationalizability (Bernheim 1984 and Pearce 1984).[14]

k-rationalizability reflects the implications of finite levels of mutual knowledge of rationality: A 1-rationalizable strategy is one for which there is a profile of others' strategies that make it a best response; a 2-rationalizable strategy is one for which there is a profile of others' 1-rationalizable strategies that make it a best response; and so on. Rationalizability is equivalent to *k*-rationalizability for all *k*, reflecting the implications of common knowledge (indefinite levels of mutual knowledge) of rationality with no further restrictions on beliefs.

Equilibrium, by contrast, reflects the implications of common knowledge of rationality plus mutual knowledge of beliefs. Any equilibrium strategy is trivially *k*-rationalizable for all *k*, but not all combinations of rationalizable strategies are in equilibrium. However, in games that are strictly dominance-solvable in *k* rounds, *k*-rationalizability implies that players have the same beliefs—with an unimportant qualification for mixed-strategy equilibrium—so that any combination of *k*-rationalizable strategies is in equilibrium.

In other games *k*-rationalizability and even rationalizability allow deviations from equilibrium. Consider Matching Pennies, in which the Row player wins by matching (on Heads or Tails) and the Column player wins by mismatching. This game has a unique equilibrium, but for either player *any* strategy, pure or mixed, is rationalizable, and therefore *k*–rationalizable.[15] To see that Heads, for instance, is rationalizable for Row or Column, note that Heads is rational for Column on the belief that Row will play Tails, which is rational for Row on the belief that

[14] Equilibrium plus noise and QRE, by contrast, restrict the *relationship* among players' strategies. The level-*k*, CH, and NI models discussed below normally make unique (though possibly probabilistic) predictions conditional on their behavioral parameters, as does equilibrium plus noise when completed by adding a refinement. Finitely iterated strict dominance and *k*-rationalizability are equivalent in two-person games; their differences in *n*–person games are unimportant for our purposes.

[15] That the equilibrium is in mixed strategies may make the example seem special, but the same point could be made in a larger game with a unique equilibrium in pure strategies.

Column will play Tails, and so on. In this way one can construct a "tower", or more precisely a "helix", of beliefs that are consistent with iterated knowledge of rationality at all levels, hence with common knowledge of rationality, to support any outcome, equilibrium or not. Importantly, however, the beliefs that support many rationalizable outcomes are behaviorally implausible in that (as in the tower/helix) they rest on rationality-based inferences at unrealistically high levels and/or they cycle from level to level. A possible remedy is to combine rationality with empirically based restrictions on beliefs, as in the level-*k* and CH models discussed below.

Finitely iterated dominance and *k*-rationalizability weaken equilibrium enough to be consistent with most of the systematic patterns in subjects' deviations from equilibrium. Their weakness in Matching Pennies, where they imply no restrictions on behavior, is not entirely typical; though this extreme weakness extends to many games with unique pure-strategy equilibria and to most coordination games. One interesting approach is to take applications such as first-price auctions or market-entry games where the weakness is less extreme, accept the set-valued restrictions implied by *k*-rationalizability, and combine them, otherwise agnostically, with an econometric error structure. We discuss such approaches in Section 8.

2.3. *Quantal Response Equilibrium ("QRE")*

To capture the sensitivity of subjects' deviations from equilibrium to a subject's out-of-equilibrium payoffs when others may deviate from their equilibrium strategies, McKelvey and Palfrey (1995) proposed the notion of QRE. In a QRE players' decisions are noisy, with the probability density of each decision increasing in its expected payoff, evaluated taking the noisiness of others' decisions into account (its key difference from equilibrium plus noise). A QRE is a fixed point in the space of decision distributions, with each player's distribution a noisy best response to the others'. As the distributions' precision increases, QRE converges to equilibrium without noise; and as the precision approaches zero, QRE converges to uniform randomization over players' feasible strategies. A QRE model is closed by specifying a response distribution, which is logit in almost all applications. The resulting logit QRE or "LQRE" usually responds to out-of-equilibrium payoffs in plausible ways.[16]

[16] McKelvey and Palfrey (1995) suggest using LQRE for both initial responses and limiting outcomes, with increasing precision as a reduced-form model of learning. But although LQRE has until recently been the most popular model of initial responses, not all researchers consider it suitable for that purpose. Goeree and Holt (2004) suggest reserving LQRE for limiting outcomes, and instead propose an NI model (Section 2.6) to describe initial responses.

In applications LQRE's precision is calibrated from previous analyses or determined by fitting the model to the data. Like equilibrium plus noise, LQRE is a general model of strategic behavior with a small number of behavioral parameters. Because it responds to all out-of-equilibrium payoffs in plausible ways, LQRE often fits subjects' initial responses better than equilibrium plus noise (McKelvey and Palfrey 1995; Goeree and Holt 2001; Georg Weizsäcker 2003; Goeree, Holt, and Palfrey 2008; Sections 4 and 5). But in some settings LQRE fits worse than equilibrium, even making errors that consistently deviate from equilibrium in the opposite direction from observed deviations (Chong, Camerer, and Ho 2005; Crawford and Iriberri 2007b, Online Appendix; Östling et al. Forthcoming, Section 5).

The probability densities of QRE decisions respond to their expected payoffs evaluated taking the noisiness of others' decisions into account. This feature, which is essential to QRE's ability to describe deviations from equilibrium, makes QRE's predictions highly sensitive to the distributional assumptions—unlike in quantal response models of individual decisions where the choice probabilities can be consistently estimated with the mean error constrained to zero but in an otherwise distribution-free way; or in other models of strategic thinking, except for NI (Section 2.6). Philip Haile, Ali Hortaçsu, and Grigory Kosenok (2008) assess the strength of this sensitivity, showing that without further distributional assumptions, QRE can "explain" any given dataset with one observation per game-player pair. This result is disturbing because there is little theory to guide the specification of error distributions (see however Lars-Göran Mattsson and Jörgen Weibull 2002), and the use of the logit distribution in QRE analyses has been guided by fit and custom rather than independent evidence. However, Goeree, Holt, and Palfrey (2005) have shown that QRE with no specific distributional assumptions but a natural monotonicity restriction on responses to payoffs does imply some distribution-free restrictions even for data from a single game-player pair; and that even without such restrictions QRE does have testable cross-game implications. Even so, QRE's point predictions are much more sensitive to distributional assumptions than level-*k*/CH predictions.

With regard to cognitive requirements, no matter how simple the structure of the game, an LQRE player must both respond to a complex probability distribution of other players' responses and find his part of a generalized equilibrium that is a fixed point in a very large space of response distributions. If equilibrium reasoning is cognitively taxing enough to make it behaviorally implausible, LQRE reasoning is doubly taxing; and LQRE is less behaviorally plausible as a model of thinking than equilibrium. Further, the mathematical complexity of LQRE means that it must usually be solved for computationally and is not easily adapted to theoretical analysis (see however Simon P. Anderson, Goeree, and Holt 2001 and Kang-Oh Yi 2003, who characterize LQRE analytically in settings like those discussed in Section 7).

2.4. *Level*-k *Models*

Motivated by these considerations and experimental evidence, a different vein of work on strategic thinking considers models that treat deviations from equilibrium as an integral part of the structure, rather than as errors or responses to errors. Although the number of logically possible nonequilibrium structures seems daunting, both experimental evidence and folk game theory support a particular class of models called level-*k* models (or CH models, Section 2.5). These models also alleviate the cognitive and computational complexity concerns noted above.

In a level-*k* model players' types are heterogeneous, but each player's type is drawn from a common distribution. Type *Lk* anchors its beliefs in a nonstrategic *L0* type, which represents players' models of others' instinctive reactions to the game. Type *Lk* then adjusts its beliefs via thought-experiments with iterated best responses: *L1* best responds to *L0*, *L2* to *L1*, and so on. These empirically motivated assumptions about beliefs rule out the rationality-based inferences at unrealistically high levels and the persistent cycling from level to level that rationalizability allows, without relying on the behaviorally implausible cross-player interactions that drive epistemic justifications of equilibrium strategic thinking.

In applications it is usually assumed that *L1* and higher types make errors, often taken to be logit as in equilibrium plus logit noise or LQRE. The population type frequencies are inferred from data-fitting exercises or calibrated from previous analyses. The estimated frequency of *L0* is usually zero or small, so that *L0* "exists" mainly as *L1*'s model of others, *L2*'s model of *L1*'s model of others, and so on. The type distribution is fairly stable across settings, with most weight on *L1*, *L2*, and perhaps *L3*.

Even though *L0* normally has a low frequency, its specification is the main issue in defining a level-*k* model and the key to its explanatory power. As illustrated below, *L0* needs to be adapted to the setting, and there is an emerging consensus about how to do this in particular applications. The instinctive reactions may follow one of several principles, such as uniform randomness as in our first illustrations in Sections 3 through 7, or attraction to salience or truthfulness as illustrated in Sections 9 through 13. By contrast, the definition of *L1*, *L2*, and *L3* via iterated best responses allows a reliable explanation of behavior across different settings.

Unlike equilibrium and LQRE players, level-*k* types have a simple recursive structure. Iterating best responses a small number of times is cognitively easy in most games, and avoids the common criticism of LQRE that finding a fixed point in the space of distributions or performing many rounds of iterated dominance is too taxing for a realistic model of strategic thinking. Moreover, a level-*k* player need not respond to the noisiness of others' choices or a nondegenerate distribution of them, except for *L1*'s response to a random *L0*, which is easy.

Like equilibrium players, *L1* and higher types are rational, with perfect models of the game. Their only departure from equilibrium is replacing its perfect model of others with a simplified model of others. *L1* and higher types make undominated decisions, and *Lk* normally respects *k* rounds of iterated dominance (without performing them), so its decisions are *k*–rationalizable.

A distribution of level-*k* types that is concentrated on low levels of *k* as the evidence suggests mimics equilibrium in games that are dominance-solvable in a few rounds. But it deviates systematically in some more complex games, in ways that are sensitive to a subject's out-of-equilibrium payoffs when others may deviate from their equilibrium strategies. This allows the deterministic structures of level-*k* and CH models (Section 2.5) to capture the sensitivity of deviations from equilibrium to out-of-equilibrium payoffs, as the stochastic structure of LQRE does. All three models also often fit initial responses better than equilibrium plus noise.

Like equilibrium plus noise with a refinement that assures uniqueness, LQRE, and CH, level-*k* models are general models of strategic behavior, applicable to "any" game, and have small numbers of behavioral parameters. Unlike QRE, level-*k* models make point (though usually probabilistic) predictions that depend only on *L0* and the estimated type distribution, and not on the distributional assumptions or estimated precisions.[17] Level-*k* models' freedom from distributional assumptions is a major advantage over QRE. We return to this issue in Section 8's discussion of Ben Gillen's (2010) nonequilibrium analysis of identification in auctions.

2.5. *Cognitive Hierarchy ("CH") Models*

In Camerer, Ho, and Chong's (2004) closely related CH model, type *Lk* best responds not to *Lk-1* alone but to an estimated mixture of lower-level types, with the type frequencies treated as

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[17] Both LQRE and CH have fewer behavioral parameters but impose more restrictions than level-*k* models, for CH on the distribution of types, which is assumed to be Poisson; and for LQRE on the error distribution.

a parameterized Poisson distribution. For an outside econometrician observer, this estimated-mixture specification seems more natural than the level-*k* specification. But which specification better describes people's strategic thinking remains an empirical question, on which the jury is still out as explained below. A CH *L1* is the same as a level-*k* *L1* by definition, but a CH *L2* or higher type may differ from its level-*k* counterpart. A CH *L1* or higher type makes undominated decisions like its level-*k* counterpart. But unlike level-*k* types, a CH *Lk* need not always comply with *k* rounds of iterated dominance and *k*–rationalizability.[18] As Camerer, Ho, and Chong (2004, Section II.A) and Chong, Camerer, and Ho (2005, Section 2.1) note, the beliefs of a CH *Lk* type converge to correct beliefs as *k* increases. By contrast, in some games the beliefs of a level-*k* type oscillate perpetually, and may converge only to the set of rationalizable beliefs.

In a CH model, unlike in a level-*k* model, *L1* and higher types are assumed not to make errors. Instead the uniform random *L0*, which the Poisson distribution constrains to have positive frequency, doubles as an error structure for higher types. Section 2.4's observations about the cognitive ease of level-*k* types apply to CH types, except that CH types above *L1* respond not to a single lower type's response but to a distribution of types' responses, in proportions determined by an estimated Poisson parameter, assumed known by the player. CH types above *L1*, like level-*k* types, need not respond to the noisiness of others' decisions or find fixed points.[19]

Like a level-*k* model, given the Poisson distribution, a CH model makes point or mean predictions that do not depend on its estimated precision. But unlike a level-*k* model, and to some extent like QRE, the form of the distribution influences the model's point predictions. In some applications the Poisson distribution is not very restrictive and a CH model fits as well as a level-*k* model and better than LQRE (Camerer, Ho, and Chong 2004, Section II.B; and Chong, Camerer, and Ho 2005); but in others the Poisson distribution seems overly restrictive (Chong, Camerer, and Ho 2005; Costa-Gomes and Crawford 2006; Crawford and Iriberri 2007ab).

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[18] The increasing rationality of CH *Lk* types is important in some settings, as illustrated in Section 6; but because types higher than *L3* are rare, we view the *k*-rationalizability of level-*k* types as more important in practice.

[19] Although we calibrate level-*k* models without errors in some of the illustrations below, this is not the usual practice in estimating them. Brian Rogers, Palfrey, and Camerer (2009) identify a link between CH models and a generalization of LQRE that allows both heterogeneity of players' precisions and truncation of their perceptions of others' precisions as in Weizsäcker (2003). They also compare the models' fits in matrix games and zero-sum betting games like those discussed in Section 5. The fits are very close, which the authors attribute to the fact that the heterogeneity and truncation that their generalization allow LQRE gives it a payoff-sensitivity that is not shared by CH in general, but happens to be shared in the games considered.

2.6. *Noisy Introspection ("NI") Models*

As noted in Section 2.3, although McKelvey and Palfrey (1995) suggest using LQRE for both initial responses and limiting outcomes, and LQRE has been the most popular model of initial responses, Goeree and Holt (2004) suggest reserving LQRE for limiting outcomes, instead proposing a noisy introspection ("NI") model to describe initial responses. Their NI model relaxes LQRE's equilibrium assumption by assuming that players form beliefs by iterating noisy best responses as in a level-*k* model, hence maintaining LQRE's assumption that players respond to a nondegenerate probability distribution of others' responses. Although Goeree and Holt motivate NI as a kind of noisy rationalizability, because it builds on iterated best responses and makes point predictions, up to errors, it is more akin to level-*k* and CH models. Higher-order beliefs are assumed to reflect increasing amounts of noise, converging to uniform randomness.

In the only applications so far, Goeree and Holt assume that the noisiness of higher-order beliefs grows geometrically with iterations, which yields beliefs similar but by no means identical to *Lk*'s; slower noise growth results in more iterated best responses, like a higher *k*. The resulting NI model is more flexible than LQRE, and cognitively less taxing because it requires no fixed-point reasoning; but it is more taxing than a level-*k* or CH model because players' choices are indefinitely iterated best responses to noisy higher-order beliefs (although for computational purposes Goeree and Holt usually truncate the iteration to ten rounds).

For given noise distributions, the NI model makes probabilistic predictions that depend on how fast the noise grows. In the extreme case where the noise does not grow with the number of iterations, NI mimics LQRE. Other extremes mimic level-*k* types: If the noise jumps immediately to infinity, NI beliefs are like *L1*'s; if it is zero for one iteration and then jumps to infinity, NI beliefs are like *L2*'s, and so on; but these extremes are ruled out by the assumption that the noisiness of higher-order beliefs grows geometrically.

3. *Keynes' Beauty Contest: Experimental Evidence from Guessing and Other Normal-Form Games*

"...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

—John Maynard Keynes, *The General Theory of Employment, Interest, and Money*

"…imagine you are partners in a private business with a man named Mr. Market. Each day, he comes to your office or home and offers to buy your interest in the company or sell you his [the choice is yours]. The catch is, Mr. Market is an emotional wreck. At times, he suffers from excessive highs and at others, suicidal lows. When he is on one of his manic highs, his offering price for the business is high as well…. His outlook for the company is wonderful, so he is only willing to sell you his stake in the company at a premium. At other times, his mood goes south and all he sees is a dismal future for the company. In fact… he is willing to sell you his part of the company for far less than it is worth. All the while, the underlying value of the company may not have changed - just Mr. Market's mood."

—Benjamin Graham,[20] *The Intelligent Investor*

The Keynes and Graham quotations evoke simultaneous-move *n*-person guessing or perhaps "outguessing" games, possibly with multiple equilibria. Like the quotations that follow, they concern games played without clear precedents. The key issue is anticipating others' strategic responses, in Keynes' case to a "landscape" of personal judgments about prettiness, which is otherwise payoff-irrelevant; and in Graham's case to the psychology of a representative uninformed investor's reaction to news. Equilibrium is not very helpful in anticipating others' responses in such settings. Instead the quotations explicitly suggest thought processes in which players anchor beliefs in a model of others' instinctive reactions and then iterate best responses a finite number of times, processes whose heterogeneity and finiteness closely resemble a level-*k*

[20] Graham, who originally became famous as the co-author of Graham and David Dodd (1934), may now be even better known as Warren Buffett's intellectual hero.

or cognitive hierarchy model. Keynes' "fourth, fifth and higher degrees" is somewhat more than the evidence we shall present suggests is realistic, but may be only a coy reference to himself.

As we shall illustrate, the level-*k*/CH features of the Keynes and Graham quotations are representative of folk game theory: One can also find quotations reflecting one or two steps of iterated (strict or weak) dominance in the normal form or of iterated (weak) dominance reflecting forward or backward induction in the extensive form. But it is difficult to find quotations involving more than one or two iterations, and at least as difficult to find quotations that illustrate the fixed-point reasoning that underlies equilibrium in games without dominance.[21]

There is now a large body of experimental research that studies strategic thinking by eliciting initial responses to games with a variety of structures. The most important studies whose designs use normal-form complete-information games with neutral framing include those of Stahl and Wilson (1994, 1995); Nagel (1995); Ho, Camerer, and Keith Weigelt (1998); Costa-Gomes, Crawford, and Broseta (2001); Antoni Bosch-Domènech et al. (2002); Camerer, Ho, and Chong (2004); Costa-Gomes and Crawford (2006); and Costa-Gomes and Georg Weizäsacker (2008).

In this section we first discuss Nagel's (1995); Ho, Camerer, and Weigelt's (1998); and Bosch-Domènech et al.'s (2002) analyses of *n*-person guessing games that were directly inspired by Keynes' beauty contest analogy. We next discuss Stahl and Wilson's (1994, 1995) and Costa-Gomes, Crawford, and Broseta's (2001) analyses of two-person matrix games. Finally, we discuss Costa-Gomes and Crawford's (2006) analysis of two-person guessing games. In the process we highlight the strengths and weaknesses of each design, and the lessons from each set of results for modeling strategic thinking.[22] We give the most emphasis to Costa-Gomes and Crawford's (2006) analysis because its design is the most powerful and comes closest to letting the data reveal subjects' strategic thinking directly, without an econometric "middleman". Its conclusions are consistent with and representative of the conclusions of most other carefully done studies of initial responses to normal-form games with neutral framing, just more precise.

[21] Tellingly, Jacob Marschak (1946), in one of the first reviews of John von Neumann and Oskar Morgenstern (1944), quotes the Keynes passage above and says (with reference to their theory of zero-sum two-person games) "…it seems to us that properly stated differences in degrees of knowledge or intelligence of individual players can also be regarded as rules of the game."

[22] In normal-form games with neutral framing it is natural in a level-*k* or cognitive hierarchy analysis to take *L0* as uniform random over the entire strategy space, as explained below. In Sections 5 and 9-13 we discuss experiments on normal-form games with incomplete information or non-neutral framing and experiments and thought-experiments on extensive-form games with preplay communication, in which different specifications of *L0* are natural as we shall explain.

3.1. *Beauty Contest Games*

In Nagel's (1995) and Ho, Camerer, and Weigelt's (1998) games, *n* subjects (*n* = 15-18 in Nagel, *n* = 3 or 7 in Ho, Camerer, and Weigelt) made simultaneous guesses between lower and upper limits (0 and 100 in Nagel, 0 and 100 or 100 and 200 in HCW). In Bosch-Domènech et al. (2002) essentially the same games were played in the field, by more than 7500 volunteers recruited from subscribers of the newspapers *Financial Times*, *Spektrum der Wissenchaft*, or *Expansión*. In each case the subject who guessed closest to a target (p = 1/2, 2/3, or 4/3 in Nagel; p = 0.7, 0.9, 1.1, or 1.3 in Ho, Camerer, and Weigelt; and p = 2/3 in Bosch-Domènech et al.) times the group average guess won a prize. There were several treatments, each with identical targets and limits for all players and games. The structures were publicly announced, to justify comparing the results with predictions based on complete information.

Although Nagel's and Ho, Camerer, and Weigelt's subjects played a game repeatedly, their first-round guesses can be viewed as initial responses if they treated their own influences on future guesses as negligible, which is plausible for all but Ho, Camerer, and Weigelt's three-subject groups. Bosch-Domènech et al.'s subjects played only once.

With complete information, in all but one treatment the game is dominance-solvable in a finite (limits 100 and 200) or infinite (limits 0 and 100) number of rounds, with a unique equilibrium in which all players guess their lower (upper) limit when $p < 1$ ($p > 1$). The rationality-based argument for this "all–0" equilibrium is stronger than many equilibrium arguments, because it depends only on iterated (though sometimes infinitely iterated) knowledge of rationality, not on the assumption that players have mutual knowledge of beliefs.

The results of these experiments vividly illustrate the failure of equilibrium as a descriptive model of initial responses, and the heterogeneity and discreteness of strategic thinking. Nagel's subjects never made equilibrium guesses initially; Ho, Camerer, and Weigelt's rarely did so, and Bosch-Domènech et al.'s (who had much more time to reflect, and who could consult with others) fairly rarely did so. In each case most subjects' initial guesses respected from 0 to 3 rounds of iterated dominance, in games where 3 to an infinite number are needed to reach equilibrium. Here we reproduce Bosch-Domènech et al.'s Figure 1, which illustrates these points most clearly; see also Nagel's Figure 1 and Ho, Camerer, and Weigelt's Figures 2A-H and 3A-B.

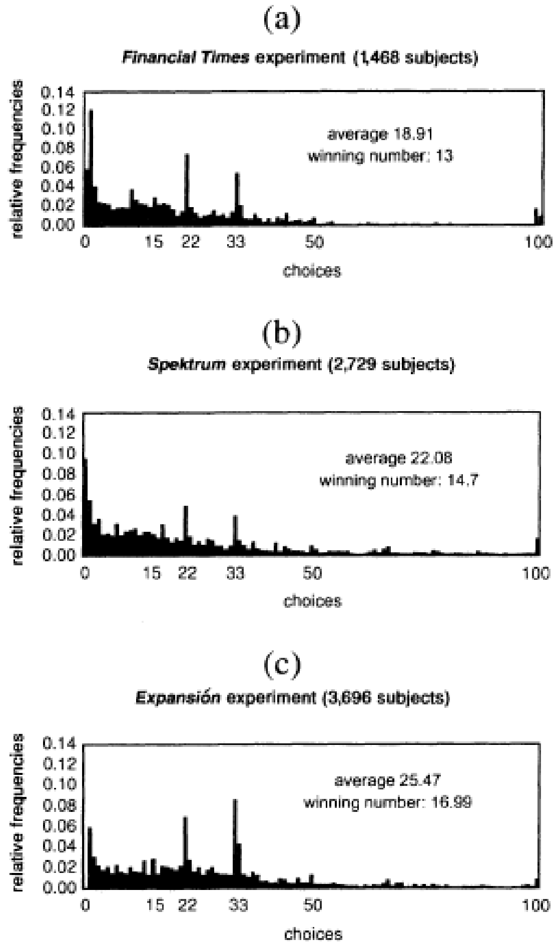

FIGURE 1. RELATIVE FREQUENCIES OF CHOICES IN THREE NEWSPAPER EXPERIMENTS

Figure 1. Bosch-Domènech et al.'s (2002) Figure 1

These data resemble neither "equilibrium plus noise" nor "equilibrium taking noise into account" as in QRE. They do suggest that subjects' deviations from equilibrium have a coherent structure. In each case the distributions of guesses have spikes that track $50p^k$ for $k = 1, 2, 3$ across the different targets p in the various treatments. Like the spectrograph peaks that foreshadow the existence of chemical elements, these spikes are evidence of a partly deterministic structure, one that is discrete and individually heterogeneous.[23]

[23] Yves Breitmoser (2010) shows that when individual players' influences are non-negligible, the tournament incentives of Nagel's design invalidate the usual characterization of *Lk* guesses. Behaviorally, this makes little difference because the usual characterization builds in heuristics most people seem to use in all but the smallest groups. In a richly parameterized econometric analysis, Breitmoser also obtains results that differ from the interpretation in the text for some subject groups. In reading the evidence, we place heavier weight on data generated by the more powerful designs discussed in Sections 3.2-4.

These designs are distinguished by their large strategy spaces, which greatly increase the informativeness of their results. But from the point of view of studying strategic thinking via initial responses—which was not these experimenters' sole purpose—they have a weakness in that each subject played only one game, so there was in effect only one observation per subject (although there was some between-subjects variation across the games used in different treatments). Even with very large strategy spaces, one observation yields limited information, and so the results leave considerable ambiguity of interpretation regarding subjects' types.

To take the most important example of this ambiguity, the spikes' locations and how they vary across treatments in these games have two plausible interpretations, which differ in some important applications. In one, subjects follow "level-*k*" rules based on an *L0* that is assumed to be uniformly random over the strategy space. Recalling that *L0* represents players' model of others' instinctive reactions to the game before they start thinking about others' incentives, it seems natural to take *L0* as derived from either ignoring strategic considerations and invoking the principle of insufficient reason, or randomly sampling the payoffs, and this is the emerging consensus for normal-form games with neutral framing.[24] *Lk* then iterates best responses *k* times, so that in these games *Lk*+1 guesses $[(0+100)/2]p^{k+1}$. In another interpretation, subjects follow rules like *Dk*, which does *k* rounds of iterated dominance for some small number, $k = 1$ or 2, and then best responds to a uniform prior over its partner's remaining strategies (thus "completing" *k*-rationalizability via a natural specific selection), so that *Dk* also guesses $([0+100p^k]/2)p$. As a result, theorists often interpret the results from these experiments as showing that subjects explicitly performed iterated dominance, though these results don't show that.

In many other games, with or without a uniform random *L0* as long as it has full support, *Dk* and *Lk*+1 respond similarly to dominance, both yielding *k*-rationalizable strategies (the different indices are only a quirk of notation). But *Dk* and *Lk*+1 are separated, although weakly, in Stahl and Wilson's (1995), Ho, Camerer, and Weigelt's (1998), and Costa-Gomes, Crawford, and Broseta's (2001) experiments; and they are strongly separated in Costa-Gomes and Crawford's (2006) experiments. The stronger the separation, the more the results favor *Lk* over *Dk* types.

We believe that more powerful and more comprehensive designs are a more likely route to further progress than progressively more sophisticated econometric analyses of existing datasets, especially those with only one observation per subject.

[24] In this paper we focus mainly on two-person games, but in *n*-person games it matters whether *L0* is independent across players or correlated, and the limited evidence (Ho, Camerer, and Weigelt 1998, Costa-Gomes, Crawford, and Iriberri 2009) suggests that most people have highly correlated models of others. For the moment we simply take *L0* to model all others' average guess, implicitly assuming perfect correlation as the evidence approximately suggests. We return to this issue in Section 6.

Nagel's (1995), Ho, Camerer, and Weigelt's (1998), and Bosch-Domènech et al.'s (2002) designs have another weakness for our purposes, in that their subject groups were so large that subjects very likely treated their own influences, except on their own payoffs, as negligible, and assumed that other subjects would do so as well.[25] Although subjects' interactions were still far from trivial from a game-theoretic point of view, such large-group results give limited insight into behavior in the many settings where players must choose their strategies while anticipating the responses of other players who do not think any one player's influence is negligible. To put it more concretely, when we (the authors) think about the stock market, we know that "it" isn't thinking about us, and this fact greatly simplifies our thinking about how the market will react to news. But when Warren Buffet thinks about the market, he knows that the market is also thinking about him, and this makes his thinking more fully strategic than ours is. In the rest of the experiments we discuss, individual players' influences are non-negligible.

3.2. *Other Normal-Form Games*

Stahl and Wilson (1994, 1995) report the results of experiments in which each subject played a series of 10 or 12 different but related 3×3 matrix games, an important advance on the common previous practice of trying to infer subjects' strategic thinking from their responses to a single game. As in all the remaining experiments discussed in this section, their subjects were randomly and anonymously paired to play the games, with no feedback, with the goal of suppressing learning and repeated-game effects in order to elicit subjects' initial responses, game by game, studying strategic thinking "uncontaminated" by learning. Stahl and Wilson's designs increase the number of observations per subject relative to Nagel's; Ho, Camerer, and Weigelt's; and Bosch-Domènech et al.'s; but they coarsen the strategy space by allowing only three decisions.

Stahl and Wilson's data analyses contained a uniform random *L0*, an *L1* as defined in Section 2.4, and an *L2* which differs from Section 2.4's in that it best responds to a noisy *L1* (which Stahl and Wilson motivate as a proxy for a weighted averaged of their uniform random *L0*s and *L1*s), much as Section 2.3's QRE players best respond to the predicted noise in others' decisions. Stahl and Wilson (1995) considered several types in addition to the level-*k* hierarchy. These include *Rational Expectations*, which best responds to the predicted choice frequencies among potential partners, a variant of the type Costa-Gomes, Crawford, and Broseta (2001) and Costa-Gomes and

[25] This excludes Ho, Camerer, and Weigelt's 3-subject groups. In experimental game theory, the smallest "large" number may be as low as 4, and is certainly not higher than 10.

Crawford (2006) call *Sophisticated*; *Worldly*, which best responds to an estimated mixture of a noisy *L1* and a noiseless *Equilibrium*; and *Naïve Nash*, which makes equilibrium decisions, possibly with noise. Stahl and Wilson found no evidence of *Rational Expectations* subjects, but there were 38 of 48 subjects for which one type had (in their Bayesian analysis) posterior probability at least 0.90: 17 *Worldly*, 9 *L1*, 6 *L0*, 5 *Naïve Nash*, and 1 *L2*.[26] Stahl and Wilson (1994) conducted a similar analysis of data generated by a design closely related to that of Stahl and Wilson (1995), allowing types *L0*, *L1*, their variant of *L2*, and *Naïve Nash* (but not *Worldly*, *Optimistic*, or *Pessimistic*). They found 35 of 40 subjects for which one type has posterior probability at least 0.90: 18 *L2*, 9 *Naïve Nash*, and 8 *L1*.[27]

Costa-Gomes, Crawford, and Broseta (2001) report experiments in which each subject played a series of 18 different but related 2×2, 2×3, and 2×4 matrix games, further increasing the number of observations per subject above Stahl and Wilson's (1994, 1995) numbers, but continuing with their coarse strategy spaces. Costa-Gomes, Crawford, and Broseta's results also show very clearly that subjects usually make undominated decisions. However, their subjects respect iterated dominance progressively less often, the more rounds of dominance are required to identify equilibrium decisions, to the point where beyond two or three rounds, equilibrium compliance is no better than random.

Costa-Gomes, Crawford, and Broseta's data analysis allows types *L1*, *L2*, and *L3* as defined in Section 2.4 (best responding to noiseless lower-level types); *D1* and *D2* as defined in Section 3.1; an *Equilibrium* type like Stahl and Wilson's *Naïve Nash*; and a *Sophisticated* type that best responds to the observed choice frequencies among potential partners, as a proxy for subjects whose understanding of strategic behavior transcends mechanical rules such as the other types.

Costa-Gomes, Crawford, and Broseta's estimates of the type distribution are quite similar to Stahl and Wilson's, with one exception: They exclude Stahl and Wilson's (1995) *Worldly* type a priori—on the grounds that it depends on estimated parameters and/or others' decision noise, and

[26] Following an early version of Costa-Gomes, Crawford, and Broseta (2001), who also looked for decision theoretic types called *Optimistic* (maximax) and *Pessimistic* (maximin), Ernan Haruvy, Stahl, and Wilson (1999) modified Stahl and Wilson's (1995) design in an attempt to identify *Optimistic* and *Pessimistic* subjects. Like Costa-Gomes et al. they found no *Pessimistic* but some *Optimistic* subjects, of whom most would have been estimated as *Worldly* in Stahl and Wilson's (1995) analysis.

[27] Rogers, Palfrey, and Camerer (2009) conduct a horse race between CH and LQRE in Stahl and Wilson's (1995) games, finding no important differences in fit.

thus implicitly assumes that subjects have prior understandings of others' responses—and as a result they identify many subjects as *L2* and some as *D1*.[28]

Although Stahl and Wilson's (1995) data analysis from a closely related experiment almost completely rejected *L2* in favor of their heavily parameterized *Worldly* type, in an econometric analysis that did not include *Worldly*, Stahl and Wilson (1994) found large numbers of *L1* and *L2* subjects, with results much closer to Costa-Gomes, Crawford, and Broseta's than to Stahl and Wilson's (1995) results, despite the similarities of the two Stahl and Wilson designs.

This instability of estimates reflects a common trade-off in the literature: To the extent that designs lack sufficient power to let the data speak for themselves (because of coarse strategy spaces or for other reasons), the data analysis rests heavily on small-sample econometrics with significant risk of specification bias and correspondingly fragile estimates. There is, therefore, a premium on designs powerful enough to let the data speak for themselves, with less reliance on econometrics, such as Costa-Gomes and Crawford's (2006) design discussed next, which suggests that Stahl and Wilson's rejection of *L2* in favor of *Worldly* was incorrect.

3.3. *Two-Person Guessing Games*

Costa-Gomes and Crawford's (2006) design combines the large strategy spaces of Nagel's (1995); Ho, Camerer, and Weigelt's (1998); and Bosch-Domènech et al.'s (2002) designs with the important strengthening feature of Stahl and Wilson's (1994, 1995) and Costa-Gomes, Crawford, and Broseta's (2001) designs that each subject played a series of different but related games, in this case 16. Again subjects were randomly and anonymously paired to play the games, with no feedback, with the goal of suppressing learning and repeated-game effects. ("Eureka!" learning was possible, but it was tested for and found to be rare.) The combination of large strategy spaces with each subject playing a series of games greatly enhances the design's power, and the profile of a subject's guesses in the 16 games forms a "fingerprint" that helps to identify his strategic thinking more precisely than is possible by observing his responses to a series of games with small strategy spaces or a single game with a large strategy space.[29]

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[28] Matthias Sutter, Simon Czermak, and Francisco Feri (2010) replicate Costa-Gomes, Crawford, and Broseta's (2001) design and results for individuals and also for three-person teams, finding teams somewhat more sophisticated than individuals.

[29] Costa-Gomes, Crawford, and Broseta's (2001) and Costa-Gomes and Crawford's (2006) designs also studied subjects' strategic thinking by monitoring their searches for hidden but freely accessible payoff information. Their data analyses rested on a simple theory of how cognition drives search as well as decisions, which implies that different types' search implications are separated, including *Dk* and *Lk*+1. The analyses of search confirmed the results of their analyses of decisions, including that the results favor *Lk* over *Dk* types (Crawford 2008 and Costa-Gomes and Crawford 2011). See also Chun-Ting Chen, Chen-Ying Huang, and Wang (2009), who add an interesting spatial dimension to the analysis of search.

In Costa-Gomes and Crawford's guessing games, each player has his own lower and upper limit, both strictly positive, which implies that the games are finitely dominance-solvable. Each player also has his own target, and his payoff increases with the closeness of his guess to his target times the other's guess. The targets and limits vary independently across players and games, with targets both less than one, both greater than one, or "mixed".[30]

These guessing games have essentially unique equilibria, determined (not always directly) by players' lower (upper) limits when the product of targets is less (greater) than one. The discontinuity of the equilibrium correspondence when the product of targets equals one stress-tests equilibrium, which responds much more strongly to the product of the targets than alternative decision rules do; and enhances the separation of equilibrium from alternative rules.

Consider a game in which players' targets are 0.7 and 1.5, the first player's limits are [300, 500], and the second's are [100, 900]. The product of targets is 1.05 > 1, and it can be shown that the equilibrium is therefore determined by players' upper limits. (When the product of targets is < 1, the equilibrium is similarly determined by their lower limits.) In equilibrium the first player guesses his upper limit 500, but the second player guesses 750 (= 500 × his target 1.5), below his upper limit 900. No guess is dominated for the first player, but any guess outside [450, 750] is dominated for the second player. Given this, any guess outside [315, 500] is iteratively dominated for the first player; any guess outside [472.5, 750] is then dominated for the second; and so on until the equilibrium at (500, 750) is reached after 22 rounds of iterated dominance.

The main difficulty in analyzing the data from such experiments is identifying subjects' decision rules, or *types*, within the enormous set of possibilities. As in previous studies, Costa-Gomes and Crawford assumed that each subject's decisions follow one of a small set of a priori plausible types, up to logit errors, and econometrically estimate which type best fit his decisions.

The types allowed include behaviorally plausible types whose relevance was suggested by previous work: *L1*, *L2*, and *L3* as defined in Section 2.4; *D1* and *D2* as defined in Section 3.1; *Equilibrium*, which makes its equilibrium decision; and *Sophisticated*, which best responds to the distribution of other subjects' responses, and is included to test whether any subject has a prior understanding of others' decisions that transcends the other simple rules. The restriction to this list was tested, and found to be a reasonable approximation to the support of subjects' types.

[30] In Nagel's (1995) and Ho, Camerer, and Weigelt's (1998) guessing experiments, by contrast, the targets and limits were always the same for both players, and they varied at most across treatments with different subject groups.

Costa-Gomes and Crawford's large strategy spaces and the independent variation of targets and limits across games greatly enhance the separation of types' implications, to the point where many subjects' types can be precisely identified from their guessing "fingerprints".

Of the 88 subjects in Costa-Gomes and Crawford's main treatments, 43 made guesses that complied exactly (within 0.5) with one type's guesses in from 7 to 16 of the games (20 *L1*, 12 *L2*, 3 *L3*, and 8 *Equilibrium*). Because their types specify precise, well-separated guess sequences in a very large space, with 200 to 800 possible exact guesses in each of 16 different games, these subjects' guesses allow one intuitively to "accept" the hypothesis that they followed their apparent types, and so rule out alternative interpretations of their behavior.[31]

In particular, because the accepted *Lk* and *Equilibrium* types build in risk-neutral, self-interested rationality and perfect models of the game, the deviations from equilibrium of the 35 subjects whose apparent types are *Lk* can be confidently attributed to nonequilibrium beliefs rather than irrationality, risk aversion, altruism, spite, or confusion. Thus, the level-*k* model is directly suggested by these subjects' data, rather than simply suggested by data-fitting exercises that impose strong structural assumptions. By contrast, in designs with coarse strategy spaces even a perfect fit does not distinguish a subject's apparent type from nearby omitted types. In designs in which each subject plays a single game, the ambiguity is even more severe, so that even in Nagel's large strategy spaces, rules as cognitively disparate as *Dk* and *Lk+1* yield identical decisions. Identification of types then necessarily rests on structural assumptions.

Costa-Gomes and Crawford's other 45 subjects made guesses that conformed less closely to a type, making structural econometrics necessary. But for all but 14 subjects, violations of simple dominance were fairly rare (less than 20%, versus 38% for random guesses), suggesting that their behavior was coherent, even if less well described by a type. Econometric type estimates are concentrated on *L1*, *L2*, *L3*, and *Equilibrium*, in roughly the same proportions as for subjects with high rates of exact compliance.

In particular, unrestricted estimates of the frequency of *L0* subjects, given the econometric model's clear separation of *L0* and the error structure, yield a frequency of zero, suggesting that *L0*s exist only in the minds of other subjects, as *L1*'s model of others, *L2*'s model of *L1*'s model of others, and so on. Low frequencies of *L0* are an important sign of health for a level-*k* model,

[31] We stress that Costa-Gomes and Crawford's Baseline subjects were taught only how to compute their payoffs and then quizzed on identifying their and their partner's best-responses, but not taught any particular decision rule. Their high rates of exact compliance with level-*k* types reflect their own thinking.

in that high frequencies would reduce the model to a parameterized distribution of responses, thus describing the data rather than explaining it. Only when the strategic iteration of best responses plays a role can the model yield a useful explanation of the data.

The econometric analysis also suggests that there are few if any *Sophisticated* subjects. The strong separation of *Dk* from *Lk*+1 allows the analysis to show convincingly that there are few if any *Dk* subjects: To the extent that subjects respect finitely iterated dominance, it is not because they explicitly perform it but because they follow level-*k* rules that respect it—a distinction that matters in many games, if not in Nagel's (1995) games where *Dk* is not separated from *Lk*+1.[32]

For those 45 subjects, there is some room for doubt about whether Costa-Gomes and Crawford's specification omits relevant types and/or overfits by including irrelevant types. The freedom to specify the possible types also raises doubts about omitted types and overfitting via accidental correlations with included but irrelevant types. Might the high estimated numbers of *L1* and *L2* subjects might be no more than proxies for altruistic, spiteful, risk-averse, or confused *Dk* or *Equilibrium* subjects; or other, entirely different omitted types? To test for this, Costa-Gomes and Crawford conducted a specification test, which reaffirms most of their identifications of *L1*, *L2*, *L3*, or *Equilibrium* subjects and supports their specification by giving no indication of significant numbers of Stahl and Wilson's (1995) *Worldly* type or any other omitted type.

3.4. *Eliciting Beliefs*

Costa-Gomes and Weizsäcker's (2008) study asks experimental subjects to choose actions and state beliefs about their partners' actions in 14 two-person 3×3 games, with beliefs elicited via a quadratic scoring rule, which is incentive compatible under the assumption that the decision-maker is a risk-neutral expected-utility maximizer. Eliciting beliefs can provide finer information about strategic thinking than eliciting actions, and beliefs are an informative supplement to actions in any case. Costa-Gomes and Weizsäcker's design strongly separates both the actions and the beliefs implied by the leading strategic decision rules *L1*, *L2*, *LQRE*, *NI*, and *Equilibrium*. In their analysis of their aggregate data, they find that although subjects' actions are best described by *L1*, their stated beliefs are closer to *L2*'s beliefs. The results suggest

[32] This last conclusion is reinforced by Costa-Gomes and Crawford's (2011) (see also Crawford 2008) analysis of subjects' searches for hidden payoff information and by their data on "robot/trained subjects," where 7 of 19 subjects, who were trained and rewarded to follow type *D1* and passed an understanding test in which *L2* answers were incorrect, then "morphed" into *L2* (*D1*'s closest *Lk* relative) in the guesses for which they were paid. Aside from the one of 19 robot/trained *D2* subjects who morphed into *L3*, this was the only kind of morphing that occurred. Although by standard measures *Dk*'s cognitive requirements are close to *Lk*+1's, and these treatments also show that most subjects were capable of learning to follow *Dk*, the morphing suggests that subjects find iterated dominance far less natural than the iterated best responses that underlie *Lk* rules.

that despite subjects' incentives, they viewed choosing actions and stating separately incentivized beliefs as largely unrelated tasks, rather than requiring actions to be best responses to beliefs as decision theory suggests; on average, their actions and beliefs are consistent in this sense 50% to 60% of the time. This in turn suggests that decision rules may be more fundamental for most subjects than beliefs. Costa-Gomes and Weizsäcker close by noting that stated beliefs are subject to error just as actions are, suggesting caution in imposing the restriction that actions have to be best responses to stated beliefs.[33]

3.5. *Level-k and CH Models versus Equilibrium plus Noise, Finitely Iterated Dominance and* k-*Rationalizability, LQRE, and NI Models*

We now summarize how the experimental evidence relates to the models of strategic thinking discussed in Section 2.

Nagel's (1995), Ho, Camerer and Weigelt's (1998), and Bosch-Domènech et al.'s (2002) results make it clear that subjects initial responses can deviate systematically from equilibrium, even in games where equilibrium reasoning requires "only" indefinitely iterated dominance, resembling neither equilibrium plus noise nor QRE for any reasonable distribution. They also show that strategic thinking is heterogeneous and falls into discrete classes (with errors), so that no model that imposes homogeneity, as equilibrium plus noise, QRE, and NI do, will do full justice to subjects' behavior.[34] Finally, although strategic thinking respects iterated dominance to some extent, this is limited to a few rounds, so rationalizability (as opposed to *k*–rationalizability for low *k*) is too strong. Stahl and Wilson's (1994, 1995), Costa-Gomes, Crawford, and Broseta's (2001) and Costa-Gomes and Crawford's (2006) results confirm these lessons.

Costa-Gomes, Crawford, and Broseta (2001) and Costa-Gomes and Crawford's (2006) results suggest that overall, a level-*k* model with a uniform random *L0* and *L1*, *L2*, *L3*, and possibly *Equilibrium* subjects explains a large fraction of subjects' deviations from equilibrium in normal-form games with neutral framing. Their conclusions are consistent with those of most other carefully done studies of initial responses to normal-form games with neutral framing, just more precise. Their results also suggest that the type distribution is fairly stable across settings, with most weight on *L1*, *L2*, and perhaps *L3*. Costa-Gomes and Crawford's (2006) specification test suggests that these types and possibly *Equilibrium* are all relevant, but that at least in this

[33] Pedro Rey-Biel (2009) partially replicates Costa-Gomes and Weizsäcker's (2008) results, showing that in constant-sum games, where equilibrium reasoning need not depend on strategic thinking, equilibrium outperforms level-*k* and other rules.
[34] Allowing heterogeneity is essential to explain the patterns of nonequilibrium behavior discussed in Sections 6, 9, and 11.

setting other deviations from equilibrium have little or no discernable structure and describe only about 1-2% of their subject population. Thus, although about half of subjects' deviations from equilibrium remain unexplained by the proposed level-*k* plus *Equilibrium* model, it may still be optimal to treat the remaining unexplained deviations as errors; and the part of the structure that can be identified can provide a sound basis for unbiased modeling of initial responses to games.

Given these conclusions, might not other models, such as equilibrium plus noise, finitely iterated strict dominance and *k*-rationalizability, LQRE, CH, or NI, do as well or even better?

Costa-Gomes and Crawford's (2006) econometric analysis nests equilibrium plus noise via their *Equilibrium* type with logit errors. Only 11 of the 88 subjects in their main treatments are estimated to be *Equilibrium* subjects, and there is clear evidence that even those subjects are following rules that only mimic *Equilibrium*, and that only in some games (pp. 1753-1754). For their remaining 77 subjects, equilibrium plus logit noise misses clear patterns in the data. Further, these subjects' "errors" neither center on 0 nor usually exhibit the sensitivity to deviation costs assumed in a logit specification. We believe this is because the errors are cognitive or structural, reflecting misspecification rather than a trade-off between effort cost and accuracy. Instead these subjects' errors have a clear deterministic structure, which is better described by the level-*k* model that emerges from Costa-Gomes and Crawford's estimates.

Because all of the types with significant estimated frequencies respect *k*–rationalizability for at least $k = 1$, except for the heterogeneity noted above there is no conflict with finitely iterated strict dominance and *k*-rationalizability, only more specific predictions that imply progressively lower and lower compliance frequencies as *k* rises above 1.

Turning to LQRE and NI, the large strategy spaces and independent variation of targets and limits across games and players of Costa-Gomes and Crawford's guessing games yields stronger separation of level-*k* types from LQRE and NI than is possible in simpler designs with unvarying games and smaller strategy spaces. At the same time, Costa-Gomes and Crawford's (footnote 34, p. 1763) "median voter" result, which stems from the piecewise linearity and symmetry of their payoff function and shows that in their games, a risk-neutral player's best response is completely determined by the median of the distribution of his partner's response, would make equilibrium plus logit noise coincide with LQRE except for small payoff asymmetries due to automatic adjustment to the limits.[35] Thus, the rejection of *Equilibrium* in favor of level-*k* types for 77 of the 88 subjects in Costa-Gomes and Crawford's main treatments strongly suggests that LQRE would be similarly rejected. This is unsurprising, because as noted above in connection with Stahl and Wilson's (1995) *Worldly* type, Costa-Gomes and Crawford's (2006, Section II.D) results cast doubt on LQRE's assumption that players respond to a nondegenerate probability distribution of others' responses.

Given that NI is a flexible parameterization that includes LQRE as a special case, it may well fit better than LQRE. Recall that in an NI model, players form beliefs by iterating best responses, with higher-order beliefs reflecting increasing amounts of noise. In Goeree and Holt's (2004) favored specification, in which the noisiness of higher-order beliefs grows geometrically, the highest-order relevant beliefs are uniform random, which we take to mean within the limits in Costa-Gomes and Crawford's games. Even assuming logit errors and geometric growth, because the NI decision is a continuous function of the noise level and its rate of growth, varying those parameters yields a wide range of possible decisions. As a result, unlike equilibrium plus logit noise, LQRE, level-*k*, or CH models, NI models seem overparameterized for applications to a single game, and they probably risk overfitting even in datasets that span multiple games.

As these comparisons illustrate, that the level-*k* model is directly grounded in evidence, rather than ambiguously suggested by data-fitting exercises that impose strong structural assumptions, is an important advantage over alternatives like LQRE and NI.

Turning finally to level-*k* versus CH models, by a quirk of Costa-Gomes and Crawford's (2006, fn. 36, p. 1763) design, level-*k* types' decisions are not separated from their CH counterparts' decisions: Level-*k* and CH *L1*s are identical by definition, and the median voter result mentioned above, for empirically plausible type distributions like those Camerer, Ho, and Chong (2004) estimate, a CH *L2* and *L3* are both identical in these games to a level-*k* *L2*.

However, to fit the data the CH model's Poisson parameter τ (the average *k*) must be approximately 1.5, which constrains the frequency of its *L0* to 0.22. Costa-Gomes and Crawford's and many other unconstrained estimates almost all assign *L0* a far lower frequency, usually zero (and this, we have argued, is a sign of health). Thus the Poisson constraint will often

[35] To see why LQRE is nonetheless separated from equilibrium plus logit noise, consider a guessing game in which players' targets are 0.7 and 1.5, the first player's limits are [300, 500], and the second's are [100, 900], in which the equilibrium, (500, 750), is reached after 22 rounds of iterated dominance. Because the first player's equilibrium guess is at her lower [upper] limit, the second players' deviations below/above 750 are less/more costly, so that LQRE players will play strategies below 750 with somewhat higher probability than strategies above 750.

be strongly binding, and with comparable error structures (though possibly not with the structure often assumed in applications of CH models, in which a uniform random *L0* doubles as the error structure for higher types), a level-*k* model will have an advantage in fit over a CH model.[36]

3.6 *Directions for Future Work*

The experimental work reviewed here has considered several different kinds of games, with an encouraging trend toward more powerful experimental designs and correspondingly less reliance on econometrics. It has yielded an increasingly clear message about the kinds of model that best describe strategic thinking, which we shall show in later sections extends to even more kinds of games. However, to date almost all of the evidence has been generated within classes of games studied in isolation; and some of the inferences have been based on econometric horse races rather than decisive experimental tests. For the models that emerge to be worthy competitors to equilibrium, they should give an account of strategic thinking as precise as possible, with the capability of tracking behavior across games with the variety of structures encountered in applications (though possibly avoiding games that are unrepresentative of realistic situations). It should be possible, via new designs, to combine this structural variety with the power of some of the designs that have focused on single classes of games. We believe that such experimental work will lead to improved specifications of level-*k*/CH and hybrid models that are portable enough to describe behavior accurately across a wide range of applications.

In one example of work that begins to assess portability, Sotiris Georganas, Paul J. Healy, and Roberto A. Weber (2010) report experiments that rerun some of Costa-Gomes and Crawford's (2006) guessing games and some new "undercutting" games (similar to Traveler's Dilemma games).[37] They find further support for level-k/CH models within games, and some positive correlation of subjects' types with cognitive ability measures; but only moderate correlation of estimated types across games within subjects, with some crossovers in the ordering of subjects by type across different games. We suspect the weak correlation is due in part to Georganas, Healy, and Weber's simplification of Costa-Gomes and Crawford's instructions regarding the structure of the games and their omission of the understanding test in which subjects were required to demonstrate understanding of how their payoffs would be determined

[36] Further, Costa-Gomes and Crawford's (2006) data on subjects' searches for hidden but freely accessible payoff information are much more consistent with the search implications of the level-*k* model than with those of a CH model, which blurs the implications of some important types (Crawford 2008).

[37] See also Ayala Arad and Rubinstein (2010), who study portability in a variety of treatments with similar games.

to continue. We believe that such understanding is crucial for results that are representative of the field, where most people seem to understand very well how their payoffs are determined.

In another such example, Konrad B. Burchardi and Stefan P. Penczynski (2010) study one of Nagel's (1995) beauty contest games and Rubinstein and Tversky's hide-and-seek game with non-neutral framing of locations (Rubinstein 1999), as also studied by Crawford and Iriberri (2007b) (Section 9) and Penczynski (2010). Burchardi and Penczynski adapt David J. Cooper and John H. Kagel's (2005) method for studying cognition by having decisions made by two-subject teams with common payoffs, whose chat deliberations were monitored along with their decisions. If the two could agree on a decision, it was implemented; if not each subject submitted a final proposed decision, which was implemented with probability one-half. The results for decisions and chats both yield further support for level-*k*/CH models. Judging from the chats, more than half of the subjects started their reasoning processes with an *L0* anchoring type, which sometimes coincided with the standard uniform random specification of *L0*. Most subjects also followed iterated best response reasoning, to the exclusion of iterated dominance. But Burchardi and Penczynski also find that subjects' apparent types are only weakly correlated within subjects across beauty contest and hide-and-seek games with non-neutral framing of locations.

In their econometric specification, Burchardi and Penczynski postulate a heterogeneous *L0* with bounded normal errors, and in the style of CH models they allow no errors in higher types' decisions beyond those implied by best responses to this *L0*, but without assuming that the type distribution is Poisson. Even so, they estimate the frequency of *L0* to be 22-37%, far more than other estimates with an unconstrained type distribution we are aware of. This result could be due to the dual role of *L0* in their model, as the anchor for higher types and as the error structure.

4. *M.M. Kaye's* Far Pavilions*:* *Responding to Payoff Asymmetries in Outguessing Games*

> "…ride hard for the north, since they will be sure you will go southward where the climate is kinder…."
>
> —M.M. Kaye's *The Far Pavilions* (1978, p. 97)

Early in M.M. Kaye's novel *The Far Pavilions*, the main male character, Ash/Ashok, is trying to escape from his pursuers along a north-south road. Both Ash and his pursuers must choose between north and south. Although Ash moves first, the pursuers must make their choice irrevocably before they learn Ash's choice, so their choices are strategically simultaneous. South is warm, but north are the Himalayas, with winter coming. Ash's mentor, Koda Dad, nonetheless advises Ash to ride north as in the quotation above. Ash overcomes his fear of freezing and follows Koda Dad's advice, the pursuers unimaginatively go south, and Ash escapes.

Examples like this are as common in experimental game theory as they are in fiction. But fiction sometimes more clearly reveals the thinking behind a decision. Here, Koda Dad is advising Ash to choose the *L3* response to a uniform random *L0*.[38] To see this, imagine that if the pursuers catch Ash, they gain two units of payoff and Ash loses two; and that they both gain one extra unit for choosing south, whether or not Ash is caught. This yields the payoff matrix:

		Pursuers	
		South (*q*)	**North**
Ash	**South (*p*)**	**3** **-1**	**0** **1**
	North	**1** **0**	**2** **-2**

Figure 2. *Far Pavilions* Escape

If the pursuers expect Ash to go south because it's "kinder", they must be modeling Ash as an *L1* response to a uniform random *L0*. For, the unspecified payoff asymmetry on which this inference rests is necessarily decisive only if north and south do not differ in the probability of being caught. Thus, Koda Dad must be modeling the pursuers as *L2* and advising Ash to choose the *L3* response to a uniform random *L0*.

Importantly, although the level-*k* model takes the inference that pursuers will expect Ash to go south literally as a best response to a uniform random *L0*, there is a behaviorally more plausible interpretation in which the inference is a model of pursuers' model of Ash ignoring his strategic considerations, and given this, based on the principle of insufficient reason. Further, in a more complex game a uniform random *L0* plausibly approximates random sampling of payoffs unstratified by the other players' strategy choices.

[38] In the HBO miniseries, Koda Dad was played by Omar Sharif. For all of Sharif's success at bridge, his character's advice may be his most enduring contribution to game theory. *L3* ties our personal best *k* for a clearly explained level-*k* type in fiction or non-game-theoretic nonfiction. We suspect that even postmodern fiction may have no clear *Lk*s higher than *L3*, because they wouldn't be credible. We also doubt that one can find examples (clear or not) of fixed-point reasoning.

How does the level-*k* model compare in predictive success with an equilibrium model? *Far Pavilions* Escape has a unique equilibrium in mixed strategies, in which Ash's Pr{South} p^* = 1/4, and the Pursuers' Pr{South} q^* = 3/4. Thus in equilibrium the novel's observed outcome {Ash North, Pursuers South} has probability $(1 - p^*)q^* = 9/16$: much better than a random 25%.

By contrast, the level-*k* model implies decisions as in Table 1. Thus the level-*k* model exactly predicts the outcome provided that Ash is either *L2* or (as we know is true from the quoted rationale) *L3*, and the Pursuers are either *L1* or (as Koda Dad believes) *L2*.

This comparison is unfair because applications seldom come with an omniscient narrator telling us how players are thinking, and equilibrium does not use such information. If we deny such information to the level-*k* model as well, we can still derive the model's implications and, with enough data, find the population type frequency distribution that fits best, as illustrated in Sections 5 and 9. Alternatively, we can calibrate the level-*k* model using previous estimates from similar settings, as illustrated in Sections 9 and 10.

Type	Ash	Pursuers
L0	Uniform random	Uniform random
L1	South	South
L2	North	South
L3	North	North
L4	South	North
L5	South	South

Table 1. *Lk* types' decisions in *Far Pavilions* Escape

Suppose, for example, that we assume that each player role is filled from a 50-30-20 mixture of *L1*s, *L2*s, and *L3*s and there are no errors.[39] Then the predicted frequencies with which Ash goes north and the pursuers go south are 1/2 and 4/5 respectively. Assuming independence, this implies that the observed outcome {Ash North, Pursuers South} has probability 2/5: less than the equilibrium predicted frequency of 9/16, but noticeably better than a random 25%.

More importantly, the level-*k* model explains a puzzling divergence between observed aggregate behavior and equilibrium predictions. In games like *Far Pavilions* Escape and perturbed Matching Pennies, the mixed-strategy equilibrium responds to the payoff asymmetry between south and north in a decision-theoretically intuitive way for the pursuers' role (q^* = 3/4

[39] This violates the unwritten laws of fiction, where protagonists are almost always more sophisticated than their opponents, but it is fully consistent with data from nonfictional games of this kind.

> 1/2, the equilibrium probability with which pursuers go south in the analogous game with no north-south asymmetry); but in a counterintuitive way for Ash's role ($p^* = 1/4 < 1/2$).[40]

Yet experimental subjects' aggregate choices in initial responses to games like this tend to reflect decision-theoretic intuition in both player roles.[41] McKelvey and Palfrey (1995) and Goeree, Holt, and Palfrey (2005) discuss the experiments with perturbed 2×2 Matching Pennies games with payoff perturbations in only one player role reported by Jack Ochs (1995), McKelvey, Palfrey, and Weber (2000), Goeree and Holt (2001), and Goeree, Holt, and Palfrey (2003), which yield initial aggregate choices that reflect decision-theoretic intuition in the role whose payoffs are perturbed. In the other role, for which the intuition is neutral, aggregate choices deviate from equilibrium in the direction that increases expected payoff, given the intuitive response in the first role. McKelvey and Palfrey (1995, Figures 6 and 7) and Goeree, Holt, and Palfrey (2005) show that LQRE with fitted precisions can fit these qualitative patterns, although it sometimes underpredicts the magnitudes of deviations from equilibrium, especially for the player whose payoff is perturbed. A level-*k* or CH model, either calibrated as described above or estimated from the data on initial responses, also predicts these qualitative patterns.

5. *Groucho's Curse: Zero-Sum Betting and Auctions with Incomplete Information*

"I sent the club a wire stating, 'Please accept my resignation. I don't want to belong to any club that will accept people like me as a member'."

—Groucho Marx (1959, p. 321), Telegram to the Beverly Hills Friar's Club

"Son," the old guy says, "No matter how far you travel, or how smart you get, always remember this: Someday, somewhere," he says, a guy is going to come to you and show you a nice brand-new deck of cards on which the seal is never broken, and this guy is going to offer to bet you that the jack of spades will jump out of this deck and squirt cider in your ear. But, son," the old guy says, "do not bet him, for as sure as you do you are going to get an ear full of cider."

—Obadiah ("The Sky") Masterson, quoting his father in Damon Runyon (1932)

[40] Crawford and Dennis E. Smallwood (1984) discuss the comparative statics of mixed-strategy equilibria in perturbed Matching Pennies games, showing that this role-asymmetric intuitiveness is general when both players' payoffs are perturbed.
[41] Ash's counterintuitive choice would not contradict this pattern if he were a subject because his revealed type is in the minority.

Although most laboratory evidence on strategic thinking comes from symmetric-information designs, most field evidence and applications involve settings with informational asymmetries. It is therefore of great importance to extend whatever can be learned about strategic thinking in complete-information games to incomplete-information games. In this section we discuss laboratory and field evidence on games with informational asymmetries, focusing on cases where the games are sufficiently simple to allow clear inferences about strategic thinking.

We begin by discussing evidence from experiments on especially clear examples of games with informational asymmetries: zero-sum betting, sealed bid auctions, and the "Acquiring a Company" game. We next discuss analyses using field data from settings with asymmetric information. We close with a brief discussion of nonequilibrium auction design, based on an extension of level-*k* models suggested by the analysis of initial responses in auction experiments.

5.1. *Zero-Sum Betting*

Experiments on zero-sum betting build on Paul R. Milgrom and Nancy Stokey's (1982) no-trade theorem, which shows that if traders are weakly risk-averse and have concordant beliefs, and the initial allocation is Pareto-efficient relative to the information available at the time, then even if traders receive new private information, no weakly mutually beneficial trade is possible. Further, if traders are strictly risk-averse, no trade at all is possible. For, any such trade would make it common knowledge that both traders had benefited, contradicting the hypothesis that the original allocation was Pareto-efficient. This result has been called the Groucho Marx theorem because its logic resembles that of our Marx quotation.

By contrast with the conclusions of the theorem, speculative zero-sum trades are common in real markets. This fact has a number of possible explanations, of which one is nonequilibrium strategic thinking. The experiments on zero-sum betting by Brocas, Carrillo, Camerer, and Wang (2010) we now discuss have the control required to distinguish between such explanations and those based on other factors such as hedging or the joy of gambling (see also Camerer, Ho, and Chong 2004, Section VI, and Rogers, Palfrey and Camerer 2009, who both build on Doron Sonsino, Ido Erev, and Sharon Gilat 2002 and Ylva Sovik 2009). Brocas et al.'s (2010) design can further distinguish between alternative nonequilibrium models of strategic thinking.

Brocas et al.'s design used simple three-state betting games, including the one in Figure 3.[42] The rules of the game and the information structure were publicly announced, with the goal of inducing common knowledge. Each of two players, 1 and 2, is given information about which of three ex ante equally likely states has occurred, A, B, or C. As indicated by the heavy borders in Figure 3, player 1 learns either that the state is {A or B} or that it is C; player 2 learns either that the state is A, or that it is {B or C}. Once informed, the players choose simultaneously between two decisions: Bet or Pass. A player who chooses Pass earns 10 no matter what the state. If one chooses Bet while the other chooses Pass, they both earn 10. If both players choose Bet, they get the payoffs in Figure 3, depending on which state has occurred.

Player/state	**A**	**B**	**C**
1	25	5	20
2	0	30	5

Figure 3. Zero-Sum Betting Game

This game has a unique trembling-hand perfect Bayesian equilibrium.[43] In this equilibrium, player 1 told C will Bet because 20 > 10, and player 2 told A will Pass because 0 < 10. Given this, player 1 told {A or B} will Pass, because player 2 will Pass if told A, so betting given {A or B} yields player 1 at most 5 < 10. Given this, player 2 will Pass if told {B or C}, because player 1 will Pass if told {A or B}, so betting given {B or C} yields player 2 at most 5 < 10. This covers all contingencies and completes the characterization of equilibrium, which shows that the game is weakly dominance-solvable in three rounds. No betting takes place in equilibrium in any state.

Despite this clear conclusion, in Brocas et al.'s and similar experiments approximately half of the subjects bet. To explain this, Brocas et al. proposed a level-*k* model with a specification like those discussed in Section 3, but with *L0* adapted to allow for incomplete information. Following Camerer, Ho, and Chong (2004, Section VI), Brocas et al. assumed that *L0* bids uniformly randomly, independent of its private information. In judging this specification, bear in mind that *L0* is meant to describe a player's model of the instinctive starting point of others' strategic thinking. It is easy enough to imagine *L0*s that take others' private information into account, but for a player who does not observe such information this would require complex

[42] Although our discussion focuses on subjects' decisions, Brocas et al. (2009) also monitored subjects' searches for hidden but freely accessible payoff information, as in Costa-Gomes, Crawford, and Broseta (2001) and Costa-Gomes and Crawford (2006). Their analysis of information search reinforces and refines the conclusions of their analysis of decisions.

[43] Trivial equilibria also exist, in which players do not bet because their partners do not bet, though this is weakly dominated.

contingent reasoning. Such reasoning is seldom consistent with results from other settings, and it seems behaviorally more plausible to assume that *L0* ignores others' private information.

As in previous level-*k* analyses, Brocas et al. took *L1* to best respond to *L0*, and *L2* to best respond to *L1*. Following Crawford and Iriberri (2007a), we call an *L1* that best responds to a random *L0* a "random *L1*" even though it is not itself random; and we call an *L2* that best responds to a random *L1* a "random *L2*". Compare Milgrom and Stokey's (1982, p. 23) Case A "Naïve Behavior," in which a player simply sticks with his prior. This refusal to draw contingent inferences from others' willingness to bet is implied by random *L1*'s random model of others. Milgrom and Stokey's Case B "First-Order Sophistication" is then equivalent to our random *L2*.

Given this specification, random *L1* player 1s will Bet if told {C} because it yields 20 > 10 if player 2 Bets, a random *L0* player 2 will bet with probability one-half in either contingency, and Betting is otherwise costless. Unlike in equilibrium, Random *L1* player 1s will Bet if told {A or B} because it yields 25 in state {A} and a random *L0* player 2 will bet with probability one-half in {A}; it yields 5 in state {B} and a random *L0* player 2 will Bet with probability one-half in {B}; and the two states are equally likely ex ante, so Betting if told {A or B} yields expected payoff (25 + 5)/2 = 15 > 10. Random *L1* player 2s will Pass if told {A}, because it yields 0 < 10. Unlike in equilibrium, random *L1* player 2s will Bet if told {B or C}, because it yields 30 in state {B} and a random *L0* player 1 will bet with probability one-half in {B}; it yields 5 in state {C} and a random *L0* player 1 will Bet with probability one-half in {C}; and the two states are equally likely ex ante, so Betting if told {B or C} yields expected payoff (30 + 5)/2 = 17.5 > 10. Similarly, it can be shown that Random *L2* or *L3* player 1s will both Pass if told {A or B} but Bet if told {C}; that Random *L2* player 2s will Pass if told {A} but Bet if told {B or C}; and that Random *L3* player 2s will Pass without regard to the state.

Thus, if all subjects were random *L1s*, 100% of player 1s would Bet and 67% of player 2s, too much in each role; and betting would occur only in states B and C, which is not the case: Although all player 1 subjects bet in state C and no player 2 subjects bet in state A, about 62% of player 1s bet in {A,B}and 34% of player 2s bet in {C,B}. Alternatively, if all player 1s were random *L2* or *L3* and if all player 2s were random *L3*, then the level-*k* models' predictions would coincide with equilibrium predictions and no betting would occur, which again is not the case. Brocas et al.'s data analysis finds clusters corresponding to random *L1*s, *L2*s, and *L3*s, and an additional cluster of apparently irrational players; and this mixture of types fits significantly better than any homogeneous model, illustrating the existence of clear, nonequilibrium types and the importance of the heterogeneity of strategic thinking.

Tomasz Strzalecki (2010) conducts a level-*k* analysis of games like Rubinstein's (1989) electronic mail game, showing that such models, with bounded depth of reasoning, make predictions that are realistically independent of tail assumptions on higher-order beliefs.

Rogers, Palfrey, and Camerer (2009) conduct a horse race between LQRE and CH for similar betting games, which is inconclusive; but on fit they favor their richly parameterized "truncated heterogeneous LQRE" model over CH or LQRE. Brocas et al.'s lookup data reinforces their level-*k* interpretation of the decision data, and argues against LQRE or even CH.[44]

5.2. *Auction Experiments*

There is a rich literature on sealed-bid incomplete information auction experiments, which has developed largely independently of the literature on game experiments, despite similar goals and methods. In these experiments, whether first-price or (to a lesser extent) second-price, independent-private-value or common-value, subjects' initial responses tend to exhibit overbidding relative to the risk-neutral Bayesian equilibrium (e.g. Kagel and Levin 1986 and Goeree, Holt, and Palfrey 2002). The literature has proposed different explanations for overbidding: "joy of winning" and/or risk-aversion for private-value auctions, and the winner's curse for common-value auctions. It is vexing that there is no overlap between the explanations proposed for private- and common-value auctions, and also that these explanations are only loosely related to explanations proposed for deviations from equilibrium in other games.

Kagel and Levin (1986) and Erik Eyster and Matthew Rabin (2005) sought to unify the explanations of nonequilibrium behavior in common-value auctions and other incomplete-information games where (unlike in private-value auctions) informational inferences are relevant. Kagel and Levin formalize the intuition behind the winner's curse in models in which "naïve" bidders do not adjust their value estimates for the information revealed by winning, but otherwise follow Bayesian equilibrium. Eyster and Rabin's notion of "cursed equilibrium", in which people do not fully take the correlation between others' decisions and private information into account, but otherwise follow Bayesian equilibrium, generalizes Kagel and Levin's model to

[44] Carrillo and Palfrey (2009) analyze a game of two-sided private information where players have privately known "strengths" and can decide whether to fight or compromise. If either chooses to fight, the stronger player receives a high payoff and the weaker a low one. If both choose to compromise, each player receives an intermediate payoff. The only equilibrium is for players to always fight, because as in zero-sum betting games, agents have opposing interests for when to compromise. In their experiments, by contrast, compromise occurs 50-70% of the time, with less fighting the higher the compromise payoff.

allow intermediate levels of value adjustment ranging from standard equilibrium with full adjustment to "fully-cursed" equilibrium with no adjustment; and also from auctions and bilateral exchange to other kinds of incomplete-information games.[45] However, Kagel and Levin's and Eyster and Rabin's models both allow players to deviate from equilibrium only to the extent that they do not draw correct informational inferences. Thus their predictions coincide with equilibrium in games where such inferences are not relevant, and their models do not address nonequilibrium behavior in private-value auctions or any complete-information game.

Crawford and Iriberri (2007a) propose a level-*k* analysis that provides a way to unify the explanation of deviations from equilibrium in initial responses to independent-private-value or common-value auctions, without invoking joy of winning, risk-aversion, or cognitive biases.[46] Their analysis establishes a connection between large bodies of experiments on auctions and experiments on strategic thinking in complete information settings.

The key issue is how to specify *L0*. In auctions there are two natural possibilities: *Random L0*, analogous to the type of the same name in the analysis of zero-sum betting above, bids uniformly on the interval between the lowest and highest possible values, independent of its own value. Thus it sometimes bids above its own value, as is plausible given its role as people's model of others' instinctive reactions to the game, given that they do not observe others' values. Alternatively, *Truthful L0* bids its expected value conditional on its own signal—a notion that is meaningful in auctions, though not in all incomplete-information games.

Crawford and Iriberri build separate type hierarchies on these *L0*s, stopping for simplicity at *L2*: *Random* (*Truthful*) *Lk* is defined by iterating best responses from *Random* (*Truthful*) *L0*; and allow each subject to be one of the types, from either hierarchy. They then explore the optimal bidding strategies for each type, in preparation for taking the model to the data from representative auction experiments for the leading environments.

For a given *Lk* type, just as in an equilibrium analysis, the optimal bid must take into account value adjustment for the information revealed by winning in common-value auctions, and in first-price auctions the bidding trade-off between the higher price paid if the bidder wins and the probability of winning. Crawford and Iriberri show that the level-*k* model allows a tractable

[45] Eyster and Rabin show that cursed equilibrium can explain zero-sum betting with a probability that is positive but less than one.

[46] See also Uri Gneezy's (2005) level-*k* analysis of complete-information auctions, in which the level-*k* model's performance is disappointing, perhaps because complete-information auctions have structures that stress-test the model.

characterization of those aspects of the bidder's problem, which closely parallels Milgrom and Robert J. Weber's (1982) equilibrium characterization.

With regard to value adjustment, Random *L1*, like Kagel and Levin's naïve bidders and Eyster and Rabin's fully cursed bidders, does not condition on winning because Random *L0* bidders bid randomly independently of their values, hence Random *L1* thinks that beating them is uninformative. Crawford and Iriberri's other types do condition on winning in various ways, but this conditioning tends to make bidders' bids strategic substitutes, in that the higher others' bids are, the greater the (negative) adjustment. Thus, to the extent that Random *L1* overbids relative to equilibrium, Random *L2* tends to underbid: If it's bad news that you beat equilibrium bidders, it's even worse news to have beaten overbidders. The bidding tradeoff, by contrast, can go either way, depending on the distribution of values, just as in an equilibrium analysis.

Overall, the analysis shows that the conclusions of equilibrium auction theory are surprisingly robust to the structured failures of the equilibrium assumption allowed by a level-*k* model: Essentially all of the results of equilibrium analysis survive, qualitatively, except those aspects that rely heavily on the ex ante symmetry across players of their model. Those last results are altered even when players are objectively symmetrical, because level-*k* players, unlike equilibrium players, have simpler models of other players than they do of themselves.

Empirically, the question is whether an estimated mixture of Random *L1* overbidding and Random *L2* underbidding fits the data better than equilibrium plus noise, cursed equilibrium, or, for private-value auctions like Goeree, Holt, and Palfrey's (2002), LQRE. In three of the four leading cases Crawford and Iriberri study, a level-*k* model does better than those alternatives. For the remaining case, Kagel and Levin's first-price auction, the most flexible cursed equilibrium specification has a small advantage over level-*k*; but this disappears when the cursed equilibrium model's number of parameters is made more comparable to that of the level-*k* model.

Except in Kagel and Levin's second-price auctions, where many subjects seem to have missed the key to optimal bidding and the results seem largely random relative to all the models Crawford and Iriberri considered, the estimated type frequencies are quite similar to those estimated for non-auction experiments: Large estimated frequencies (59-65%) of random *L1*, smaller but significant frequencies of random *L2* (4-9%), truthful *L1* (9-18%), *Equilibrium* (4-16%), and truthful *L2* (1-16%), and zero or very low frequencies of Random *L0* or Truthful *L0*.

Overall, Crawford and Irriberi's analysis shows how to extend level-*k* analysis to an important class of incomplete-information games, establishes the robustness of most of the conclusions of equilibrium auction theory to level-*k* failures of the equilibrium assumption, and gives a more unified explanation for the systematic patterns of nonequilibrium behavior in private- and common-value auctions and other games.

5.3. *Acquiring a Company and the Winner's Curse*

Gary Charness and Dan Levin (2009) report experiments that stress-test existing explanations of experimental subjects' failure to take adverse selection effects like those behind the winner's curse and related deviations from equilibrium adequately into account. Their experiments are based on William F. Samuelson and Max H. Bazerman's (1985) "Acquiring a Company" game, a game-theoretic analog of George A. Akerlof's (1970) "lemons" market. The game has two players, a bidder and a responder, both risk-neutral. The responder owns a single indivisible object (the company) and bidder makes a single bid for it. If the proposer accepts the bid the company is transferred at the bid price, and if not, there is no deal. In either case the game is over. The value of the company to the responder is an integer between 0 and 100 inclusive, with each of these values equally likely. Only the responder observes his value, before he must decide whether to accept; but the bidder knows that, whatever the value, it is 50% larger for him than for the responder; and this fact and the value distribution are common knowledge.

This game has an essentially unique perfect Bayesian equilibrium, in which the proposer bids zero and the responder rejects that bid, but would accept any offer greater than his value. The reasoning rests on the proposer drawing a simple contingent inference from the responder's willingness to accept that is like the inference required to overcome the winner's curse or to avoid losing money in zero-sum betting games. Suppose the bidder offers $x > 0$. The responder will then accept if and only if his value is less than x, so that given the uniform distribution, the responder's expected value conditional on acceptance is $x/2$. Thus the expected value to the bidder conditional on acceptance is $3x/4$, in which case the transaction makes him lose $x/4$ on average. The optimal bid is therefore 0, and no transfer will occur even though it is common knowledge that there are prices at which a transfer is mutually beneficial.

Equilibrium of course assumes that people get this inference right, but many people seem to find it difficult; and as a result, they make unprofitable positive offers in Acquiring a Company. It is natural to ask whether cursed equilibrium or a level-*k* model can explain this behavior.

Charness and Levin address this issue via a clever design with "robot" treatments in which subjects' decisions determine their payoffs in a way that is logically identical to the way a rational responder's decisions determine the bidder's payoffs in Acquiring a Company, but in which the robot responder is framed not as another player but as part of the rules of the game. The problem that determines the proposer's optimal decision is then identical to the proposer's problem in the original Acquiring a Company treatment when he assumes the responder will make a rational acceptance decision, and so involves the same probabilistic inferences. But because the new problem no longer involves other's decisions, cursed equilibrium or level-*k* players, taken literally, are predicted to get it right. Thus Charness and Levin's design sharply separates explanations of the winner's curse and related phenomena based on cursed equilibrium or level-*k* thinking from explanations based on nonstrategic failures of probabilistic judgment. Their main finding, that their subjects are cursed as much as in a standard Acquiring a Company design, suggests that cursed equilibrium or level-*k* models miss part of what is going on.

Asen Ivanov, Levin, and Muriel Niederle (2010) use subjects' initial responses to a different auction game to further investigate whether the winner's curse is driven primarily by judgment failures or instead by deviations from equilibrium beliefs as in cursed equilibrium or a level-*k* analysis. Ivanov, Levin, and Niederle's design is based on Jeremy Bulow and Paul Klemperer's (2002) Maximum Game, a second-price common-value auction in which bidders' common value equals the maximum of their independent and identically distributed value signals. The Maximum Game is weakly dominance-solvable in two steps, with truthful bidding as its unique equilibrium. Ivanov, Levin, and Niederle run three treatments, Baseline, ShowBidFn, and MinBid, but most importantly they focus on two different parts of each treatment. In part I subjects are randomly paired with other subjects to play the Maximum Game for 11 periods, with their value signals sampled without replacement from a set of 11 possible values (so that a subject's 11 bids reveal the entire function mapping his values into bids). In part II subjects again play the Maximum Game, but now against a computer "robot" that draws values in random order, again sampled without replacement, and uses the subject's own bidding function from part I to map them into bids. Thus in part II each subject effectively plays against his own past self, and he knows that; although he is not reminded of his bidding function from part I. ShowBidFn is identical to the Baseline except that in its part II subjects are explicitly reminded of their part I bidding functions. Min-Bid is identical to the Baseline except that subjects are not allowed to bid below their own value signals; this makes truthful bidding a weakly dominant strategy.

Ivanov, Levin, and Niederle find that many subjects make weakly dominated bids in part I of the Baseline, most overbidding as in other auction experiments. Much of this overbidding persists in part II, where less than a quarter of the subjects even approximately best respond to their own part I bidding behavior. These patterns mostly persist in the ShowBidFn and MinBid treatments, with the frequency of overbidding substantially higher in the MinBid treatment (where underbidding was not allowed) than in the Baseline and ShowBidFn treatments.

Ivanov, Levin, and Niederle then argue that if bidding behavior is driven by non-equilibrium beliefs, for subjects who overbid in part I there should be less overbidding in part II of each treatment, where there is no strategic interaction so beliefs are known, relative to that treatment's part I, where beliefs must be predicted. They also argue that there should be less overbidding in part I of MinBid, where truthful bidding is weakly dominant, than in parts I of the Baseline and ShowBidFn, where truthful bidding is the only strategy that survives two steps of iterated weak dominance. Because there were widespread violations of weak dominance and there was *not* significantly less overbidding in either case, they claim that their results are evidence against non-equilibrium beliefs-based models such as level-*k* or cursed equilibrium.

In our view Ivanov, Levin, and Niederle's conclusion is not well supported. First, parts II of their three treatments are decision environments, not games; and part I of MinBid tests only reliance on simple weak dominance. Thus only parts I of Baseline and ShowBidFn have much to say about how subjects' beliefs are formed. Parts II of all three treatments simply stress-test *all* beliefs-based optimizing models, and part I of MinBid tests only a minimal restriction on beliefs. Charness and Levin (2009) showed that few subjects are up to the probabilistic inferences needed in a game with a winner's curse as simple as that in Acquiring a Company, yet the Maximum Game requires far more subtle inferences: As Bulow and Klemperer (2002) said of its equilibrium predictions, "…the Maximum Game, provides a good illustration of how a different choice of value function…can make it easy to obtain extreme 'perverse' results." Ivanov, Levin, and Niederle's rejection of beliefs-based models simply reconfirms Charness and Levin's finding that settings where rational behavior requires complex inferences can make subjects deviate from equilibrium, without suggesting an alternative model of what subjects are doing.

Second, Ivanov, Levin, and Niederle rest their rejection of beliefs-based models entirely on violations of simple dominance and indirect tests comparing game-theoretic and decision-theoretic settings, without explicitly analyzing any specific model.[47] Costa-Gomes and Makoto Shimoji (2010) also question the suitability of Ivanov, Levin, and Niederle's design to test belief-based theories in general and level-*k* models in particular. They also show that a standard level-*k* model fares quite well in direct tests using their data, approximately accounting for over 90% of subjects' bids in the MinBid treatment—albeit with a predominance of *L2* subjects rather than the *L1* subjects whose behavior Ivanov, Levin, and Niederle focused on in their analysis.[48]

To sum up, in our view neither Charness and Levin's nor Ivanov, Levin, and Niederle's results undermine the strong experimental support for level-*k*/CH or cursed equilibrium models. Those models were formulated for settings in which the main difficulty is predicting and responding to other players' decisions, and Eyster and Rabin (2005) and Crawford and Iriberri (2007a) follow the common practice of simplifying other aspects of the problem to make their central points as clearly as possible. There is no reason to expect a model so specified to translate unmodified to settings in which the complexity has been shifted from the "other people" part of the problem to the "own decisions" part; although it is clear than one can falsify existing specifications by making the own decisions part sufficiently complex.

Instead, we read Charness and Levin's and related results as pointing out the need for a model general enough to encompass both nonequilibrium strategic thinking and nonstrategic failures of judgment. Before specifying such a model, we need to know more about why subjects have so much trouble with Bayesian updating and best responding in designs like Charness and Levin's and Ivanov, Levin, and Niederle's. We suspect that people have trouble with reasoning that is contingent on future events, even in settings where the contingency is as simple and immediate as it is in their designs. Possibly people don't update correctly because they have "representativeness" bias (underweighting the prior) or "conservatism" bias (overweighting the

[47] Aside from their indirectness, such comparisons are less than usually reliable because decision environments shed no light on whether subjects' bids are best-responses to the beliefs they would hold in an analogous strategic situation. As Ivanov, Levin, and Niederle say (p. 1436), "…it is possible that subjects employ very different cognitive mechanisms in interactions with other players; such interactions may trigger all sorts of thought processes about others' reasoning, beliefs, and intentions." They go on to say "In our study, subjects play against other people"; but the robots in the part II treatments are not equivalent to other people for this argument, because subjects' bidding functions are predetermined and known.

[48] Even in much simpler settings, Costa-Gomes and Weizsäcker (2008) found that subjects' actions often deviate from the best responses to their incentivized elicited beliefs, while also finding that a level-*k* model with suitable allowance for errors fits the data better than the leading alternatives.

prior). Or possibly they update correctly but don't always choose rationally, given their posteriors. More experiments are needed to discriminate sufficiently among these alternatives.

5.4. *Naive and Sophisticated Traders in Speculative Markets*

Ricardo Serrano-Padial (2010) conducts an illuminating analysis of the interaction between continua of naïve and sophisticated traders in prediction and other speculative markets. Naïve traders include any whose trading decisions can be expressed as functions of the data of the game and their own private values, without solving a fixed-point problem such as that required to compute an equilibrium. Sophisticated traders follow equilibrium in the market, but unlike equilibrium traders, taking the frequency and behavior of naïve traders rationally into account.

Serrano-Padial's analysis proceeds by plugging in the behavior of the naive traders to reduce the market to a "reduced market" whose equilibrium is a fixed point in the supply and demand behavior of the sophisticated traders, taking the behavior of the naïve traders as given. He then characterizes the equilibrium in the reduced market using fairly standard methods.

Depending on the frequency of sophisticated traders, the market can be in one of three phases. When there are enough sophisticated agents to counteract naive agents' deviations from equilibrium, the usual rational-expectations equilibrium ensues, even if there is a nonnegligible frequency of naïve traders. With an intermediate frequency of sophisticated traders, the market segments into intervals of the space of possible valuations in which sophisticated traders never bid; and disjoint intervals in which both naïve and sophisticated traders bid. In the "naïve" intervals, naïve traders have the pivotal influence on pricing, which deviates systematically from equilibrium. In the "sophisticated" intervals, pricing is just as predicted in the standard model.

An arbitrage argument shows that the deviations from equilibrium in naïve intervals must involve a local quasi-"favorite-longshot" bias, in which in a given interval, relatively low valuations are overpriced but high valuations are underpriced. (This is only a "quasi-bias" because it follows from sophisticated trader's equilibrium responses to the behavior of naïve traders, not directly from any explicit decision-theoretic bias.) Finally, with a low frequency of sophisticated traders, pricing is almost everywhere determined by the behavior of naïve traders.

Serrano-Padial's methods can be viewed as market analogs of Crawford's (2003) game-theoretic analysis of the interaction between sophisticated and level-*k* players in a model of strategic communication (Section 11), and the two analyses have parallel transitions between phases as the frequency of sophisticated agents varies. The fact that similar methods yield similar results in these disparate settings suggests that the methods will be of more general usefulness.

5.5. *Field Studies: Movie Opening and Lowest Unique Positive Integer Games*

Alexander Brown, Camerer, and Dan Lovallo (2010) use field data to study an incomplete-information signaling game with verifiable signals. Film distributors face a choice between "cold opening" a movie and pre-releasing them to critics in the hope that favorable reviews will increase profits. In perfect Bayesian equilibrium, cold-opening should not be profitable, because moviegoers will infer low quality for cold-opened movies and the process will unravel. Yet distributors sometimes cold-open movies, and in a set of 856 widely released movies, cold opening increased domestic box office revenue by 15% over movies of similar quality that were reviewed before release (though not in foreign markets and DVD sales). This is consistent with the hypothesis that some moviegoers did not infer low quality from cold opening. However, movie distributors do not appear to take advantage of moviegoers lack of sophistication, since only 7% of movies were opened cold despite the expected-profit advantage.

After preliminary tests that rule out more conventional explanations, Brown, Camerer, and Lovallo seek to explain these results by comparing variants of cursed equilibrium, LQRE, and CH models.[49] The best fitting cursed-equilibrium model has moviegoers almost fully cursed (drawing no inferences regarding cold-opened movies) but studios not cursed at all; given the specification, the resulting model is like a partially cursed (moviegoers but not distributors) version of LQRE. The best fitting CH model again has moviegoers almost fully cursed (τ, the average k, is 1.12 where 1 is fully cursed, which given the assumed Poisson distribution for k implies that the population frequency of $L0$s is 33%) but studios very sophisticated ($\tau = 8.5$). The best fitting CH model has a significant advantage in fit over the best fitting cursed-equilibrium or LQRE model. Overall, neither model really explains the behavior of studios, given the mismatch needed between the degree of strategic thinking of moviegoers and distributors. This may be unsurprising, because the simple model the authors sketch and estimate at the end is static. There has been a huge recent trend in the percentage of cold-opened movies (Brown, Camerer, and

[49] In the CH model, *Lk* best responds to *Lk*-1 rather than an estimated mixture of all lower-level types as it would in a CH model; but *Lk* responds to *Lk*-1's decision noise as in an LQRE model, a choice that is not standard in level-*k* or CH models. Further, *L0* for moviegoers assumes a uniform distribution over the whole range of possible qualities; although sample-mean quality might seem more natural here, the authors say that that does not work well either.

Lovallo 2010, Figure 2), complicated by changes in technology but probably still significant, which suggests that static models cannot span the full sample period.

Östling, Wang, Eileen Chou, and Camerer (Forthcoming) study a novel set of field data from a Swedish gambling company, which ran a competition for a short period of time involving a "lowest unique positive integer" or LUPI game. (They also studied experimental data from parallel treatments.) In the LUPI game, players strategically simultaneously pick positive integers and the player who chose the lowest unique (not chosen by anyone else) number wins a prize. Except for the uniqueness requirement, the game is like a first-price auction.

The game would have complete information except that participants had no way to know how many others would enter in a given week. The authors deal with this by adapting Myerson's (2000) Poisson games model, in which fully rational players face Poisson-distributed uncertainty about the number of players. They characterize the LUPI game's unique symmetric Poisson-Nash equilibrium, and compare it to the predictions of versions of QRE and CH models, using both the field data and data from experiments using a scaled-down version of the LUPI game.

Both the field and laboratory data show participants choosing very low and very high numbers too often, relative to the Poisson-Nash equilibrium, and avoiding round and/or salient numbers.[50] However, initial responses are surprisingly close to the equilibrium, given that the setting makes it almost inconceivable that participants could be computing it. Learning brings them even closer in subsequent periods.

In comparing the data to the predictions of versions of QRE and CH, Östling et al. assume that both have power rather than the usual logit error distributions, and they allow the CH types to best respond to the noise in others' decisions.[51] They find that relative to the Poisson-Nash equilibrium, power QRE predicts too few low-number choices while CH predicts too many—the pattern observed in the field data. Thus QRE gets the deviations from equilibrium qualitatively wrong. However, the experimental data discriminate much less sharply between the theories.

5.6. *Level*-k *Auction Design*

A number of recent papers reconsider mechanism design taking a "behavioral" view of individual decisions or probabilistic judgment, but to date there are very few analyses of design

[50] Salience plays a similar role in Crawford and Iriberri's (2007b) analysis of hide-and-seek games (Section 9).

[51] A standard CH model would not fit the LUPI data at all well: *L1* would choose 1, *L2* 2, *L3* 3 or less, and *Lk k* or less. But best responding to power errors allows *L2*'s modal choice to be as high as 5 (Östling et al. Forthcoming, Figure 3)). This is not a criticism of the Östling et al.'s CH analysis per se: rather, the LUPI game reveals a general limitation of the structural features of thinking steps models like level-*k* or CH.

outside the equilibrium paradigm. Yet design inherently involves the creation of new games, and it may be important for an application to work the first time. Further, assuming equilibrium can yield theoretically optimal designs that are too complex for confidence in equilibrium behavior.

Replacing equilibrium with a model that better describes people's responses to novel games should allow us to design more effective mechanisms. It also suggests an evidence-based way to assess the robustness of mechanisms, something previously left to intuition. A mechanism that is robust in the sense that it implements the desired outcome in dominant strategies or after a small number of rounds of iterated dominance will evoke the desired response from most or all level-*k* types that are empirically likely to be observed. It may therefore perform better in practice than a mechanism that can theoretically implement better outcomes, but only in equilibrium.

Crawford, Tamar Kugler, Zvika Neeman, and Ady Pauzner (2009) explore relaxing the equilibrium assumption in mechanism design by conducting a level-*k* analysis of optimal auction design. They consider the leading case of an expected-revenue maximizing single-object sealed-bid auction with two symmetric bidders who have independent private values, for which Myerson (1981) gives a complete equilibrium-based analysis. To focus on strategic behavior, they maintain standard rationality assumptions regarding decisions and probabilistic judgment.

They model strategic behavior via a level-*k* model that follows Crawford and Iriberri's (2007a) analysis of data from leading auction experiments, with either a random *L0* that bids uniformly over the natural range of bids or a truthful *L0* that bids its private value. They assume that bidders are drawn from a given population of level-*k* types, known to the designer. In examples, they consider what reserve prices are optimal and how much revenue they yield in first-price auctions. They also consider the optimality of auction forms and the use of exotic auctions that exploit bidders' nonequilibrium beliefs to exceed Myerson's revenue bound.

Crawford et al. show, trivially and unsurprisingly, that with independent private values, revenue-equivalence breaks down. Because a second-price auction makes the equilibrium bid a dominant strategy, level-*k* bids coincide with equilibrium bids, hence a second-price auction yields only the equilibrium expected revenue. By contrast, level-*k* bidders in a first-price auction can deviate from equilibrium, and they give an example to show that such an auction with a suitable reserve price can yield higher expected revenue than the best second-price auction. They also give examples in which the optimal reserve price is large with equilibrium bidders but small with level-*k* bidders, and vice versa. Interesting open questions are when a reserve induces more aggressive bidding for equilibrium than level-*k* bidders, and the extent to which this makes optimal level-*k* reserves higher than optimal equilibrium reserves.

Finally, Crawford et al. give an example to show that in theory, a designer can use exotic auction forms to exploit level-*k* bidders' nonequilibrium beliefs to obtain very large expected revenues. They note, however, that their formulation of the design problem takes the level-*k* model's specification as given, independent of the auction design, just as the standard formulation assumes that bidders will play an equilibrium for any design. Although the specification is based on substantial experimental evidence, there is reason to doubt the exogeneity assumption, particularly for exotic auctions that go beyond the evidence on which our specification is based. A general formulation of the design problem must take a position on how the design influences the rules that describe bidders' behavior and develop new methods to deal mathematically with that influence.

Even without such influences, the heterogeneity of level-*k* beliefs and behavior greatly complicates the characterization of optimal auctions. In the standard analysis there is no loss of generality in using the revelation principle to restrict attention to direct mechanisms because, if equilibrium is assumed (with a selection rule in case of multiple equilibria), a bidder's private value is all that is needed to predict his behavior. Given the restriction to direct mechanisms, the design problem is well-behaved enough that it is guaranteed to have a solution. The example given in the paper shows that this is no longer the case with level-*k* bidders, even if their level-*k* types are all the same, and even if this is known to the designer. With a heterogeneous population of types, the problem becomes more complex. Bidders with the same private values but different level-*k* types have different beliefs and will generally behave differently. It appears that Myerson's (1981) methods can be used to characterize an optimal auction if the designer knows that the population is homogeneous, and knows its type; and if the class of possible designs is restricted to rule out those that are too exotic for an optimal auction to exist. But if the population is heterogeneous the problem becomes multidimensional and much more difficult; and the high-dimensional reporting mechanisms one would consider for this case complicate the specification of *L0* and the influence of design on behavior.

Behaviorally optimal auction design poses interesting challenges, and meeting them should increase the practical usefulness of design.

6. *Kahneman's Entry Magic: Coordination via Symmetry-Breaking*

"…to a psychologist, it looks like magic."

—Kahneman (1988), quoted in Camerer, Ho, and Chong (2004)

Kahneman's "magic" refers to the fact that subjects in his own and others' market-entry experiments (see also Amnon Rapoport et al. 1998 and Rapoport and Daryl A. Seale 2002) achieve systematically better coordination ex post than in the natural equilibrium benchmark.[52]

In these experiments, *n* subjects choose simultaneously between entering ("In") and staying out ("Out") of a market with given capacity. In yields a given positive profit if no more subjects enter than a given market capacity; but a given negative profit if too many enter. For simplicity, Out yields 0 profit, no matter how many subjects enter. Because players cannot distinguish their roles, it is not sensible to predict systematic differences across roles in behavior. Thus, the natural equilibrium benchmark is the unique, symmetric mixed-strategy equilibrium, in which each player enters with a given probability that makes all players indifferent between In and Out. This mixed-strategy equilibrium yields an expected number of entrants approximately equal to market capacity, but there is a positive probability that either too many or too few will enter. Even so, subjects in market-entry experiments have systematically better coordination ex post (number of entrants stochastically closer to market capacity) than in the symmetric equilibrium.

In these games, efficient coordination requires breaking the symmetry of players' roles. The same issues arise in field settings such as those studied using incomplete-information models by Goldfarb and Yang (2009) and Goldfarb and Xiao (2011) (Section 6.2); and those considered in our discussion of the nonequilibrium econometrics of such games (Section 8).

6.1. *A level-*k *Analysis of Two-Person Entry/Battle of the Sexes Games*

Camerer, Ho, and Chong (2004, Section III.C) explain Kahneman's magic via a CH model, in which the heterogeneity of strategic thinking allows some players to mentally simulate others' entry decisions and accommodate them, which in entry games yields coordination benefits for all. Here we illustrate Camerer, Ho, and Chong's analysis in a two-person Battle of the Sexes

[52] Kahneman's statement does not qualify as folk game theory because only a game theorist would be surprised by these results.

game, which is like a two-person market-entry game with capacity one. For simplicity, we also substitute a level-*k* model for their CH model.[53]

Consider a two-person Battle of the Sexes game with $a > 1$, as in Figure 4. The unique symmetric equilibrium is in mixed strategies, with $p \equiv \Pr\{\text{In}\} = a/(1+a)$ for both players. The mixed-strategy equilibrium expected coordination rate is $2p(1 - p) = 2a/(1+a)^2$, and players' equilibrium expected payoffs are $a/(1+a)$. This expected coordination rate is maximized when $a = 1$, where it takes the value ½. With $a > 1$ the expected payoffs are $a/(1+a) < 1$: worse for each player than his worst pure-strategy equilibrium. As $a \to \infty$, $2a/(1 + a)^2 \to 0$ like $1/a$.

	In	Out
In	0, 0	*A*, 1
Out	1, *a*	0, 0

Figure 4. Battle of the Sexes

Now consider a level-*k* model in which each player follows one of four types, *L1*, *L2*, *L3*, or *L4*, with each role filled by a draw from the same distribution. For simplicity assume the frequency of *L0* is 0, and that *L0* chooses uniformly randomly, with Pr{In} = Pr{Out} = 1/2.

Type pairings	***L1***	***L2***	***L3***	***L4***
L1	**In, In**	**In, Out**	**In, In**	**In, Out**
L2	**Out, In**	**Out, Out**	**Out, In**	**Out, Out**
L3	**In, In**	**In, Out**	**In, In**	**In, Out**
L4	**Out, In**	**Out, Out**	**Out,In**	**Out, Out**

Table 2. Outcomes in Battle of the Sexes

*L1*s mentally simulate *L0*s' random decisions and best respond, thus, with $a > 1$, choosing In; *L2*s choose Out; *L3*s choose In; and *L4*s choose Out. The predicted outcome distribution is determined by the outcomes of the possible type pairings (Table 2) and the type frequencies. If both roles are filled from the same distribution, players have equal ex ante payoffs, proportional

[53] Camerer, Ho, and Chong (2004, Section III.C) and Chong, Camerer, and Ho (2005, Section 2.1) argue that in this context, CH models fit better than level-*k* models because they yield smooth monotonicity of entry rates as market capacity increases, as in the data, while a level-*k* model implies a step function; and because CH models imply that beliefs converge to correct beliefs in the limit as *k* increases, unlike level-*k* models which cycle perpetually in these games. However in most of the datasets Camerer, Ho, and Chong consider, unlike in their stylized CH model, there are congestion effects that allow payoff-sensitive logit errors like those in a typical level-*k* analysis, which smooth things as well. Further, cycling or correctness of beliefs in the limit have little or no relevance when *k* seldom exceeds 3. One question we do not consider here is whether a level-*k* model can explain the fact that entry rates are too high for low capacities and too low for high, which the CH model explains by estimating a high frequency of a random *L0* type; logit errors for higher level-*k* types can probably explain this as well.

to the expected coordination rate. *L3* behaves like *L1*, and *L4* like *L2*. Lumping *L1* and *L3* together and letting v denote their total probability, and lumping *L2* and *L4* together, the expected coordination rate is $2v(1 - v)$, maximized at $v = ½$ where it takes the value ½. Thus for v near ½, which is behaviorally plausible, the coordination rate is near ½. For more extreme values the rate is worse, converging to 0 as $v \to 0$ or 1. But because the equilibrium rate of $2a/(1 + a)^2 \to 0$ like $1/a$, even for moderate values of a, the level-k coordination rate is higher.

This analysis highlights a drawback of level-k/CH models, in that in the absence of payoff-sensitive errors, their predictions are independent of a as long as $a > 1$, while in experiments with similar games behavior is often sensitive to such parameter variations. Adding payoff-sensitive errors, particularly when starting with a CH model, would help to remedy this, but probably not enough to make the models fully descriptive of observed behavior.

The level-k analysis suggests a view of tacit coordination profoundly different from the traditional view, and illustrates the importance of the heterogeneity of strategic thinking the model allows. With level-k thinking, equilibrium and selection principles such as risk- or payoff-dominance (Harsanyi and Selten 1987) play no direct role in players' thinking. Coordination, when it occurs, is an almost accidental (though statistically predictable) by-product of the use of nonequilibrium decision rules. Even though players' decisions are simultaneous and there is no communication or observation of the other's decision, the predictable heterogeneity of strategic thinking allows more sophisticated players such as *L2*s to mentally simulate the decisions of less sophisticated players such as *L1*s and accommodate them, just as Stackelberg followers would. This mental simulation doesn't work perfectly, because an *L2* is as likely to be paired with another *L2* as an *L1*. Neither would it work if strategic thinking were homogeneous. But it's very surprising that it works at all.

6.2. *Field Studies: Cognitive Hierarchy Analyses of Entry Games*

In this section we review two field studies of incomplete-information entry games, which both use CH models. These studies provide only limited comparison of alternative models of strategic thinking, but they are of particular interest because they are among the first studies of nonequilibrium models of strategic thinking using field data.

Goldfarb and Yang (2009) apply an incomplete-information CH model to explain choices by managers at 2,233 Internet Service Providers (ISP) in 1997 whether or not to offer their customers access through 56K (versus the standard then, 33K) modems. There were two possible

56K technologies: Rockwell Semiconductor's *K56Flex* and US Robotics's *X2*. Thus an ISP manager could make one of four choices: (i) adopt neither technology, (ii) adopt Rockwell's, (iii) adopt US Robotics's, or (iv) adopt both. Controlling for market and ISP-specific characteristic, Goldfarb and Yang, adapted the CH model to describe the heterogeneity in ability or strategic sophistication among the SPI managers in these decisions. They assumed (departing from the usual *L0* specification) that an *L0* manager maximizes profits on the assumption that he will be a monopolist; an *L1* manager on the assumption that his competitors will be *L0*s; an *L2* manager on the assumption that his competitors will be an estimated mixture of *L0*s and *L1*s, and so on.

Goldfarb and Yang found significant evidence of heterogeneity of sophistication among managers, with an estimated τ, the average *k* in a CH model, of 2.67—seemingly higher than most previous estimates, but their *L0* is in some respects akin to an *L1*, which would bring it more in line with previous estimates. The CH model fits no better than a Bayesian equilibrium plus noise model, but the CH estimates have interesting and plausible implications. Interestingly, they suggest that relative to Bayesian equilibrium, heterogeneity of strategic thinking slowed the diffusion of the new 56K technology, with more strategic managers *less* likely to adopt, anticipating more competition. Managers behaved more strategically, in the sense of higher estimated *k*s, if they competed in larger cities, with more firms, or in markets with more educated populations. Finally, those managers estimated to be more strategic in 1997 were more likely to survive through April 2007. We note however that in a CH model, though not a level-*k* model, a higher *k* implies a more accurate model of others, hence higher expected profits. Thus, in a CH model a firm that does well in the market must have had a higher *k*; and the model rules out the possibility that a firm might err by perceiving others as being of a higher level than in reality. Only a model that allows the latter possibility allows independent inferences about a firm's level of sophistication and its beliefs about others' sophistication.

Goldfarb and Xiao (2011) applied an incomplete-information CH model to explain managers' choices whether or not to enter local U. S. telecommunications markets after the *Telecommunications Act* of 1996, which allowed free competition in such markets. They use Goldfarb and Yang's (2009) specification of *L0*. They found that more experienced and/or better educated managers did better, in the sense of entering markets with fewer competitors, on average; having better survival rates; and having higher revenues, conditional on survival. Estimated strategic thinking goes up from 1998 to 2002. The CH model fits much better than a

Bayesian equilibrium plus noise model in 1998, but only slightly better in 2002, in keeping with the view that models like CH are well suited to initial responses to novel situations, but are less relevant once players have had enough experience to converge to equilibrium.

7. *Bank Runs: Coordination via Assurance*

> "A crude but simple game, related to Douglas Diamond and Philip Dybvig's (1983) celebrated analysis of bank runs, illustrates some of the issues involved here. Imagine that everyone who has invested \$10 with me can expect to earn \$1, assuming that I stay solvent. Suppose that if I go bankrupt, investors who remain lose their whole \$10 investment, but that an investor who withdraws today neither gains nor loses. What would you do? Each individual judgment would presumably depend on one's assessment of my prospects, but this in turn depends on the collective judgment of all of the investors.
>
> Suppose, first, that my foreign reserves, ability to mobilize resources, and economic strength are so limited that if any investor withdraws I will go bankrupt. It would be a Nash equilibrium (indeed, a Pareto-dominant one) for everyone to remain, but (I expect) not an attainable one. Someone would reason that someone else would decide to be cautious and withdraw, or at least that someone would reason that someone would reason that someone would withdraw, and so forth. This…would likely lead to large-scale withdrawals, and I would go bankrupt. It would not be a close-run thing. …Keynes's beauty contest captures a similar idea.
>
> Now suppose that my fundamental situation were such that everyone would be paid off as long as no more than one-third of the investors chose to withdraw. What would you do then? Again, there are multiple equilibria: everyone should stay if everyone else does, and everyone should pull out if everyone else does, but the more favorable equilibria seems much more robust."
>
> —Lawrence H. Summers (2000).

Summers here views bank runs as an *n*-person coordination game with Pareto-ranked equilibria, a kind of generalized Stag Hunt game as in Diamond and Dybvig's (1983) model. A simplified version of his game can be represented by a payoff table as in Figure 5.

		Summary statistic	
		In	**Out**
Representative player	**In**	**1**	**-10**
	Out	**0**	**0**

Figure 5. Bank Runs

The summary statistic is a measure of whether or not the required number of investors stays In. In Summers's first example, all investors must stay In to prevent the bank from collapsing, so the summary statistic takes the value In if and only if all but the representative player stay In. In his second example two-thirds of the investors need to stay In, so the summary statistic takes the value In if and only if (adding in the representative player) this is the case.

These games pose the problem of coordination via assurance in a starkly simple form. In each example there are two pure-strategy equilibria: "all-In" and "all-Out".[54] All-In is better for everyone, but when the summary statistic is as extreme as in Summers's first example and n is large enough, all-In is also sufficiently fragile to make it risky for an individual player to play In.

Summers' discussion presumes that some equilibrium will describe people's responses to the one-shot game of his model. Given that few people can have had enough experience with closely analogous bank runs to make a learning justification for equilibrium plausible, we judge his arguments from the standpoint of modeling strategic thinking rather than learning.

Here as elsewhere, models that make unique predictions have important advantages and are accordingly favored in the literature. Refinements such as risk- or payoff-dominance (Harsanyi and Selten 1987) are the traditional route to unique predictions in coordination games.

In Summers' examples, payoff-dominance—in this context, Pareto-efficiency within the set of equilibria—uniquely favors the all-In equilibrium for any population size n; but as Summers suggests, the coordination it requires is behaviorally implausible here, even for small n.

Risk-dominance is more plausible. Here it selects the equilibrium with the largest "basin of attraction"—the set of initial beliefs that yield convergence of best responses to that equilibrium, assuming independence of players' beliefs about others' strategies. In two-person games with the structure of Summers' examples (Figure 5), risk-dominance therefore selects the equilibrium that results if each player best responds to a uniform prior over others' strategies, just as random *L1* does (and thus just as random *Lk* does for $k > 1$). In n-person games like Summers' examples, given independence of players' beliefs, risk-dominance again selects the "all-*L1*" equilibrium. For Summers' payoffs (Figure 5), whether all investors or only two-thirds must stay In to prevent collapse, the *L1* decision is Out and risk-dominance selects the all-Out equilibrium for any n. Even with much less extreme payoffs, say with -1.5 replacing -10, and with only two-

[54] There is also a mixed-strategy equilibrium in which the probability that the summary statistic equals In just balances the benefits of In and Out; but this equilibrium is behaviorally implausible.

thirds In needed to prevent collapse, risk-dominance favors all-Out for all *n*, because no *n* makes the probability at least 0.6 that at least two-thirds of *n*-1 independent Bernoulli trials yield In. Thus, risk-dominance does not adequately reflect Summers' intuition (or ours) in his examples.

Many people are skeptical of risk-dominance as a positive model of strategic thinking in initial responses to games. As Summers' discussion illustrates, many of them take comfort in the fact that in simple bank runs games, risk-dominance can be justified via a "global games" analysis as in Stephen Morris and Hyun Song Shin (1998) (see also Morris and Shin 2003, Section 2.3.3), which has become the workhorse model of behavior in bank runs games. A global games model replaces the original complete-information game with an incomplete-information version with stochastic payoff perturbations that satisfy particular distributional assumptions. Those assumptions make the perturbed version, unlike the original game, dominance-solvable, with a unique equilibrium that in sufficiently simple games coincides with the risk-dominant equilibrium of the original game.[55] A global-games analysis then implies unique equilibrium selection via iterated dominance or iterated knowledge of rationality (Bernheim 1984 and Pearce 1984), without recourse to a behaviorally questionable risk-dominance refinement.

The experimental analyses of strategic thinking reviewed here (Section 3) suggest that the comfort provided by the global games/iterated dominance rationale for risk-dominance is illusory. First, the perturbed game is a device chosen not because it is supported by any evidence that it accurately models players' initial responses to any uncertainty they perceive, but simply to enable the iterated dominance argument. Second, even granting the realism of the perturbed game, the evidence stops far short of supporting the indefinite (though usually finite) reliance on iterated dominance that is needed to make a global games analysis yield a precise prediction.

That said, with a uniform random *L0*, a level-*k*/CH analysis yields predictions in Summers' examples strikingly similar to those of the global games approach. As already noted, if random *L1* responses are in equilibrium against each other, they select the risk-dominant/global games

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[55] In large-population versions of such games, Morris and Shin (2003, p. 57) advocate initiating a global games analysis via a common, naïve "Laplacian" prior, specified so "the prescription for each player is to hypothesize that the proportion of other players who will opt for a particular action is a random variable that is uniformly distributed over the unit interval and choose the best action under these circumstances"; note the similarity to the starting point of Harsanyi and Selten's (1986) tracing procedure). In large populations a Laplacian prior is a useful shortcut to the results of a global games analysis, and often yields results similar to risk-dominance, but with nonlinear payoff functions in finite populations there may be substantive differences. For example, with two pure strategies per player, whose payoffs are determined by the aggregate strategy frequencies, Laplacian frequencies average 50%, while for an independent *L0* the population frequencies are approximately Normal, and for a perfectly correlated *L0* they are certain to be 50%. Frankel, Morris, and Pauzner (2003) generalize these results to large numbers of actions, but show that in games other than potential games, the uniqueness of the limiting equilibrium fails in general.

equilibrium. But then, in a CH model as well as a level-*k* model, the responses of higher types are the same as random *L1*'s. Thus the level-*k*/CH approach selects the risk-dominant/global games equilibrium as well (see for example Camerer, Ho, and Chong 2004, Section III.B).[56] In settings where it is not possible to improve upon the naïve, mathematically neutral prior that underlies risk-dominance, a level-*k*/CH analysis lends predictions based on risk dominance or global games some evidence-based support.

However, unlike the mathematically motivated risk dominance/global games analyses, which "black-box" players' anchoring in a way that draws attention away from market psychology and common causes of beliefs, a level-*k*/CH analysis immediately suggests ways to take such factors into account and gracefully accommodates them. First, unlike the risk-dominance/global games approach, it is easily combined with an *L0* in the style of Graham's Mr. Market, which models the psychology of a representative player's (or players') instinctive reaction to news (Section 3). Such judgments about market psychology are plainly of central importance in applications, but combining them with the purely payoff-structure-based risk-dominance/global games approach poses formidable challenges. And because such an *L0* is a psychological rather than a strategic concept, it is easier to extrapolate its specification across games (as illustrated in Section 9).

Second, a level-*k*/CH approach highlights the issue of how players model the correlation of others' strategy choices in *n*–person games, which is of great potential importance but to our knowledge not considered in the traditional game theory or global-games literatures.

Ho, Camerer, and Weigelt (1998) found, in level-*k*/CH analyses of initial-response data from their Nagel-style guessing games, that a model in which players' models of others are highly correlated fit the data better than one in which their models assume independence. Costa-Gomes, Crawford, and Iriberri (2009) use the initial-response data from Van Huyck, Battalio, and Beil's (1990, 1991) *n*–person coordination experiments, with games like Stag Hunt but with seven efforts per player and seven Pareto-ranked pure-strategy equilibria, to conduct a horse race between equilibrium plus noise with risk- or payoff-dominance (in turn), LQRE, level-*k*, CH, and NI models, with or without correlation in players' models of others'. They found that correlated versions of these models almost always do as well or better than independent versions. Among the equilibrium selection criteria, payoff-dominance fits at least as well as the

[56] Note that the level-k/CH approach does not justify *Laplacian* beliefs, which in 2×2 games hypothesize that the proportion of others who choose a decision is uniformly distributed over the unit interval.

alternatives, and often better. Among the individualistic models, level-*k* and CH perform comparably well; level-*k* usually does better than NI or LQRE; CH does slightly better than NI or LQRE; and NI does slightly better than LQRE. Overall, payoff-dominant equilibrium fits best, noticeably better than level-*k* and CH. These conclusions are based on limited evidence; but the idea that people rely on representative-agent models of others even when it's inappropriate is behaviorally plausible when one considers the subtlety of the probabilistic judgments needed to do otherwise and the cognitive difficulty of having diverse models. The conclusion that players' models of others are correlated is also suggested, less directly, by subjects' notorious inability to anticipate group-size effects in settings where they are relevant.

A level-*k*/CH model is easily modified to allow correlation of players' models of others via the specification of *L0*.[57] Perfect correlation makes players perceive examples like Summers' as quasi-two-person games. Depending on the payoffs and the fragility of the all-In equilibrium, this can make all-In more or less likely than when players' models of others are independent.

A level-*k*/CH approach to bank runs games has further advantages. In a level-*k*/CH model players use the same rules to choose their strategies with or without multiple equilibria. As in Section 6's market-entry games, neither equilibrium nor refinements play any role in players' thinking; and coordination when it occurs is an accidental but statistically predictable by-product of how players' nonequilibrium decision rules interact with the game—though this time symmetry-breaking is not required, and there is no "magic". In these symmetric coordination games the higher payoffs of equilibria attract level-*k*/CH as well as equilibrium players, so the likely outcome is some equilibrium, which as we have seen is the risk-dominant equilibrium in simple games. Importantly, however, a level-*k*/CH model also predicts the likelihood of coordination failure and the forms it may take. Further, in more complex games level-*k*/CH predictions may deviate from those of global games or risk-dominant equilibrium selection (Crawford 1995; Costa-Gomes, Crawford, and Iriberri 2009). In our opinion this is a richer and more plausible view of strategic thinking than the one that underlies the global-games approach.

[57] The correlation of players' models of others is irrelevant in defining payoff-dominance. Risk-dominance is traditionally defined assuming independence, but its definition could easily be modified to allow such correlation.

8. *Nonequilibrium Econometrics: Structural Alternatives to Incomplete Models*

How might the availability of structural nonequilibrium models that reliably describe initial responses to games change the way we think about data? In recent econometric work on auctions, market entry games, and other kinds of coordination games, attention centers on identification and estimation of the parameters that represent individual players' payoff idiosyncrasies, which are normally unobservable in the field.[58] If equilibrium can reasonably be assumed, and if a precise mechanism for selection among any multiple equilibria can be specified, then it is often possible to identify and estimate the distributions of the payoff idiosyncrasies, even without imposing parametric restrictions.

Building on the equilibrium-based work, recent work has considered auction, market entry, and coordination settings where equilibrium is implausible, and other settings where *some* equilibrium is plausible but it is hard to specify the selection mechanism with confidence. In the former settings, the leading approach has been to accept set-valued restrictions such as those implied by *k*-rationalizability (Section 2) as the model's only implications and use them to identify and estimate the resulting incomplete model (Andres Aradillas-Lopez and Elie Tamer 2008, who call *k*-rationalizability "level-*k* rationality").[59] This is natural for settings involving initial responses, because just as the experimental evidence suggests that equilibrium and even rationalizability are not reliable models of initial responses (Section 3), it also suggests that initial responses often respect *k*–rationalizability for sufficiently low values of *k*.

In the latter settings, a common approach has been to estimate a model that imposes equilibrium but is incomplete model in that it does not restrict equilibrium selection (Bresnahan and Reiss 1991, Federico Ciliberto and Tamer 2009, and Federico Echenique and Ivana Komunjer 2009). This is natural for settings where it is plausible that players have learned to play an equilibrium, but the learning process cannot be specified with confidence.

In the absence of evidence to guide reliable precise specifications of nonequilibrium behavior or equilibrium selection, incomplete models may be the only way to avoid misspecification. However, although Charles F. Manski (2007) and others have shown that incompleteness need not seriously reduce the econometric usefulness of decision-theoretic models, incompleteness

[58] In some applications it is reasonable to assume that these idiosyncrasies are commonly known among the players, though not to the analyst; in others they are taken to be privately observed.

[59] Recall that in games that are not sufficiently dominance-solvable, *k*-rationalizability is incomplete in that it does not specify a unique (though probabilistic) prediction conditional on the value of the behavioral parameters; and that in games with multiple equilibria, equilibrium plus noise but without refinements is incomplete in the same general sense (Section 2).

can have severe identification and estimation costs in game-theoretic models, where individual ambiguity can "multiply up" across players to yield much greater ambiguity.

Aradillas-Lopez and Tamer (2008) show, for example, that in two-person entry games, weakening equilibrium to *k*-rationalizability for low *k* implies much weaker identification of the distributions of players' payoff perturbations, with individuals' *k*'s often unidentified. In an extreme case, in complete-information entry games 1-rationalizability implies that even unlimited data can rule out only a tiny fraction of possible parameter values (their Figure 3).[60]

Aradillas-Lopez and Tamer (2008) also compare the identifying powers of equilibrium and *k*-rationalizability in first-price private-value auctions. Following Pierpaolo Battigalli and Marciano Siniscalchi (2003), they note that *k*-rationalizability implies only a weak upper bound on bids, which shrinks with *k* but allows bids both above and below equilibrium for any *k*. This ambiguity leads to weak bounds on bidders' value distributions and limits their identifiability.

These analyses suggest that it may be helpful to complete *k*-rationalizability, and/or to model equilibrium selection, by postulating a structural level-*k* model with enough behavioral parameters to limit the risk of misspecification. *k*-rationalizability allows some beliefs that, though consistent with finitely iterated knowledge of rationality, are behaviorally outlandish (Section 2.2). Further, there is now a large body of experimental evidence that, to the extent that initial responses respect *k*-rationalizability, they do so because people follow level-*k* decision rules that respect it, not because they explicitly perform finitely iterated dominance (Section 3). Thus, the cost in descriptive accuracy of adding a level-*k* structure may be quite small.

The benefits of completing *k*-rationalizability or modeling equilibrium selection via a structural level-*k* model can be considerable. Benjamin Gillen (2010) studies a level-*k* model of private-value first-price auctions based on Crawford and Iriberri (2007a) (Section 5). He shows that under a reasonable but not unrestrictive assumption on the separation of level-*k* types' bidding functions, and with enough variation in the number of bidders, both bidders' value distributions and their types are identified, parametrically or nonparametrically. Thus Gillen's

[60] Costa-Gomes and Crawford's (2006, footnote 42, p. 1766) makes a similar point, noting that in their maximum likelihood estimation of a model of subjects' guesses and searches for hidden payoff information, the guess part of the log-likelihood is nearly six times larger than the search part. As they explain, this is because their theory makes precise predictions of a subject's decisions, given his type; but their theory of cognition and search imposes only weak, set-valued restrictions on a subject's searches, given his type. Because their theory of decisions is complete while their theory of search is incomplete, the search restrictions are much more likely to be satisfied by chance, which causes the disparity in likelihood weights.

level-*k* model makes identification just as strong as it is assuming equilibrium, with the bonus that one can identify bidders' level-*k* types as well.[61]

Another potential application revisits Ciliberto and Tamer's (2009) analysis of airline entry decisions into U.S. markets. Following Bresnahan and Reiss (1991), Ciliberto and Tamer assume equilibrium but use an incomplete model that is agnostic about equilibrium selection. Even allowing for prediction ambiguity, their estimated model correctly predicts the entrants in a given market less than 35% of the time, with participants often coordinating better ex post than in any equilibrium. This feature of their analysis is strongly reminiscent of Kahneman's "entry magic" (Section 6), which suggests that replacing their agnostic model of equilibrium selection with a level-*k* structure might complete the model in a way that yields more accurate predictions.

9. *Yushchenko and Lake Wobegon: Non-neutral Framing in Outguessing Games*

> "Any government wanting to kill an opponent…would not try it at a meeting with government officials."
>
> —comment, quoted in Chivers (2004), on the poisoning of Ukrainian presidential candidate—now ex-president—Viktor Yushchenko

> "…in Lake Wobegon, the correct answer is usually 'c'."
>
> —Garrison Keillor (1997) on multiple-choice tests (quoted in Attali and Bar-Hillel (2003)

The Yushchenko and Lake Wobegon quotations refer to simultaneous-move zero-sum two-person games with unique mixed-strategy equilibria. In the first, the players are an assassin choosing one of several dinners at which to try to poison Yuschenko, only one of which is with officials of the government suspected of wanting to poison him; and an investigator who has the resources to check only one of the dinners. In the second, the players are a test designer deciding where to hide the correct answer and a clueless test-taker trying to guess its hiding place.

[61] This completion would not work in the same way with a CH model, because CH types do not always choose *k*-rationalizable strategies. Brendan Kline (2010) also studies the problems that arise in identifying and estimating econometric level-*k*/CH models. He gives conditions for large-sample identification and robustness of estimators to sampling variation when types' predicted behaviors are known and each agent makes only one choice, and parallel conditions for identification when types' predicted behaviors are unknown but each agent makes many choices.

In each case the key issue is how players react to framing of decisions that is psychologically non-neutral but does not directly affect payoffs. Equilibrium in zero-sum two-person games leaves no room for such framing to affect outcomes, but people often react to it anyway. The thinking reflected by the quotations is plainly strategic, but nonequilibrium: To the first, for example, any game theorist worth his salt would respond, "If that's what people think, a meeting with government officials is exactly where *I* would try to poison Yushchenko."

9.1. *Hide and Seek Experiments*

Rubinstein and Tversky ("RT"; e.g. Rubinstein 1999) conducted experiments with zero-sum, two-person "hide-and-seek" games with non-neutral framing of locations. A typical seeker's instructions were: "Your opponent has hidden a prize in one of four boxes arranged in a row. The boxes are marked as shown below: A, B, A, A. Your goal is, of course, to find the prize. His goal is that you will not find it. You are allowed to open only one box. Which box are you going to open?" A hider's instructions were analogous.

RT's design is important as a tractable abstract model of a non-neutral cultural or geographic frame, or "landscape." The frame has no direct payoff consequences; all that matters is whether or not the hider finds the seeker, not where. But the frame is non-neutral in two ways: The "*B*" location is distinguished by its label, and the two "*end A*" locations may be inherently focal. This gives the "*central A*" location its own brand of uniqueness as the "least salient" location.[62] In our quotations, Yuschenko's meeting with government officials is analogous to RT's B location, and the physiology of poison may have created something like RT's end locations. Although there is nothing as uniquely salient in Lake Wobegon as the dinner with government officials, psychologists think that with four possible answers, both the *a* and *d* end locations and location c are inherently salient (with the jury still out on which is more salient; see Christenfeld, 1995).

Traditional game theory rules out any influence of the landscape by fiat, and RT's hide-and-seek game has a clear equilibrium prediction, which leaves no room for framing to influence the outcome. Moreover, the rationale for playing one's equilibrium strategy is immune to most of the usual counterarguments in a zero-sum two-person game. Even so, framing had a strong and systematic effect in RT's experiments, qualitatively the same in six experiments around the world, with *Central A* or its analogs in other treatments most prevalent for hiders (37% in the

[62] Mathematically this "negative" uniqueness is analogous to the "positive" uniqueness of "*B*", but Crawford and Iriberri's (2007b) analysis shows that its psychological effects are quite different.

aggregate) and *Central A* even more prevalent for seekers (46%).[63] These results pose two puzzles. On average hiders are as smart as seekers, so hiders tempted to hide in *central A* should realize that seekers will be just as tempted to look there. Why do hiders allow seekers to find them 32% of the time when they could hold it down to 25% via the equilibrium mixed strategy? And why do seekers choose *central A* (or its analogs) even more often (46%) than hiders (37%)?

RT took the nonequilibrium patterns in their data as evidence that their subjects did not think strategically (see the quotations in Crawford and Iriberri 2007b, p. 1733, footnote 3). But as Crawford and Iriberri argued, responses to such simple games are unlikely to be completely non-strategic and the fact that subjects' patterns of behavior were qualitatively the same in six experiments suggests that they have a common structure, even if it is a nonequilibrium one.

What kind of model can explain the main patterns in the data? First, although the payoff structure of RT's game is asymmetric, all models that focus on payoffs to the exclusion of labeling—equilibrium, QRE, and level-*k* with a uniform random *L0*—imply role-symmetric responses (QRE here coincides with equilibrium, for any distribution and precision) and so miss the strong role-asymmetric patterns in the results.

Crawford and Iriberri (2007b) accordingly compared versions of equilibrium, QRE, and level-*k*/CH models that all incorporate the effects of labeling— for equilibrium or QRE, by adding payoff perturbations that plausibly describe hiders' and seekers' instinctive reactions to salience (seekers get extra payoff credit for salient locations, hiders lose credit); and for level-*k*, by making *L0* role-independent but probabilistically favoring salient locations.

In particular, Crawford and Iriberri (2007b, online appendix) found that LQRE and equilibrium with payoff perturbations both miss the strong role-asymmetric patterns in the results; and that LQRE with estimated perturbations either gets the main patterns qualitatively wrong or estimates an infinite precision and thereby turns itself back into an equilibrium with payoff perturbations model, which itself fits significantly less well than a level-*k* model.

By contrast, a level-*k*/CH model responds to the role-asymmetric payoff structure in a role-asymmetric way, and a level-*k* model with a role-independent *L0* that probabilistically favors

[63] This statement depends on identifying analogies among RT's treatments as explained in Crawford and Iriberri (2007b, Section 1). One might argue that because any strategy, pure or mixed, is a best response to equilibrium beliefs, deviations do not violate the theory. But systematic deviations from equilibrium choice frequencies must (with high probability) have a cause that is partly common across players. They are therefore symptomatic of systematic deviations from equilibrium probabilities.

salient locations can gracefully explain RT's results.[64] Assume that *L0* hiders and seekers both choose A, B, A, A with probabilities $p/2$, q, $1-p-q$, $p/2$ respectively, with $p > ½$ and $q > ¼$, so that *L0* favors both the end locations and the B location equally for hiders and seekers. Then for behaviorally plausible type distributions (estimated 0% *L0*, 19% *L1*, 32% *L2*, 24% *L3*, 25% *L4*—almost hump-shaped), a level-*k* model explains the prevalence of *central A* for hiders and its even greater prevalence for seekers. Given *L0*'s attraction to salient locations, *L1* hiders choose *central A* to avoid *L0* seekers and *L1* seekers avoid *central A* searching for *L0* hiders (the data suggest that end locations are more salient than B). For similar reasons, *L2* hiders choose *central A* with probability between 0 and 1 (breaking payoff ties randomly) and *L2* seekers choose it with probability 1. *L3* hiders avoid *central A* and *L3* seekers choose it with probability between zero and one (breaking payoff ties randomly). *L4* hiders and seekers both avoid *central A*.[65] The role asymmetry in aggregate behavior follows naturally from the asymmetry of the game's payoff structure, via hiders' and seekers' asymmetric responses to *L0*'s *role-symmetric* choices.

Note that only a heterogeneous population with substantial frequencies of *L2* and *L3* as well as *L1* (estimated 0% *L0*, 19% *L1*, 32% *L2*, 24% *L3*, 25% *L4*) can reproduce the aggregate patterns in the data. Crawford and Iriberri estimate that the salience of an end location is greater than that of the *B* location ($p > 2q$). Given this, a 50-50 mix of *L1*s and *L2*s in both player roles would imply (their Table 2) 75% of hiders but only 50% of seekers choosing *central A*, in contrast to the 37% of hiders and 46% of seekers who did choose *central A*.

Crawford and Iriberri's analysis suggests that RT's subjects were quite strategic and in fact more than usually sophisticated (with many *L3*s and even some *L4*s, even though in most settings *L1*s and *L2*s are more common)—they just didn't follow equilibrium logic. Their analysis suggests that the Yushchenko quotation is not unusually sophisticated: it reflects the reasoning of an *L1* poisoner, or equivalently an *L2* investigator reasoning about an *L1* poisoner.[66]

9.2. *Portability*

Although prior intuitions about the likely hump shape and location of the type distribution impose some discipline in specifying a level-*k* model, the freedom to specify *L0* leaves room for

[64] Defining *L0* as uniform random would be unnatural, given that *L0* describes others' instinctive responses to the non-neutral framing of decisions. It would also make *Lk* coincide with *Equilibrium* for all $k > 0$.

[65] Even though there is a nonnegligible estimated frequency of *L4*s, they don't really matter here because they never choose *central A* (Table 2 above), hence they are not implicated in the major aggregate patterns. For the same reason, their frequency is not well identified in the estimation.

[66] In a more detailed analysis of Burchardi and Penczynski's (2010) data on these games, including their chat deliberations, Penczynski (2010) finds support for a level-*k* model, but one with role-asymmetric *L0* and type distribution.

doubts about overfitting and portability, the extent to which a model estimated from responses to one game can be extended to predict or explain responses to different games. Crawford and Iriberri (2007b) tested for overfitting, and found that the test also favored their level-*k* model. But here we focus on their test for portability, which has instructive general implications.

Crawford and Iriberri compared the ability of the leading alternative models, when estimated from RT's data, to "predict" subjects' initial responses to the two closest relatives of RT's games in the literature, Barry O'Neill's (1987) famous card-matching game, and Amnon Rapoport and Richard B. Boebel's (1992) closely related game. These games both raise the same strategic issues as RT's games, but with more complex patterns of wins and losses, different framing, and in the latter case five locations. Here we discuss only O'Neill's game, in which players simultaneously choose one of four cards: A, 2, 3, J. One player wins if there is a match on J or a mismatch on A, 2, or 3; otherwise the other wins. The game is thus like hide-and-seek, but with each player a hider for some locations and a seeker for others.

There is a uniquely natural way to adapt Crawford and Iriberri's *L0* specification from RT's hide-and-seek games to O'Neill's game: A and J, "face" cards and end locations, are more salient than 2 and 3, but either A or J could be more salient. Although this specification appears to add two degrees of freedom, because all that matters about *L0* is what it makes *L1*s do in each player role, effectively it adds only a single discrete choice between two alternative models.

This specification also illustrates an important point: Because *L0* is "only" a psychological issue, it is easy to gather evidence on it from different settings, and such evidence is more likely to yield consensus on a general definition of *L0* than if it were combined with strategic issues.

It may appear that the flexibility of the type frequencies gives level-*k* models considerable freedom to overfit the data, but empirically plausible frequencies often imply severe limits on what decision patterns a level-*k* model can generate. Discussions of O'Neill's data, for example by McKelvey and Palfrey (1995), have been dominated by an "Ace effect": Aggregated over all 105 rounds, row and column players played A with frequencies 22.0% and 22.6%, slightly but significantly above the equilibrium 20%. Yet no plausible level-*k* model can make a row player play A more than the equilibrium 20%.[67] Thus, despite the apparent flexibility, the level-*k* model's structure and the principles that guide the specification of *L0* imply a strong restriction.

[67] Crawford and Iriberri's (2007b, online appendix) Tables A3 and A4 show that, excluding *L0*s (which normally have 0 estimated frequencies) and restricting attention to row players (Player 1s), when A is more salient ($3j - a < 1$) only *L4* chooses A, and that with probability at most 1/3 (Table A3); and that when A is less salient ($3j - a > 1$) only *L3* chooses A, and that

Crawford and Iriberri did not have O'Neill's data before they carried out their portability test; but based on the success of the level-*k* model in explaining RT's results, they speculated that O'Neill's subjects' *initial* responses must not have had an Ace effect. In fact for initial responses there was no Ace effect: only a Joker effect, a full order of magnitude stronger, in which rows played J 56% of the time and columns played it 64% of the time.

Unlike the putative Ace effect, this Joker effect and the other observed frequencies *can* be gracefully explained by a level-*k* model with an *L0* that probabilistically favors the salient A and J cards, in the spirit of Crawford and Iriberri's analysis of RT's data.[68] By contrast, equilibrium or LQRE with payoff perturbations are well-defined for O'Neill's game, but they both fit significantly worse than Crawford and Iriberri's favored level-*k* model.

Importantly, the analysis traces the superior portability of the level-*k* model to the fact that *L0* is psychological rather than strategic, and that it is based on simple and universal intuition and evidence. If *L0* were strategic, it would interact with the strategic structure in new ways in each new game, and it would be a rare event when one could extrapolate a specification from one game to another. Thus, the definition of *L0* as an instinctive, nonstrategic response is more that a convenient cognitive categorization: it is important for portability.

10. *Mr. Schelling Goes to Chicago: Coordination via Payoff Asymmetries and Non-neutral Framing*

Perhaps the most famous examples of framing effects in economics are Schelling's (1960) classic "meeting in New York City" experiments. Crawford, Gneezy, and Yuval Rottenstreich (2008) randomly paired subjects to play games with commonly observed, non-neutral decision labels like Schelling's, but except for a game with the payoff symmetry of Schelling's games, they used payoff-asymmetric games like Battle of the Sexes.

In unpaid pilots run in Chicago, Crawford, Gneezy, and Rottenstreich used naturally occurring labels, pitting the world-famous Sears Tower versus the little-known AT&T Building across the street. The salience of Sears Tower makes it easy and, in principle, obvious for subjects to coordinate on the "both-Sears" equilibrium; and they almost all do this in the

with probability at most 1/3 (Table A4). This is logically possible, but in the first case it would require a population of 60% or more *L4*s, and in the second case it would require 60% or more *L3*s: in each case behaviorally extremely unlikely.

[68] Thus, although O'Neill speculated that the Ace effect in the time-aggregated data occurred because "…players were attracted by the powerful connotations of an Ace" the analysis suggests that it was an accidental by-product of how subjects learned.

symmetric version of the game. Since Schelling’s experiments with symmetric games, people have assumed that slight payoff asymmetry would not interfere with this. However, even with slight payoff asymmetry, the game poses a new strategic problem because both-Sears is one player’s favorite way to coordinate but not the other player’s. Just as in a society of men and women playing Battle of the Sexes, in which Ballet is more salient than Fights, there is a tension between the “label salience” of Sears and the “payoff-salience” of a player’s favorite way to coordinate: Payoff salience reinforces label salience in one player role (P2s) but opposes it for players in the other (P1s). This tension may lead players to respond asymmetrically, which in this game is bad for coordination.

As Crawford, Gneezy, and Rottenstreich suspected, although the Chicago Skyscrapers results replicated Schelling’s results in the symmetric version of the game, there was a substantial decline in coordination with even slight payoff asymmetry (Figure 6). To investigate the reasons for the decline, they conducted more formal, paid treatments using abstract decision labels, pitting X against Y, with X presumed (and shown) to be more salient than Y. Like the salience of Sears Tower, the salience of the X label makes coordinating on the “both-X” equilibrium the obvious thing to do; and subjects do coordinate on “both-X” in the symmetric version of the game. But payoff asymmetry again introduces a tension between the “label salience” of X and the “payoff-salience” of a player’s favorite way to coordinate, which reinforces label salience for P2s but opposes it for P1s. This tension again had a large and surprising effect (Figure 7).

Even tiny payoff asymmetries caused a large drop in the expected coordination rate, from 64% ($0.64 = 0.76\times0.76 + 0.24\times0.24$) in the symmetric game to 38%, 46%, and 47% in the asymmetric games. Perhaps more surprisingly (and unlike in the unpaid Chicago Skyscrapers treatment), the pattern of miscoordination reversed as asymmetric games progressed from small to large payoff differences: With slightly asymmetric payoffs, most subjects in both roles favored their partners’ payoff-salient decisions. But with moderate or large asymmetries, most subjects in both roles switched to favoring their own payoff-salient decisions.

There are two things to explain here: Why didn’t subjects in the asymmetric games ignore the payoff asymmetry, which cannot be used to break the symmetry as required for coordination, and use the salience of Sears Tower to coordinate? Why did the pattern of miscoordination reverse as the asymmetric games progressed from small to large payoff differences? Standard notions such as equilibrium plus noise and QRE ignore labeling, and so cannot help.

P1 (90% Sears)		P2 (90% Sears): Sears	P2 (90% Sears): AT&T
	Sears	100,100	0,0
	AT&T	0,0	100,100

Symmetric

P1 (61% Sears)		P2 (58% Sears): Sears	P2 (58% Sears): AT&T
	Sears	100,101	0,0
	AT&T	0,0	101,100

Slight Asymmetry

P1 (50% Sears)		P2 (47% Sears): Sears	P2 (47% Sears): AT&T
	Sears	100,110	0,0
	AT&T	0,0	110,100

Moderate Asymmetry

Figure 6. Chicago Skyscrapers

P1 (76% X)		P2 (76% X): X	P2 (76% X): Y
	X	5,5	0,0
	Y	0,0	5,5

Symmetric

P1 (78% X)		P2 (28% X): X	P2 (28% X): Y
	X	5,5.1	0,0
	Y	0,0	5.1,5

Slight Asymmetry

P1 (33% X)		P2 (61% X): X	P2 (61% X): Y
	X	5,6	0,0
	Y	0,0	6,5

Moderate Asymmetry

P1 (36% X)		P2 (60% X): X	P2 (60% X): Y
	X	5,10	0,0
	Y	0,0	10,5

Large Asymmetry

Figure 7. X-Y Treatments

A level-*k* model can gracefully explain the patterns in the data, but again it's important to have an *L0* that realistically describes people's beliefs about others' instinctive reactions to the tension between label- and payoff-salience that seems to drive the results. Assume that *L0* is the same in both player roles, and that it responds instinctively to both label and payoff salience; but with a "payoffs bias" that favors payoff over label salience, other things equal: In symmetric games *L0* chooses X with some probability greater than ½. And in any asymmetric game, (for simplicity only) whether or not label-salience opposes payoff-salience, *L0* chooses its payoff-salient decision with probability $p > ½$.[69]

Under these assumptions, *L1*'s and *L2*'s choices in roles P1 and P2 are determined by p, the extent of *L0*'s payoff bias. Except in symmetric games, even though *L0*'s choice probabilities are the same for P1s and P2s, they imply *L1* and *L2* choice probabilities that differ across player roles due to the asymmetric relationships between label and payoff salience for P1s and P2s.

Simple calculations show that a level-*k* model can track the reversal of the pattern of miscoordination between the slightly asymmetric game and those with moderate or large payoff asymmetries if (and only if) 0.505 (= 5.1/[5.1+5]) $< p <$ 0.545 (= 6/[6+5]), so that *L0* has only a modest payoff bias. If p falls into this range and the population frequency of *L1* is 0.7 and that of *L2* is 0.3, close to most previous estimates, the model's predicted choice frequencies differ from the observed frequencies by more than 10% only in the symmetric game, where the model somewhat overstates the homogeneity of CGR's subject pool because it predicts 100% play of X.

More work is needed in this area, particularly with regard to the precise specification of *L0* and the interaction between level-*k*/CH thinking and "team reasoning", in which players do what would be best for both or all, if both or all did it (Nicholas Bardsley, Judith Mehta, Chris Starmer and Robert Sugden 2010). The full explanation will plainly have a level-*k*/CH component, but more work is needed to see how it interacts with team reasoning.

[69] These assumptions are consistent with Crawford and Iriberri's (2007b) assumptions, because their games had no payoff-salience. However, there remain some unresolved issues about how to generalize these assumptions.

11. *Huarangdao and D-Day: Communication of Intentions in Outguessing Games*

> “Have you forgotten the tactic of ‘letting weak points look weak and strong points look strong’?”
>
> — General Kongming, in Luo Guanzhong’s historical novel, *Three Kingdoms*.

> “Don’t you know what the military texts say? ‘A show of force is best where you are weak. Where strong, feign weakness.’”
>
> — General Cao Cao, in *Three Kingdoms*.[70]

> "Lord, what fools these mortals be!"
>
> —Puck, in *A Midsummer Night’s Dream*, Act 3

In the Huarongdao story, set around 200 A.D., fleeing General Cao Cao, trying to avoid capture by pursuing General Kongming, chose between two escape routes, the easier Main Road and the awful Huarong Road (http://en.wikipedia.org/wiki/Battle_of_Red_Cliffs). Thus the game closely resembles *Far Pavilions* Escape (Section 4). But there is an added wrinkle: Before Cao Cao’s choice Kongming had an opportunity to send a message by lighting campfires along one of the roads. This message had an obvious literal meaning, but it was scarcely more costly to send a false message than a true one: The message was approximately cheap talk. Kongming, having sent his message before Cao Cao’s choice, then chose which road to wait in ambush on.

In the event Kongming lit his campfires along the Huarong Road and waited in ambush there, sending a deceptively truthful message. Cao Cao, misjudging the extent of Kongming’s deviousness, inverted the message, took the Huarong Road, and was caught.

Huarongdao closely resembles the organizing example in Crawford’s (2003) level-*k* analysis of deceptive preplay communication, Operation Fortitude South, the Allies’ attempt to deceive the Germans regarding where they planned to invade Europe on D-Day (6 June 1944; http://en.wikipedia.org/wiki/Operation_Fortitude; see also Ken Hendricks and R. Preston McAfee, 2006). As in Huarongdao, the Allies’ message was approximately cheap talk and the

[70] Evidently Cao Cao had bought a used, out-of-date edition! Thanks to Duozhe Li for the reference.

underlying game was an outguessing game with conflicting interests; but in this case made zero-sum to sharpen the point.[71] There were two possible attack or defense locations, Calais and Normandy. The greater ease of crossing to Calais is reflected in the payoff assumptions, which imply that attacking an undefended Calais is better for the Allies than attacking an undefended Normandy, hence better for the Allies if the Germans are equally likely to defend each place; and that defending an unattacked Normandy is worse for the Germans than defending an unattacked Calais, hence worse for the Germans if the Allies are equally likely to attack each place.

In the event the Allies faked preparations for invasion at Calais, sending a deceptively *deceptive* message. The Germans, misjudging the extent of the Allies' deviousness, defended Calais and left Normandy lightly defended; and the Allies then invaded Normandy.

In each case the key strategic issue is how the sender—Kongming or the Allies—should choose his message and how the receiver—Cao Cao or the Germans—should interpret it, knowing that the sender is thinking about the message from the same point of view.

Moreover, in each case essentially the same thing happened: In D-Day the message was literally deceptive but the Germans were fooled because they "believed" it—either because they were credulous or because they inverted the message one too many times. Kongming's message was literally truthful but Cao Cao was fooled because he inverted it. Although the sender's and receiver's message strategies and beliefs were different, the outcome in the underlying game was the same: The sender won, but in the less beneficial of the two possible ways. Why did the receiver allow himself to be fooled by a costless (hence easily faked) message from an *enemy*? And if the sender expected his message to fool the receiver, why didn't he reverse it and fool the receiver in the way that would allow him to win in the *more* beneficial way?

Equilibrium analysis does not help to explain these puzzles. In real interactions with preplay communication, a receiver's thinking often assigns a prominent role to the literal meanings of messages, without necessarily taking them at face value. Yet equilibrium analysis precludes a role for the literal meanings of cheap-talk messages.[72] Further, in real interactions the sender's message and action are part of a single, integrated strategy: He tries to anticipate which message

[71] The game differs from Huarongdao in the relation between payoffs and labeling, in that both Cao Cao and Kongming prefer the Main Road, holding the probability of being outguessed equal; while the Allies and the German have opposing preferences about where the invasion takes place, holding the probability of being outguessed equal. But the analysis will show that in a level-*k* analysis, as in a traditional analysis, this difference is inessential. In a traditional analysis any effect of labeling is ruled out by fiat. We have seen in Section 9 that labels may matter in a level-*k* analysis, but here their effect is overridden by the fact that in communication games, *L0* is anchored in truthfulness, and communication overrides the effects of labeling.

[72] But see Joseph Farrell (1993), whose notion of neologism-proofness sometimes allows literal meanings influence, but not here.

will fool the receiver, and his action may differ from the one he would have chosen with no opportunity to send a deceptive message. Yet with conflicting interests there is no equilibrium (refined or not) in which cheap talk conveys information or the receiver responds to the message. In such an equilibrium, if the receiver found it optimal to respond to the message, the response would help the receiver and therefore hurt the sender, who would then prefer to make his message uninformative (Crawford and Sobel, 1982). Thus communication can have no effect, and the underlying game must be played according to its unique mixed-strategy equilibrium.

As the quotations from Kongming and Cao Cao suggest, these puzzles can be resolved via a level-*k* analysis, as in Crawford (2003). In specifying *L0* for games with communication, a uniform random *L0* seems quite unnatural. For sender or receiver, the instinctive reaction to a message in a language one understands is surely to focus on its literal meaning, even if one ends up lying or not taking the message at face value. Accordingly, Crawford (2003, Table 1, but with the types renamed and renumbered here to conform more closely to later usage) assumed that players anchor their beliefs in truthful literal meanings, with an *L0* sender telling the truth and an *L0* receiver credulously believing whatever he is told.[73] Given this, iterating best responses as in other level-*k* models: an *L1* receiver believes what he is told; an *L1* sender lies; an *L2* receiver inverts what he is told; an *L2* sender lies; an *L3* receiver inverts; an *L3* sender tells the truth (anticipating an *L2* receiver's inversion); and so on. Thus it appears that Cao Cao was *L2*, while Kongming was *L3*. Similarly, it appears that the Allies were *L2*, while the Germans were *L1*, or perhaps (inverting one too many times) *L4*.[74]

If there is no omniscient narrator telling us how the players are thinking, we can create an outcome table as in our analyses of *Far Pavilions* Escape (Section 4) or Entry Magic (Section 6), and combine it with an estimate of the type distribution to generate a statistical prediction of the outcome. The model's implications then follow mechanically from estimates of the frequencies of sender types who tell the truth, or lie; and of receiver types who believe, or invert messages. In such settings, trivially, receivers sometimes misread senders' messages and are deceived.

[73] The literature has not converged on how types should be numbered, or on whether *L0* receivers should be defined as credulous or as uniform random—compare Ellingsen and Östling (2010)—but the issue is partly semantic because truthful *L0* senders imply credulous *L1* receivers. Here we take *L0* receivers to be credulous; and given this, we define *Lk* in either player role as the type that iterates best responses *k* times. Note that unlike in equilibrium cheap-talk analyses where the meaning of messages is determined endogeneously (Crawford and Sobel 1982), the definition of *L0* resolves that indeterminacy.

[74] As this last possibility illustrates, in a level-*k* model, unlike a CH model, it can be just as costly to be too clever as to be not clever enough, which we view as a realistic feature of level-*k* models.

It is more interesting and potentially more useful to ask what happens if some participants follow level-*k* decision rules, but others (like Costa-Gomes et al.'s 2001 and Costa-Gomes and Crawford's 2006 *Sophisticated* type) understand both game theory and how real people think about strategy better than any mechanical rule. Although *Sophisticated* subjects are rare in experiments, we presume they are more common in field settings. And despite the occurrence of deception in the analysis with only level-*k* types, it is far from clear that a *Sophisticated* sender can deceive a *Sophisticated* receiver in the presence of level-*k* types. Aside from shedding additional light on strategic communication, such an analysis might yield a deeper understanding of settings such as financial markets with some experienced participants, where the standard distinction between rational and "noise" traders seems oversimplified as a model of people's reactions to news (compare Section 3's Graham quotation).

We discuss this extension in the context of Crawford's (2003) analysis of strategic communication, but see also Ricardo Serrano-Padial's (2010) innovative analysis of the interaction between naïve and sophisticated traders in prediction and other speculative markets. We also continue to restrict attention to novel situations, so that experience can teach people to predict other people's general behavior patterns, but not their specific strategies.

Crawford (2003) assumed that with positive probabilities, each player role is filled either by one of the various possible kinds of level-*k* types, for which his generic term was *Mortal* types (following Puck); or by a *Sophisticated* type. As suggested by experimental evidence from other kinds of games, he assumed the frequencies of *L0* senders and receivers are zero. As also suggested by the evidence, higher-level *Mortal* types avoid fixed-point reasoning (recall footnote 7's quotation), and instead use step-by-step procedures, which normally determine unique pure strategies. *Sophisticated* types, by contrast, know everything about the game, including the distribution of *Mortal* types; and are capable of fixed-point reasoning.

The perfect Bayesian equilibria of the game between possibly *Mortal* or *Sophisticated* senders and receivers can be characterized as follows. Given *L0*, *Mortal* players' strategies are determined mechanically and independently of each other's and *Sophisticated* players' strategies. They can therefore be treated as exogenous, even though they affect others' payoffs. We can then plug in the distributions of *Mortal* senders' and receivers' strategies to obtain a "reduced game" between possible *Sophisticated* senders and receivers, taking *Mortals*' strategies as given.

Because *Sophisticated* players' payoffs are influenced by *Mortal* players' decisions, the reduced game is no longer zero-sum, its messages are no longer cheap talk, and it no longer has complete information. The sender's message, ostensibly about his intentions, is in fact read by a *Sophisticated* receiver as a signal of the sender's type. Thus, the possibility of *Mortal* players completely changes the character of the game between *Sophisticated* players, which is what gives the model the ability to explain the effectiveness of communication in a zero-sum game and the possibility of deception between *Sophisticated* players.

In the equilibrium, *Mortal* (non-*L0*) senders' simplified models of others always make them expect to fool receivers, which depending on the sender's type (via whether he believes his message will be believed or inverted) he thinks he can do either by lying (as the Allies did) or by telling the truth (as Kongming did). Accordingly, each *Mortal* sender type sends the message that it expects to maximize the gain from fooling receivers, and then chooses the corresponding strategy in the underlying game. For example, a *Mortal* Allied type sends the message it expects to make the Germans think it will attack Normandy, and then attacks Calais.

Given this, the equilibria of the reduced game are determined by the population frequencies of *Mortal* and *Sophisticated* senders and receivers. When *Sophisticated* senders and receivers are common—not the most plausible case—the reduced game has a mixed-strategy equilibrium whose outcome duplicates that of the game without communication. In this equilibrium, *Sophisticated* senders' and receivers' mixed strategies offset *Mortal* senders' and receivers' deviations from equilibrium, and so eliminate *Sophisticated* senders' gains from fooling *Mortal* receivers, so that *Sophisticated* and *Mortal* players in each role have equal expected payoffs.

By contrast, when *Sophisticated* senders and receivers are rare—the plausible case, judging by experimental evidence—the game has an essentially unique pure-strategy equilibrium. In this equilibrium, *Sophisticated* senders can predict *Sophisticated* receivers' strategies perfectly, and vice versa. Speaking for concreteness of D-Day, *Sophisticated* Germans always defend Calais because they know that *Mortal* Allied types, who predominate when *Sophisticated* Allies are rare, will all attack Calais. *Sophisticated* Allies, knowing that they cannot affect the behavior of *Sophisticated* Germans, send the message that fools the most frequent type of *Mortal* German (feinting at Calais or Normandy depending on whether more *Mortal* Germans believe than invert messages) and then attack Normandy. Thus the model explains why *Sophisticated* Germans might allow themselves to be "fooled" by a costless message from *Sophisticated* Allies: It is an unavoidable cost of exploiting the mistakes of *Mortal* Allies, who are more common.

Surprisingly, there never exists a pure-strategy equilibrium in which *Sophisticated* Allies feint at Normandy and then attack Calais. In such an equilibrium, any deviation from *Sophisticated* Allies' equilibrium message would lead *Sophisticated* Germans to infer that the Allies were *Mortal*, making it optimal for *Sophisticated* Germans to defend Calais and suboptimal for *Sophisticated* Allies to attack there. If, in the equilibrium, *Sophisticated* Allies feinted at Normandy and attacked Calais, then their message would fool only the most likely kind of *Mortal* German— in a pure-strategy equilibrium *Sophisticated* Germans can never be fooled, and a given message cannot fool both *Mortal* German believers and inverters—with expected payoff gain equal to the frequency of the most frequent *Mortal* German type times the payoff of attacking an undefended Normandy. But such *Sophisticated* Allies could reverse both their message and attack location, again fooling the most frequent *Mortal* German type, but now with expected payoff gain equal to the frequency of that type times the payoff of attacking an undefended Calais, which is higher than the payoff of attacking an undefended Normandy. This contradiction shows that in any pure-strategy equilibrium, *Sophisticated* Allies must feint at Calais and then attack Normandy.

Thus, in the pure-strategy equilibrium that exists when *Sophisticated* players are rare, the model explains why, in both of our examples, the sender won but in the less beneficial of the two possible ways. The sender's message and decision are part of a single, integrated strategy; and the decision to seek a win in the less beneficial way has much higher probability than if no communication was possible.

Nonetheless, *Sophisticated* players in either role do strictly better than their *Mortal* counterparts. Their payoff advantage comes from the ability to avoid being fooled and/or to choose which *Mortal* type(s) to fool. This suggests that in an adaptive analysis of the dynamics of the type distribution, the frequencies of *Sophisticated* types will grow until the population is in or near the region of mixed-strategy equilibria in which types' expected payoffs are equal. Thus, somewhat surprisingly, *Sophisticated* and *Mortal* players can coexist in long-run equilibrium.

12. *Alphonse and Gaston: Communication of Intentions in Coordination Games*

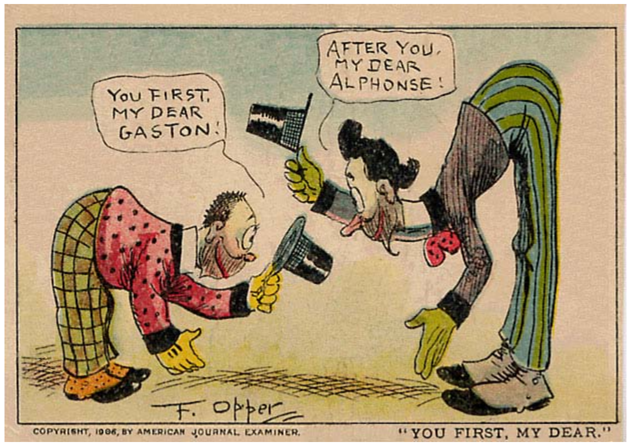

Figure 8. *Alphonse and Gaston*

—Frederick B. Opper's comic strip, *Alphonse and Gaston* (http://en.wikipedia.org/wiki/Alphonse_and_Gaston)

If level-*k* models allow preplay communication of intentions to affect the outcomes of zero-sum two-person games, it is no surprise that they also allow effective communication in coordination games. Here the stylized experimental facts (Crawford 1998) are that when coordination requires symmetry-breaking (Section 6), one-sided communication is more effective; that when coordination requires assurance (Section 7), two-sided communication is more effective; and that when coordination requires symmetry-breaking and communication is two-sided, more communication is better than less. In this section we consider level-*k*/CH explanations that have been proposed for these facts. In each case the power of the analysis stems from the use of a model that does not assume equilibrium, but which imposes a realistic structure less agnostic than rationalizability or *k*-rationalizability.

12.A. *Coordination via One Round of Communication*

Tore Ellingsen and Robert Östling (2010) adapt Crawford's (2003) level-*k* analysis to study the effectiveness of a single round of one- or two-sided preplay communication in games where communication of intentions plays various roles.

Here the central puzzle turns on Joseph Farrell and Matthew Rabin's (1996) distinction between messages that are self-committing in the sense that if the message convinces the receiver, it's a best response for the sender to do as he said he would do, and those that are self-signaling, in that they are sent when, and only when, the sender intends to behave accordingly. For example, in a two-person Stag Hunt game each player, without regard to his own intentions, does (weakly) better if his partner chooses high effort, so the message "I intend to play High Effort" is self-committing but not self-signaling. Robert J. Aumann (1990) argued on this basis that such messages are not credible. But Gary Charness (2000) and others have shown experimentally that messages that are self-committing but not self-signaling are quite effective in practice (but see Kenneth Clark, Stephen Kay, and Martin Sefton 2001). Theoretical explanations for this effectiveness have been elusive.

Ellingsen and Östling's (2010) analysis makes significant progress explaining this and other puzzles. Importantly, they depart from Crawford's (2003) analysis by assuming that *L0* receivers are uniform random rather than credulous and that all types have a preference for honesty when they are otherwise indifferent about which message to send. They show that in their model, one-sided communication solves the coordination problem in games like Battle of the Sexes where it requires symmetry-breaking, and is therefore more effective than two-sided communication, as is usually found in experiments. They also show that their model can explain why two-sided communication is more effective than one-sided communication in games where coordination requires assurance as it does in Stag Hunt, as is also found in experiments. More generally, they show that in common interest games when both players are *L2* or higher, either one- or two-way communication assures efficient coordination. But this tendency is far from universal: In some games players have incentives to misrepresent that erode coordination.

12.B. *Coordination via Multiple Rounds of Communication*

Crawford (2007) reconsiders Farrell's (1987) and Rabin's (1994) analyses of the effectiveness of one or more rounds of simultaneous, two-sided cheap-talk messages about players' intentions. Farrell's and Rabin's analyses assume equilibrium, sometimes weakened to rationalizability; and they further restrict attention to outcomes that satisfy plausible behavioral restrictions defining which combinations of messages create agreements, and whether and how agreements can be changed. Within this framework they address two conjectures regarding complete-information games: that preplay communication will yield an effective agreement to play an equilibrium in the underlying game; and that the agreed-upon equilibrium will be Pareto-efficient within that game's set of equilibria (henceforth "efficient"). They show that rationalizable preplay communication need not assure equilibrium; and that, although communication enhances coordination, even equilibrium with “abundant” (Rabin’s term for “unlimited”) communication does not assure that the outcome will be Pareto-efficient.

More specifically, Farrell (1987) uses Battle of the Sexes to study symmetry-breaking via one or more rounds of two-sided preplay communication with conflicting preferences about how to coordinate. He focuses on the symmetric mixed-strategy equilibrium in the entire game, including the communication phase, in which the first pair of messages in the same communication round that identify a pure-strategy equilibrium in Battle of the Sexes are treated as an agreement to play that equilibrium, ignoring all previous messages. He calculates the equilibrium rate of efficient coordination with one or more rounds of communication, showing that the rate increases steadily with the number of rounds but converges to a limit less than one even with abundant communication. Rabin (1994) extends Farrell’s analysis to a wide class of underlying games while dropping Farrell’s symmetry restriction; augmenting Farrell’s restrictions on how players use language to allow them to make interim agreements, which can be improved upon in subsequent agreements; and considering the implications of rationalizability as well as equilibrium. Rabin defines notions called *negotiated equilibrium* and *negotiated rationalizability* that combine the standard notions with his restrictions on how players use language. He shows that with abundant communication, each player’s negotiated equilibrium expected payoff is at least as high as in his worst efficient equilibrium in the underlying game. He then shows, replacing negotiated equilibrium by negotiated rationalizability, that even without equilibrium, each player expects (perhaps incorrectly) a payoff at least as high as in his worst efficient equilibrium.

Despite Farrell’s and Rabin’s partly negative conclusions, the conjectures that preplay communication will yield an agreement to play an equilibrium in the underlying game, and that the agreed-upon equilibrium will be efficient within the set of equilibrium are still widely held. Further, although equilibrium and rationalizability are natural places to start in analyses like theirs, it is also natural to test their robustness by replacing equilibrium and rationalizability with a structural nonequilibrium model based on level-*k* thinking—thus making the analysis less agnostic than rationalizability, while relaxing equilibrium in a way that has empirical support.

Crawford (2007) adapts his 2003 level-*k* analysis of strategic communication of private information to study the effectiveness of multiple rounds of simultaneous, two-sided cheap-talk messages about intentions, focusing on Farrell’s analysis of Battle of the Sexes. His analysis partly supports Farrell's and Rabin's assumptions about how players use language, but suggests that their "agreements" do not reflect a full meeting of the minds. Instead they reflect either one player's perceived credibility as a sender or the other's perceived credulity as a receiver, never both at the same time. As a result, a level-*k* analysis may not fully support the assumptions about agreements in Rabin's analysis of negotiated rationalizability.

A level-*k* analysis also yields very different conclusions about the effectiveness of one- or multi-round two-sided communication. A level-*k* analysis suggests that coordination rates in Battle of the Sexes will be largely independent of the difference in players' preferences, while in Farrell's equilibrium analysis coordination rates are highly sensitive to this difference. Further, with one round of communication, the level-*k* rate is well above the rate without communication, and is likely to be higher than the equilibrium rate with one round of communication unless preferences are very close. Finally, with abundant communication, the level-*k* coordination rate is likely to be higher than the equilibrium rate unless preferences are moderately close. The model's predictions with abundant communication are consistent with Rabin's bounds based on negotiated rationalizability, but their precision yields additional insight into the causes and consequences of breakdowns in negotiations.

13. *October Surprise:*
Communication of Private Information in Outguessing Games

> "…The news that day was the so-called 'October Surprise' broadcast by bin Laden. He hadn't shown himself in nearly a year, but now, four days before the [2004 presidential] election, his spectral presence echoed into every American home. It was a surprisingly complete statement by the al Qaeda leader about his motivations, his actions, and his view of the current American landscape. He praised Allah and, through most of the eighteen minutes, attacked Bush,… At the end, he managed to be dismissive of Kerry, but it was an afterthought in his 'anyone but Bush' treatise….
>
> Inside the CIA…the analysis moved on a different [than the presidential candidates' public] track. They had spent years, as had a similar bin Laden unit at FBI, parsing each expressed word of the al Qaeda leader…. What they'd learned over nearly a decade is that bin Laden speaks only for strategic reasons…. Today's conclusion: bin Laden's message was clearly designed to help the President's reelection."
>
> —Ron Suskind, *The One Percent Doctrine*, 2006, pp. 335-6 (quoted in Jazayerli 2008 http://www.fivethirtyeight.com/2008/10/guest-column-will-bin-laden-strike.html).

13.1. *October Surprise*

The situation described in the quotation can plausibly be modeled as a zero-sum two-person game of incomplete information between bin Laden and a representative American voter. The American knows that he wants whichever candidate bin Laden doesn't want, but only bin Laden knows which candidate he wants. Bin Laden has a one-sided opportunity to send a cheap talk message about what he wants and, talk being cheap, he will say whatever he believes is most likely to bring about his desired outcome. The key strategic issues are how bin Laden should relate his statement to what he really wants and how the American voter should interpret bin Laden's statement, knowing that bin Laden is choosing his message strategically.

Once again, the literal meanings of messages are likely to play a prominent role in applications, but equilibrium analysis precludes such a role. By the argument given in Section 11, there is again no equilibrium in which cheap talk conveys information, or in which the receiver responds to the sender's message.

However, Crawford's (2003) analysis is easily adapted to model the CIA's conclusion that bin Laden's verbal attack on George W. Bush was intended to aid Bush's reelection. Consider a level-*k* model in which *L0* is again anchored on truthfulness for the sender (bin Laden) and credulity for the receiver (American). An *L0* or *L1* American believes bin Laden's message, and therefore votes for whichever candidate bin Laden attacks. An *L0* bin Laden who wants Bush to win attacks Kerry, but an *L1* (*L2*) bin Laden who wants Bush to win believes that "reverse psychology" will be effective, and so attacks Bush to induce *L0* (*L1*) Americans to vote for him. Given bin Laden's attack on Bush, an *L0* or *L1* American ends up voting for Bush, and an *L2* American ends up voting for Kerry. A *Sophisticated* bin Laden, recognizing that he cannot fool *Sophisticated* Americans, would choose his message to fool the most prevalent kind of *Mortal* American—believer or inverter—as in Crawford (2003).

13.2. *Experiments*

Wang, Spezio, and Camerer (2010), building on the experiments of Hongbin Cai and Wang (2006), studied communication of private information via cheap talk in discretized versions of Crawford and Sobel's (1982) sender-receiver games (see also Toshiji Kawagoe and Hirokazu Takizawa 2009). In Wang, Spezio, and Camerer's design, the sender observes a state, S = 1, 2, 3, 4, or 5; and sends a message, M = 1, 2, 3, 4, or 5. The receiver then observes the message and chooses an action, A = 1, 2, 3, 4, or 5. The receiver's choice of A determines the welfare of both:

The receiver's ideal outcome is $A = S$ and his von Neumann-Morgenstern utility function is $110 - 20|S–A|^{1.4}$; and the sender's ideal outcome is $A = S + b$ and his von Neumann-Morgenstern utility function is $110 - 20|S+b–A|^{1.4}$. They varied the parameter representing the difference in preferences across treatments: $b = 0, 1$, or 2.

The key issue is how much information can be transmitted in a Bayesian equilibrium, and how the amount is influenced by the difference between the sender's and the receiver's preferences. Crawford and Sobel characterized the possible equilibrium relationships between sender's observed *S* and receiver's choice of *A*, which determines the informativeness of communication. They showed, for a class of models with continuous state and action spaces that generalizes Wang, Spezio, and Camerer's examples (except for their discreteness), that all equilibria are "partition equilibria", in which the sender partitions the set of states into contiguous groups and tells the receiver, in effect, only which group his observation lies in. Importantly, the receiver's beliefs on hearing the sender's message *M* are an unbiased—though noisy—estimate of *S*: In equilibrium there is no lying or deception, only intentional vagueness.

For any given difference in sender's and receiver's preferences (*b*), there is a range of equilibria, from a "babbling" equilibrium with one partition element to more informative equilibria that exist when *b* is small enough. Under reasonable assumptions there is a "most informative" equilibrium, which has the most partition elements and gives the receiver the highest ex ante (before the sender observes the state) expected payoff. As the preference difference decreases, the amount of information transmitted in the most informative equilibrium increases (measured by the correlation between *S* and *A*, or by the receiver's expected payoff).

Previous experiments, summarized by Crawford (1998), have confirmed the key comparative statics result that closer preferences allow more informative information transmission, while at the same time revealing systematic deviations from equilibrium. The puzzle is then, why does the comparative statics result hold even though equilibrium fails? A natural conjecture is that the comparative statics result holds for a wider class of nonequilibrium models, hence is robust to deviations from equilibrium; this is strongly confirmed by Wang, Spezio, and Camerer's results.

The unambiguous part of Crawford and Sobel's characterization of equilibrium concerns the possible relationships between *S* and *A*. Because messages are "cheap talk", with no direct effect on payoffs, there is nothing to tie down their meanings in equilibrium. As a result, any equilibrium relationship between S and A can be supported by any sufficiently rich language, with the meanings of messages determined by players' equilibrium beliefs.

Behaviorally, however, in experiments with a clear correspondence between state and message, as here, or where communication is in a common natural language, the interpretations of messages are dictated by their literal meanings. Thus messages are always understood—even if not always believed. Wang, Spezio, and Camerer's data analysis therefore fixes the meanings of senders' messages at their literal values. Even with this restriction, when the sender's and receiver's preferences are close enough ($b = 0$ or 1), there are multiple equilibria. Wang Spezio, and Camerer's analysis then focuses on the "most informative" equilibrium.

When $b = 0$, the most informative equilibrium has $M = S$ and $A = S$: perfect truth-telling, credulity, and information transmission, as is intuitively plausible when the sender and receiver have identical preferences. When $b = 2$, the most informative equilibrium has senders sending a completely uninformative message $M = \{1, 2, 3, 4, 5\}$ for any value of S; and receivers ignoring it, hence choosing $A = 3$, which is optimal given their prior beliefs, for any value of M. (A babbling equilibrium also exists when $b = 0$ or 1, but then it is not the most informative equilibrium.) When $b = 1$, the most informative equilibrium has senders sending $M = 1$ when $S = 1$ but $M = \{2, 3, 4, 5\}$ when $S = 2, 3, 4$, or 5; and receivers choosing $A = 1$ when $M = 1$ and $A = 3$ or 4 when $M = \{2, 3, 4, 5\}$.[75] In this case, the difference in preferences causes noisy information transmission even in the most informative equilibrium.

When $b = 0$ sender subjects almost always set $M = S$ and receivers almost always set $A = M$: The result is near the perfect information transmission predicted by the most informative equilibrium. As Wang , Spezio, and Camerer's Figures 1-3 show, as b increases to 1 or 2, the amount of information transmitted decreases as predicted by Crawford and Sobel's equilibrium comparative statics, but there are also systematic deviations from the most informative (or any) equilibrium, and lying and successful deception occur. Most senders exaggerate the truth, apparently trying to move receivers from receivers' ideal action toward senders' ideal action (or 5, whichever is smaller). Even so, there is some information in senders' messages, which are positively correlated with the state. Receivers are usually deceived to some extent.

[75] The sender's message $M = \{2, 3, 4, 5\}$ is the simplest way to implement the intentional vagueness of this partition equilibrium. Another way would be for the sender to randomize M uniformly on $\{2, 3, 4, 5\}$ when $S = 1$.When $b = 1$, there's another, more informative equilibrium, found by David Eil, in which Senders send $M = \{1, 2\}$ when $S = 1$ or 2 but $M = \{3, 4, 5\}$ when $S = 3$, 4, or 5; and Receivers choose $A = 2$ when M = $\{1, 2\}$ and $A = 4$ when $M = \{3, 4, 5\}$. But this equilibrium is not "robust", in that Senders who observe $S = 2$ are indifferent between $M = \{1, 2\}$ and $M = \{3, 4, 5\}$.

Wang, Spezio, and Camerer propose a level-*k* explanation of these results, based on Crawford's (2003) analysis (see also Navin Kartik, Marco Ottaviani, and Francesco Squintani 2007). They anchor beliefs in a truthful sender *L0*, which sets $M = S$; and a credulous receiver *L0* (which also best responds to an *L0* sender), setting $A = M$. *L1* senders best respond to *L0* receivers by inflating their messages by b: $M = S + b$ (up to $M = 5$), so that *L0* receivers will choose $S + b$, yielding the sender's ideal action given S. *L1* receivers (as they define them; the numbering is only a convention) best respond to *L1* senders (and not to *L0* senders as in Crawford 2003) by discounting the message, normally setting $A = M – b$, yielding receivers' ideal action given $M = S + b$ of S. *L2* senders best respond to *L1* receivers by inflating their messages by $2b$: $M = S + 2b$ (up to $M = 5$), so that *L1* receivers will set $A = M – b = S + b$, yielding senders' ideal action given S. *L2* receivers best respond to *L2* senders by discounting the message, normally setting $A = M – 2b$, yielding receivers' ideal action given $M = S + 2b$ of S. Econometric estimation classifies 18% of 16 Sender subjects as *L0*, 25% as *L1*, 25% as *L2*, 14% as *Sophisticated*, and 18% as *Equilibrium*, broadly consistent with earlier results.[76]

13.3 *Field Studies*

Ulrike Malmendier and Devin Shanthikumar (2007, 2009) discuss the interaction between stock analysts and traders. Analysts issue recommendations on individual stocks that range from "strong sell" and "sell" to "hold", "buy", and "strong buy"; and they also issue earnings forecast. In managing their portfolios, traders are presumed to use all the information available on the market, of which analysts' recommendations are a major source.

An analyst's recommendation or forecast is like a message in a sender-receiver game (Section 13.2). Particularly when an analyst is affiliated with the underwriter of a particular stock, he has an incentive to distort such messages. Malmendier and Shanthikumar (2007) find that analysts tend to bias their stock recommendations upward, the more so when they are affiliated with the underwriter of the stock. They also find two main patterns of responses to recommendations among receivers: Large investors tend to buy following "strong buy" recommendations, but not to sell following "hold" recommendations, thus discounting recommendations somewhat. Small traders, by contrast, are credulous enough to follow

[76] Wang, Spezio, and Camerer focus on sender subjects because they, but not receiver subjects, were eye-tracked. For comparison, Cai and Wang (2006) in a closely related design classified 6% of senders and 9% of receivers as *L0*, 25% of senders and 9% of receivers as *L1*, 31% of senders and 34% of receivers as *L2* or *Equilibrium*, and 13% of senders and 28% of receivers as *Sophisticated*. They also state that a logit agent QRE model fits their data well.

recommendations almost literally. Malmendier and Shanthikumar (2009) find somewhat different patterns of responses to earnings forecasts. Large investors tend to react strongly and in the direction suggested by forecast updates, without regard to whether the forecast came from an affiliated analyst. Small investors, by contrast, react insignificantly to the forecasts of unaffiliated analysts and significantly negatively to the forecasts of affiliated analysts.

Malmendier and Shanthikumar (2007, 2009) use these and other patterns in the data to distinguish between explanations of the bias in recommendations based on optimism-driven selection effects and those based on strategic distortion. They conclude that strategic distortion is the more important factor. Their analyses, which rest mainly on qualitatively patterns in the data, might be sharpened and refined by an explicit model of strategic distortion and its effects along the lines of a multidimensional generalization of the level-*k*/CH analyses in Sections 13.1-2.

14. Conclusion

This paper has reviewed recent theoretical, experimental, and empirical work on models of strategic thinking and surveyed their applications in economics. Better models of strategic thinking are important because they can improve predictions of people's responses to games played only once. And when a game is played repeatedly, in a setting where learning will plausibly converge to equilibrium, better models of initial responses can yield better predictions of the limiting outcomes of history-dependent learning.

Although Nash equilibrium can be, and often has been, viewed as a model of strategic thinking, experimental research shows with progressively increasing clarity that subjects' responses to novel games often deviate systematically from equilibrium, and that the deviations have a large structural component that can be modeled in a simple way. Subjects' thinking tends to avoid the fixed-point reasoning or indefinitely iterated dominance reasoning that equilibrium sometimes requires, in favor of rules of thumb that anchor beliefs in an instinctive reaction to the game and then adjust them via a small number of iterated best responses. The resulting level-*k* or cognitive hierarchy models share the generality and much of the tractability of equilibrium analysis, but can in many settings systematically out-predict equilibrium.

Although level-*k* or cognitive hierarchy models are alternatives to equilibrium analysis, they generalize equilibrium rather than replacing it. In sufficiently simple games the low-level types that describe most subjects' behavior mimic equilibrium strategy choices, even as they deviate from equilibrium thinking. But in more complex games some or all such types may deviate systematically from equilibrium choices, in which case the models identify which settings are likely to evoke deviations; what forms they are likely to take; and with what frequencies. These conclusions are based mainly on experimental analyses, but a growing number of empirical studies using field data from settings where the game can be identified find similar patterns.

The paper has also illustrated several ways in which a level-*k*/cognitive hierarchy analysis can help in applications. In settings where the types that best describe most people's behavior mimic equilibrium choices, such an analysis can establish the robustness of equilibrium predictions. In settings where it is implausible to assume equilibrium, a level-*k*/CH analysis can challenge equilibrium predictions and resolve empirical puzzles by explaining the deviations from equilibrium some games evoke. The paper illustrates these possibilities in applications ranging from zero-sum betting and auctions with private information, where a level-*k*/CH analysis explains systematic deviations from equilibrium predictions; to coordination via symmetry-breaking, where such an analysis can explain the results of experiments in which subjects do systematically better than in the best symmetric equilibrium; coordination via assurance, where such an analysis helps to resolve some more subtle puzzles; hide-and-seek, outguessing, and coordination games played on non-neutral salience landscapes, where the analysis explains systematic patterns in subjects' strategic uses of salience; and strategic communication in "outguessing" and coordination games, where equilibrium gives an inadequate account of communication but a level-*k*/CH analysis some commonly observed patterns.

We hope that this survey has shown that structural nonequilibrium models of strategic thinking deserve a place in the analyst's toolkit.

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