LOOK-UPS AS THE WINDOWS OF THE STRATEGIC SOUL: STUDYING COGNITION VIA INFORMATION SEARCH IN GAME EXPERIMENTS[1]

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1. Introduction

Because human decisions are the result of cognitive processes, theories of human behavior rest at least implicitly on assumptions about cognition. Neuroeconomics reflects the belief that using evidence on neural correlates of cognition will lead us to better theories of decisions.

Gul and Pesendorfer (2005, henceforth "GP") argue that, on the contrary, because economic theory was intended to explain only decisions it should only be tested by observing decisions. They view neuroeconomics as a radical departure from economics in part because neural data concern involuntary, unconscious processes. Such processes are not decisions, so "our" theories cannot be about them. Moreover, they argue, trying to extend our theories to explain neural data would require sacrificing important strengths of rational-choice analysis.

This paper attempts to narrow the gap between these views by discussing some recent experiments that elicit subjects' initial responses to games with the goal of identifying the structure of their strategic thinking—subjects' attempts to predict others' decisions by taking their incentives into account.[2] Strategic thinking can of course be studied in experiments that elicit decisions alone, via designs in which different models of cognition imply different decisions, as for example in Stahl and Wilson (1994, 1995) or "SW"; Nagel (1995); and Ho, Camerer, and Weigelt (1998) or "HCW". But the experiments I discuss study strategic thinking more directly, by monitoring and analyzing subjects' searches for hidden but freely accessible payoff information, as

[1] This paper is based on joint work with Miguel Costa-Gomes, University of York, and Bruno Broseta, Red de Institutos Tecnológicos de la Comunidad Valenciana, particularly on Costa-Gomes and Crawford (2006, 2007). I thank Miguel Costa-Gomes for many helpful discussions and comments. The experiments and analysis on which this paper is based were funded in part by the U.S. National Science Foundation under grant SES- 0100072 and the U.K. Economic & Social Research Council under grant R/000/22/3796.

[2] Why study strategic thinking when with enough experience in a stationary environment, even amoebas—or human reinforcement learners, who need not even know that they are playing a game—usually converge to equilibrium? Many applications of game theory involve situations with no clear precedents. (Do you sell airline stocks when the market re-opens after 9/11, or buy them on the anticipation that others will overreact? How will Microsoft respond when your start-up enters one of "its" markets?) Comparative statics and design questions inherently involve new games with new equilibria, which players cannot reach by copying behavior from analogous games. In such situations subjects' initial responses are often plainly "strategic" but nonetheless deviate from equilibrium. Even in settings in which players can be expected to converge to equilibrium, the structure of strategic thinking can influence the rate of convergence and equilibrium selection.

in Camerer, Johnson, Rymon, and Sen (1993) and Johnson, Camerer, Sen, and Rymon (2002), henceforth collectively "CJ"; Costa-Gomes, Crawford, and Broseta (2001), "CGCB"); and Costa-Gomes and Crawford (2006, 2007), "CGC". My discussion draws extensively on CGC (2007), which reports and analyzes the information search data from CGC (2006).

CJ's, CGCB's, and CGC's analyses of search rest on explicit models of cognition and therefore raise some of the same issues that GP raise about neuroeconomics. But the clarity of the insights into behavior they yield is an important "proof of concept" that shows how much can be gained by expanding the domain of analysis beyond decisions. Further, unlike neural correlates of cognition, search is a voluntary, conscious process. Rational-choice analysis can therefore be used to describe it, eliminating one source of resistance to studying cognition. Although the analysis of search data sidesteps some important issues raised by studying neural data, I hope considering these analyses will bring us closer to agreement on how, and whether, to do neuroeconomics.

These analyses suggest a concrete answer to GP's challenge, Why study cognition if our goal is only to understand and predict decisions? In CGC's decision data, for instance, most subjects deviate systematically from equilibrium. To the extent that their non-equilibrium decisions can be distinguished from randomness, which is considerable, they are almost entirely the decisions of rational, self-interested players who understand the game but base their beliefs on simplified models of others' decisions. In other words, subjects' deviations from equilibrium have mainly to do with how they think about others, not preferences or irrationality. (This conclusion is consistent with SW's, Nagel's, HCW's, and CGCB's results, but their evidence is less clear.)

One could still choose to use subjects' decisions alone to model their behavior via revealed preference, as in GP's proposal. But in games, given the lack of a rational-choice model of non-equilibrium beliefs, such an analysis would have to impose equilibrium. Such a reduced-form approach would not predict reliably well beyond sample: In different games, subjects' non-equilibrium models of others would yield different patterns of deviation from equilibrium; and only by coincidence would they be well described by equilibrium with subjects' previously inferred preferences. Thus, no empirically serious model of initial responses to games can ignore cognition. And as I will show, using a model of cognition to analyze search allows more precise identification of subjects' decision rules, sometimes even directly revealing the algorithms subjects use to process payoff information into decisions and distinguishing intended decisions from errors.

The rest of the paper is organized as follows. Section 2 begins by reviewing CJ's and CGCB's designs and results, with particular attention to design desiderata for studying cognition via search and the associated modeling issues. Section 3 reviews CGC's (2006) use of decision data to identify subjects' decision rules and the evidence that the main source of their deviations from equilibrium is cognitive, not preference-based. Section 4 introduces CGCB's and CGC's (2006, 2007) model of cognition, search, and decisions; discusses specification issues; and uses CGC's search data to illustrate the model's use in interpreting CGC's search data. Section 5 highlights questions raised by CGC's (2006) analysis of decisions that search analysis might answer, but seem likely to resist analysis via decisions alone. Section 6 outlines an explanation of the assumptions that underlie CGCB's and CGC's model of cognition and search, which views search strategies as rational decisions under plausible assumptions about the benefits and costs of search and constraints on working memory.[3] Section 7 is the conclusion. Throughout the paper I assume that subjects are rational, risk-neutral decision-makers; but when they seem important, as indicated below, I allow "social" preferences that reflect altruism, spite, fairness, or reciprocity.

2. Early Experiments that Studied Cognition in Games by Monitoring Information Search

In this section I review CJ's and CGCB's experimental designs and results. Their and CGC's experiments randomly and anonymously paired subjects to play series of different but related two-person games, with different partners each play and no feedback between plays. The goal was to suppress learning and repeated-game effects in order to elicit subjects' responses, game by game, to each as if played in isolation, and so to reveal strategic thinking as clearly as possible.[4]

The structure of the games was publicly announced except for hidden, varying payoff parameters, to which subjects were given free access, game by game, one at a time, before making their decisions.[5] With low search costs, free access made the entire structure effectively public knowledge, allowing the results to be used to test theories of behavior in complete-information

[3] The proposed explanation differs greatly from classical search theory in purpose, but only slightly in methods.

[4] "Eureka!" learning remains possible, but it can be tested for and seems to be rare. Initial responses yield insights into cognition that also help us think about how to model learning from experience, but that is another story.

[5] Subjects were not allowed to write, and the frequencies with which they looked up the parameters made clear that they did not memorize them. Subjects were taught the mechanics of looking up targets and limits and entering guesses, but not information-search strategies. Access was via a MouseLab interface that automatically recorded information searches along with decisions. MouseLab is an automated way to track search as in eye-movement studies of individual decisions (Payne, Bettman, and Johnson (1993), http://www.cebiz.org/mouselab.htm). Wang, Spezio, and Camerer (2006) illustrate the use of a modern, more powerful eye-tracking method.

versions of the games.[6] Varying the payoff parameters makes it impossible for subjects to remember the current game's parameters from previous plays, and so gives them incentives to search for the information their decision rules require. It also allows stronger separation of the decisions implied by equilibrium and leading alternative decision rules than in designs such as Nagel's or HCW's, in which subjects play the same game over and over again.

2a. Camerer, Johnson, Rymon, and Sen's alternating-offers bargaining experiments

CJ (1993, 2002) pioneered the use of search for hidden payoff parameters to study cognition in games, eliciting subjects' initial responses to series of three-period alternating-offers bargaining games.[7] Previous experiments yielded large, systematic deviations from the subgame-perfect equilibrium offer and acceptance decisions when players have pecuniary preferences, like those observed in ultimatum experiments. The deviations were attributed to cognitive limitations preventing subjects from doing the required backward induction, or believing that their partners would; to subjects having "social" preferences that modify their pecuniary payoffs; or both. Most researchers now agree that both factors are important, but in the early 1990s this was less clear.

CJ addressed the cognitive aspect of this question more directly by creating a design to study cognition via search and by deriving cognitive implications of alternative models of behavior and using them to analyze the search data. Within a publicly announced structure, they presented each bargaining game to subjects in extensive form as in Figure 1, as a sequence of three pies and the associated offer and acceptance decisions. Discounting was simulated by shrinking the pies over time, from roughly \$5.00 in round 1 to roughly \$2.50 in round 2 and \$1.25 in round 3; but the pies were varied slightly from game to game, to preserve subjects' incentives to search.

The pies were normally hidden in boxes as for rounds 2 and 3 in Figure 1, but subjects were allowed to look them up as often as desired, one at a time. In Figure 1 the subject has opened the box to look up the \$5.00 round-1 pie.[8] Subjects' knowledge of the structure of the games and their

[6] A partial exception is that CJ's experiments evoked nonpecuniary, "social" preferences, and these and subjects' risk aversion are uncontrolled and privately known. Privately known social preferences are easily accommodated in the analysis of CJ's results, and risk aversion was unlikely to have been significant.

[7] CJ's (1993) and CJ's (2002) designs differed in some ways, for example framing in losses versus in gains, that are not important for my purposes and are not discussed here. At roughly the same time in the early 1990s, CJ (2004) did a MouseLab study of forward induction in extensive-form games. Algaze [Croson] (1990) reported a very brief study of search for hidden payoff information in matrix games. Neither is discussed here.

[8] CJ used a "rollover" option in MouseLab, in which subjects could open the box that concealed a pie by moving the cursor into it, revealing the pie for as long as the cursor was in the box. Subjects could also use the interface to look up their roles in each round; but these were known and those look-ups were not reported or analyzed.

free access to the pies allowed them to evaluate their own and their partners' pecuniary payoffs for any combination of offer and acceptance decisions.

If free access to the pies induces public knowledge of pecuniary payoffs, and if it is also public knowledge that subjects maximize their own expected pecuniary payoffs, then the results can be used to test theories of behavior in complete-information versions of the game, which has a unique subgame-perfect equilibrium whose offer and acceptance decisions are easily computed by backward induction. Even if players have privately observed social preferences, the incomplete-information version of the game has a generically unique sequential equilibrium whose strategies are easily computed by backward induction. In each case, the subgame-perfect or sequential equilibrium initial offer depends on both the second- and third-round pies, so that the search requirements of equilibrium are mostly independent of preferences.[9] From now on I use "subgame-perfect equilibrium" to subsume pecuniary payoff-maximization.

In CJ's baseline treatment, in which subjects untrained except in the mechanics were rewarded according to their payoffs playing the games against each other, subjects' decisions were far from the subgame-perfect equilibrium, replicating the results of previous studies and suggesting that requiring subjects to look up the pies did not significantly affect their decisions.

CJ took the analysis a step further by using a model of cognition and search to analyze the search data. They first noted that 10% of their baseline subjects never looked at the third-round pie and 19% never looked at the second-round pie. Thus, even if those subjects' decisions conform to equilibrium (given some specification of preferences, with or without a social component), they cannot possibly be making equilibrium decisions for the reasons the theory assumes. In a non-magical world their compliance with equilibrium cannot be expected to persist beyond sample.

This observation motivates a basic general restriction on how cognition drives search, which anticipating CGCB's term for it I call "Occurrence": If a subject's decision rule depends on a piece of hidden payoff information, then that piece must appear in her/his look-up sequence. Occurrence, as a cognitive restriction, goes against GP's proposal, but it is still uncontroversial enough to be widely accepted by theorists. In this case at least, the epistemic foundations of equilibrium have implications for the interpretation of decisions it seems hard to justify ignoring.

[9] Only "mostly" because with only pecuniary preferences, the first-round pie, as long as it is large enough, does not affect the equilibrium initial offer. With social preferences the first-round pie may be relevant because it may influence the responder's acceptance decision.

If Occurrence were the whole story, there would be little to gain from studying cognition via search. Because CJ's subjects who never looked at the second- or third-round pies tended to make decisions far from subgame-perfect equilibrium, there is little risk of misinterpreting them; even so, Occurrence helps by ruling out explanations in which subjects' decisions are in sequential equilibrium for extreme distributions of social preferences. Inferences based on Occurrence are sometimes useful in CGCB's and, as we will see, CGC's analyses as well, but the full power of monitoring search depends on analyzing the order, and perhaps the duration, of subjects' look-ups.

CJ's analysis of order and duration is based on the argument that in their design the backward induction that is the easiest way to compute sequential or subgame-perfect equilibrium decisions has a characteristic search pattern, in which subjects first look up the third-round pie, then the second-round pie (possibly re-checking the third), and so on, with most transitions from adjacent later- to earlier-round pies. Their argument rests on the empirical generalization that most subjects use the interface as a computational aid, making the comparisons or other operations on which their decisions are based via adjacent look-ups and relying on repeated look-ups rather than memory. This observation motivates another basic restriction, which again anticipating CGCB's term I call "Adjacency": The hidden parameters associated with the simplest of the operations on which a subject's decision rule depends will appear as adjacent look-ups in his look-up sequence.[10]

Adjacency, unlike Occurrence, requires assumptions that not all theorists find compelling. It is theoretically possible for a subject to scan the pies in any order, memorize them, and then "go into his brain" to figure out what to do, in which case the order and duration of his look-ups will reveal nothing about cognition. (Here, brain imaging has a potential advantage over monitoring search because involuntary correlates of such a subject's thinking may still be observable.)

Fortunately, subjects' searches in designs like CJ's, CGCB's, and CGC's exhibit strong regularities that make Adjacency a reasonable working hypothesis. When challenged, CJ defended their Adjacency-based characterization of backward-induction search by running a "robot" treatment with the same games as their baseline, in which subjects were told that they were playing against a computer that simulated a rational, self-interested player. This was followed after four periods by a "robot/trained subjects" treatment in which the same subjects received training in

[10] This informal definition, like the one for Occurrence, is intentionally vague regarding how often look-ups or operations appear to accommodate variations in CJ's, CGCB's and CGC's use of Occurrence and Adjacency. The notions are made more precise in CGCB's analysis and, as explained below, CGC's. Note that both are general restrictions on how cognition drives search, which can be applied for a variety of games and decision rules.

backward induction (but not search) and continued to play against robots as before. The latter subjects' search patterns were close to the backward-induction pattern (CJ 2002, Figure 6). Although the shift in search was small prior to training, these results provide support for CJ's characterization, Adjacency, and of course Occurrence. As illustrated below, further (and sometimes stronger) support for Adjacency is provided by CGCB's trained subjects, CGC's robot/trained subjects with high compliance with their assigned decision rule's guesses (Table 2), and CGC's baseline subjects with high compliance with their apparent rule's guesses (Table 3).

CJ's robot subjects' offer and acceptance decisions were shifted away from the baseline patterns toward subgame-perfect equilibrium, but were still far from it. Their robot/trained subjects' decisions were approximately in subgame-perfect equilibrium (CJ (2002, Table II)). These shifts can be attributed to the robot treatment's "turning off" social preferences, assuming subjects don't think of experimenters or their funding agencies as "people"; the robot treatment's eliminating strategic uncertainty; and/or cognition. CJ suggest that the deviations from equilibrium in the baseline are due to a combination of social preferences and cognition, with both important.

Returning to cognition and search, CJ's baseline subjects' searches were almost the opposite of the searches of robot/trained subjects and CJ's characterization of backward induction search: baseline subjects spent 60-75% of the time looking up the first-round pie and only 20-30% looking up the second-round pie and 5-10% looking up the third-round pie, with most transitions forward, from earlier to later rounds. Importantly, subjects who looked up the second- and third-round pies more often, or had more backward transitions, also had a weak tendency to make, or accept, offers closer to the subgame-perfect equilibrium (CJ (2002, Figures 4-5)). Thus, CJ's baseline subjects' deviations from backward induction search were correlated with their deviations from subgame-perfect equilibrium decisions, in the direction that an epistemic, procedural view of subjects' decision-making would suggest. Although the correlation is weak, this result is an exciting first indication that subjects' search patterns might reveal something about their strategic thinking.

2b. Costa-Gomes, Crawford, and Broseta's matrix-game experiments

CGCB adapted CJ's methods, building on SW's (1994, 1995) designs, to study cognition via search in a series of 18 2×2, 2×3, or 2×4 matrix games with unique pure-strategy equilibria, some of which can be identified by iterated dominance and some without pure-strategy dominance. The games were designed to turn off social preferences, and CGCB's results show little evidence of them. I therefore assume that subjects maximized their own expected pecuniary payoffs.

Within a publicly announced structure, CGCB presented each game to subjects as a matrix with players' payoffs spatially separated to ease cognition and clarify inferences from search. The payoffs were hidden but subjects were allowed to look them up as often as desired. Instead of the rollover option CJ used, CGCB used a MouseLab "click" option, in which subjects could open a box by moving the cursor into it and left-clicking the mouse.[11] In the 2×2 game in Figure 2 the subject, framed as the row player, has opened the box with his own payoff, 42, when he chooses decision # and his partner chooses @. If free access induces public knowledge of the payoffs and it is public knowledge that subjects maximize them, then the structure is public knowledge and the results can be used to test theories of behavior in complete-information versions of the games.

Although there are close connections between epistemic analyses of equilibrium decisions in extensive- and normal-form games, their cognitive foundations are very different. The different presentation of payoff information in CGCB's matrix games allow them to explore aspects of strategic thinking that do not come into play in CJ's bargaining games. Moreover, although CGCB's games have small strategy spaces, their sequence of 18 games creates a large space of possible decision histories, which allows their design to separate the implications of leading normal-form theories of decisions more strongly than in previous designs in which subjects play series of different matrix games with small strategy spaces as in SW (1994, 1995), or in which they repeatedly play the same normal-form game with large strategy spaces as in Nagel and HCW.

Finally, and most importantly here, the 8-16 hidden payoffs in CGCB's design create a large space of possible information searches, which allows the design to separate leading theories' implications for search as well as decisions. In CJ's design, a subject's searches can vary in only one important dimension: backward or forward in the pies. Measuring a subject's searches in this dimension can convey a limited amount of information about his strategic thinking—though as we have seen, this information can be quite revealing. In CGCB's games, by contrast, a subject's searches can vary in three important dimensions: up-down (or not) in his own payoffs, left-right (or not) in his partner's payoffs, and the frequency of transitions from his own to his partner's payoffs. With the subject framed as the row player in Figure 2, it is clear that, assuming Adjacency, the first of these dimensions is naturally associated with decision-theoretic rationality, the second with using others' incentives to predict their decisions, and the third with interpersonal payoff comparisons. It would be difficult to imagine an empirically successful theory of initial

[11] Before he could continue, a subject had to close the box by right-clicking, which could be done from anywhere.

responses to this kind of game in which those three traits were not independently variable and important. Only a design with a search space as rich as CGCB's can separate the implications of alternative theories for both search and decisions strongly enough to identify their relationships.

In addition to a baseline treatment that paired subjects to play the 18 games with other subjects, CGCB conducted a trained subjects treatment, identical to the baseline except that each subject was trained and rewarded for identifying equilibrium decisions. This treatment confirms that subjects trained and motivated to find equilibrium guesses could do so; and provides data on *Equilibrium* search behavior that are helpful in evaluating the model of cognition and search.

CGCB's games have unique equilibria that are easily identified by direct checking, best-response dynamics (which always converges in their games), or (in most of their games) iterated pure-strategy dominance. Yet, as in previous studies of initial responses to matrix games, CGCB found systematic patterns of deviation from equilibrium, with high equilibrium compliance in games solvable by one or two rounds of iterated dominance but much lower compliance in games solvable by three rounds or the circular logic of equilibrium without dominance (CGCB, Table II). These patterns are not well explained by noisy generalizations of equilibrium such as McKelvey and Palfrey's (1995) quantal response equilibrium. CGCB explained them via a structural non-equilibrium model of initial responses in the spirit of SW's, Nagel's, and HCW's models, in which each subject's decisions are determined by one of a small set of decision rules or *types*, which determines his decisions, with error, in each game. The types are general principles of strategic decision-making, selected for behavioral plausibility and theoretical interest.

The leading types in CGCB's analysis include *L1* (for Level 1, first named by SW), called *Naive* in CGCB and *L1* here from now on, which best responds to a uniform random *L0* "anchoring type"; *L2*, which best responds to *L1*; *Equilibrium*, which makes its equilibrium decision; *D1* (Dominance 1), which does one round of deletion of dominated decisions and then best responds to a uniform prior over the other's remaining decisions; *D2*, which does two rounds of iterated deletion and then best responds to a uniform prior over the other's remaining decisions; and *Sophisticated*, which best responds to the probabilities of other's decisions, as estimated from subjects' observed frequencies, included to test whether subjects have prior understanding of others' decisions that transcends simple rules.[12] Because CGCB gave first priority to separating strategic from nonstrategic types, *L1*'s decisions were perfectly confounded with those of a maximax type CGCB called *Optimistic*. CGCB's econometric analysis of decisions alone estimated high frequencies of *L1*, *L2*, and *D1*. Because those types mimic equilibrium in simple games but deviate systematically in more complex games, this estimated type distribution allows the model to explain the aggregate relationship between complexity and equilibrium compliance.

Turning to CGCB's analysis of search, the main difficulty was imposing enough structure on the enormous spaces of possible decision and search histories to describe subjects' behavior in a comprehensible way. Although CJ identified a correlation (and a "right" direction for it) between subjects' decision and search deviations from subgame-perfect equilibrium in their alternating-offers bargaining games, their analysis does not show how to define or identify such a relationship in the higher-dimensional spaces of possible decisions and searches created by CGCB's design.

CGCB addressed this issue by using the types as models of cognition and search as well as decisions. They took an explicitly procedural view of decision-making, in which a subject's type and the associated cognitive process determine his search, and his type and search then determine his decision, game by game.[13] They characterized the link between cognition and search via the Occurrence and Adjacency restrictions described above, which generalize the ideas behind CJ's characterization of backward-induction search to a much wider class of games, patterns of hidden payoff information, and types. With these restrictions on cognition and search, the types provide a kind of basis for the spaces of possible decision and search histories, imposing enough structure to make it meaningful to ask whether subjects' decisions and searches are related in a coherent way.

Incorporating search into the econometric analysis yields a somewhat different view of subjects' deviations from equilibrium than previous analyses of decisions. It shifts CGCB's estimated type distribution toward *L1* at the expense of *Optimistic* and *D1*, leaving *L1* and *L2* as the only empirically important types. Part of this shift occurs because *L1*'s searches, unlike *L1*'s decisions, are clearly separated from *Optimistic*'s, and *L1*'s search implications explain more of

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[12] *Lk*'s and *Dk*-1's decisions both survive *k* rounds of iterated elimination of dominated decisions and so in two-person games are *k*-rationalizable (Bernheim (1984)). Although *Dk*-1 types are closer to how theorists analyze games, *Lk* types seem more natural and predominate in applications.

[13] Because a type's search implications depend not only on what decisions it specifies, but why, something like a types-based model seems necessary here. In CJ (1993) types are implicit in the discussion and limited to two, which might be called "subgame-perfect equilibrium" and "other". CJ (2002) adapted CGCB's analysis by defining extensive-form "types" modeled after CGCB's and SW's normal-form types, using them to construct a more structured data analysis than CJ's (1993).

the variation in subjects' searches and decisions than *Optimistic*'s, which are too unrestrictive to be useful. Another part of the shift occurs because *L1*'s search implications explain more of the variation in subjects' searches and decisions than *D1*'s, which are much more restrictive than *Optimistic*'s but too weakly correlated with subjects' observed decisions. *D1* loses frequency to *L2* as well, even though their decisions are only weakly separated in CGCB's design, because *L2*'s search implications explain more of the variation in subjects' searches and decisions.

Overall, CGCB's analysis of decisions and search gives a strikingly simple view of behavior, with *L1* and *L2* making up 90% of the population. This type distribution and the clear relationships between subjects' cognition as revealed by search and their decisions support my claim that their deviations from equilibrium in these games are due mainly to how they think about others.

3. Costa-Gomes and Crawford's Two-Person Guessing Game Experiments

CGC (2006, 2007) adapted CGCB's methods to elicit subjects' initial responses to a series of 16 dominance-solvable two-person guessing games, cousins of Nagel's and HCW's *n*-person guessing games. In this section I review CGC's design and their results for decisions, which provide even stronger evidence that the deviations from equilibrium in initial responses to games are due mainly to strategic thinking. In Section 4 I review CGC's analysis of cognition and search.

3a. CGC's design

In CGC's games, newly designed for the purpose of studying cognition via decisions and search, two players make simultaneous guesses. Each player has his own lower and upper limit, both strictly positive, as in some of HCW's games, to ensure finite dominance-solvability. Unlike in previous designs, however, players are not required to guess between their limits: To enhance the separation of types via search, guesses outside the limits are automatically adjusted up to the lower limit or down to the upper limit as necessary. Thus, the only thing about a guess that affects the outcome is the adjusted guess it leads to. Each player also has his own target, and (unlike in Nagel's and HCW's "winner-take-all" games) his payoff is higher, the closer his adjusted guess is to his target times his partner's adjusted guess. In the most important departure from previous guessing designs, the targets and limits vary independently across players and games, with the targets either both less than one, both greater than one, or (unlike in previous designs) mixed.[14] The resulting games are asymmetric and dominance-solvable in 3 to 52 rounds, with essentially

[14] In previous designs the targets and limits were the same for both players, and varied only across treatments.

unique equilibria determined (but not always directly) by players' lower limits when the product of the targets is less than one or their upper limits when the product is greater than one.

From the point of view of studying decisions, CGC's design combines the main strength of SW's and CGCB's designs, with subjects playing sequences of different but related games, and the main strength of Nagel's and HCW's designs, games with very large strategy spaces. This combination greatly enhances the separation of equilibrium and other leading types' decisions.

CGC's games explore different aspects of strategic thinking than CJ's, CGCB's, or Nagel's and HCW's games. Of particular note is the subtle way in which the location of the equilibrium is determined by the product of players' targets, which adds greatly to the power of the design to distinguish equilibrium from boundedly rational strategic thinking. The only important difference between some of CGC's games is whether the product of targets is slightly greater or slightly less than one. *Equilibrium* responds very strongly to this difference, but low-level *Lk* or *Dk* types, whose guesses vary continuously with the targets, respond much less. Also noteworthy is the strong separation of *Lk*'s and *Dk*-1's decisions, which are perfectly confounded in most of Nagel's and HCW's treatments and only weakly separated in their other treatments and in CGCB's design.

In addition to a baseline treatment that paired subjects to play the 16 games with other subjects, CGC conducted six different robot/trained subjects treatments, identical to the baseline except that each subject was trained and rewarded as a type: *L1*, *L2*, *L3*, *D1*, *D2*, or *Equilibrium*. These treatments assess the types' cognitive demands, confirming for instance that subjects trained and motivated to make equilibrium guesses could do so; and provide data on the search behavior of subjects of known types that are helpful in evaluating the model of cognition and search.

In all treatments, within a publicly announced structure, CGC presented each game to subjects as an array of targets and limits, with these payoff parameters hidden but subjects allowed to look them up as often as desired, one at a time, using MouseLab's click option as in CGCB. In Figure 3 the subject has opened the box to look up his own ("Your") lower limit, 100.

3b. CGC's analysis of decisions

The strong separation of types' implications for guesses (CGC 2006, Figure 5) and the clarity of CGC's baseline subjects' responses allow many of their types to be confidently identified from guesses alone. Of 88 subjects, 43 have clear "fingerprints" in that they made guesses that complied *exactly* (within 0.5) with one type's guesses in 7-16 of the games (20 *L1*, 12 *L2*, 3 *L3*, and 8

Equilibrium).[15] Figure 4 (CGC (2006, Figure 2)) shows the fingerprints of the 12 whose apparent types were *L2*. Of their 192 (= 12×16) guesses, 138 (72%) were exact, which means they tracked the complex pattern of the games' *L2* guesses with a remarkable degree of accuracy. I stress that these baseline subjects, unlike the robot/trained subjects, were taught nothing about strategic thinking: The models of others' guesses implicit in their apparent types were self-generated.

Given how strongly CGC's design separates types' guesses, and that guesses could take 200-800 different rounded values, these 43 subjects' compliance is far higher than could occur by chance. Further, because the types specify precise, well-separated guess sequences in a very large space of possibilities, their compliance rules out alternative interpretations of their guesses.[16] In particular, because the types build in risk-neutral, self-interested rationality and perfect models of the game, the deviations from equilibrium of the 35 whose apparent types are *L1*, *L2*, or *L3* can be attributed to non-equilibrium beliefs, not irrationality, risk aversion, altruism, spite, or confusion.

CGC's other 45 subjects' types are less apparent from their guesses; but *L1*, *L2*, and hybrids of *L3* and/or *Equilibrium* are still the only types that show up in econometric estimates.[17] The fact that most subjects follow low-level *Lk* types, which mimic equilibrium in games that are dominance-solvable in small numbers of rounds but deviate systematically in some more complex games, also explains the inverse relationship between strategic complexity and equilibrium compliance observed in CGCB and previous experiments (CGCB, Table II).

CGC's results for decisions provide very strong evidence that subjects' deviations from equilibrium in initial responses to games are due mainly to non-equilibrium strategic thinking, not preferences or irrationality. As noted in the Introduction, one could still use subjects' guesses alone

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[15] 11 of these subjects were from an "open boxes" treatment, not discussed here, identical to the baseline but with the parameters continually visible. The results of this treatment (and analogous treatments in CJ and CGCB) confirm that making subjects look up the parameters does not significantly affect their decisions, so that the data can be pooled with baseline decision data, as here. CGC's open boxes subjects have numbers that begin with a 5.

[16] By contrast, in SW's or CGCB's matrix-game designs, even a perfect fit does not distinguish a subject's best-fitting type from nearby omitted types; and in Nagel's and HCW's guessing-game designs, with large strategy spaces but with each subject playing only one game repeatedly, the ambiguity is worse.

[17] Nagel's results are often viewed as evidence that subjects perform finitely iterated dominance, as in *Dk*-1. However, *Lk*'s and *Dk*-1's decisions are perfectly confounded in Nagel's main treatments and weakly separated in Nagel's and HCW's other treatments and in CGCB's design. CGC's clear separation of *Lk* from *Dk*-1 allows them to conclude that *Dk* types don't exist in significant numbers, at least in this setting, and thus that subjects respect low levels of iterated dominance as a by-product of following *Lk* types, not because they explicitly perform it. *Sophisticated*, which is clearly separated from *Equilbrium* here, also doesn't exist in significant numbers. CGC's (2006, Section II.D) specification test rules out significant numbers of other types, omitted from the specification.

to model their behavior via revealed preference, but such a model would misattribute the cause of the deviations, and so would predict well beyond sample only by coincidence.

4. Costa-Gomes and Crawford's Analysis of Cognition and Search

CGC's (2006, Section II.E; 2007) model of cognition and search refines CGCB's model, adapting their Occurrence and Adjacency restrictions to give a tractable characterization of each type's search requirements. With regard to search, CGC's design combines the strengths of CJ's presentation of games as functions of a small number of hidden parameters within an intuitive common structure, which allows subjects to focus on predicting others' responses without getting lost in the details of the structure; and CGCB's high-dimensional search spaces, which make search more informative and allow greater separation via search. CGC's design strongly and independently separates the implications of leading types for search and decisions, which makes it easier to identify relationships between them and multiplies the power of the design. Finally, it makes each type's implications search independent of the game, which simplifies the analysis.[18]

This section begins with a discussion of the issues that arise in specifying a model of cognition and search. It then presents CGC's leading types' search requirements and illustrates how they are derived. Finally, it presents sample search data for some of CGC's robot/trained and baseline subjects. As these data will be used to show, CGC's design and characterization of types' search implications make it possible to read the algorithms a large minority of subjects used to choose their guesses directly from their search sequences. Other subjects' cognition is not apparent from their searches, but CGC's (2006) measures of their compliance with leading types' search implications have considerable discriminatory power in the econometric analysis, often allowing those subjects' types to be reliably estimated from searches alone, without regard to guesses.

4a. Specification issues

Studying cognition via search requires a model of how cognition determines subjects' look-up sequences. Previous papers have taken quite different positions on this issue. CJ's analysis gave roughly equal weight to look-up durations and total numbers of look-ups ("acquisitions") of each pie and to the numbers of transitions between look-ups of adjacent pies. Rubinstein's (2005) analysis considered only durations. Gabaix et al.'s (2006) focused on total numbers of look-ups

[18] By contrast, the lack of a simple common structure in CGCB's design makes rules' search implications vary from game to game in ways so complex you need a "codebook" to identify them.

rather than durations, but also considered some aspects of the order of look-ups. CGCB's and CGC's analyses focused instead on which look-ups subjects make, in the sense of Occurrence, and on the order of look-ups in the sense of Adjacency, relegating durations to a secondary role.[19]

On another dimension, CJ's and Rubinstein's analyses and most of Gabaix et al.'s aggregated search data across subjects and over time, while CGCB and CGC took the position that cognition and search are so heterogeneous that it is essential to study them at the individual level.

CGCB's and CGC's focus on Occurrence and Adjacency follows naturally from a procedural view of decision-making and the empirical tendency, now confirmed by a large body of MouseLab data, of most subjects to perform the operations on hidden parameters on which their decisions are based via adjacent look-ups, relying on repeated look-ups rather than memory. In this view, duration is unimportant per se because the information content of a look-up is independent of its length as long as the length suffices for cognition; look-ups too short for comprehension, less than 0.18 seconds, were filtered out in all the analyses discussed here. Although duration might still be correlated with time spent thinking about a particular parameter, which might be important in a more refined model of cognition, search, and decisions, a procedural view does not suggest such a correlation, and CGCB's and CGC's subjects sometimes left boxes open for long periods while staring out the window, etc., which would weaken any such correlations.[20] Total numbers of look-ups are important, but are captured indirectly through CGC's notion of search compliance.

4b. CGC's model of cognition and search

CGC's model of cognition and search, like CGCB's, is based on a procedural view of decision-making. Each leading type implies a generically unique, pure adjusted guess in each game, which maximizes its expected payoff given the beliefs regarding others' guesses implicit in the type. (The leading types all specify best responses to some beliefs.) Each type is thereby naturally associated with algorithms that process hidden payoff information into decisions, which CGC use as models of cognition. Given the need to go beyond Occurrence (Section 2a) and the lack of a well-developed theory of cognition and search, the goal was to add enough restrictions to extract the signal from subjects' look-up sequences but not so many that they distort its meaning. CGC derived types' minimal search implications under conservative assumptions, based on Occurrence and Adjacency, about how cognition determines search (CGC (2006, Section I.B)).

[19] CGCB and CGC made no claim that durations are irrelevant, just that durations don't deserve priority. CGCB (Table IV) present some results on durations under the heading of "gaze times".

[20] Spezio, Wang, and Camerer's (2006) eye-tracking methods have an advantage in avoiding this ambiguity.

The leading role in these derivations is played by a type's *ideal guesses*, those that would be optimal given the type's beliefs, ignoring its limits. Given the quasiconcavity of CGC's payoff functions, a subject can enter his ideal guess and know that his adjusted guess will be optimal without checking his own limits. Thus, a type's ideal guess not only determines its adjusted guess and the resulting outcome, it also determines the type's minimal search implications.

The left-hand side of Table 1 (CGC (2006, Table 4)) lists the formulas for the leading types' ideal guesses in CGC's games, which are easily derived as in CGC (2006, Section I.B), using CGC's notation for the limits and targets, a^i for the player's own lower limit, b^i for the player's own upper limit, and p^i for the player's own lower target, with analogous notation using superscript *j*'s for the player's partner's limits and target. The right-hand side of Table 1 lists the leading types' minimal search implications expressed as sequences of parameter look-ups, first in CGC's notation and then in terms of the associated box numbers (1 for a^i, 2 for p^i, 3 for b^i, 4 for a^j, 5 for p^j, 6 for b^j) in which MouseLab records subjects' look-up sequences in our design. Table 1 shows look-ups in the order that seems most natural; but that order is not required in the analysis.[21]

The search implications are derived as follows. Evaluating a formula for a type's ideal guess requires a series of arithmetic *operations*, some of which—the innermost operations, whose parameters are in square brackets in the right-hand side of Table 1, like $[a^j,b^j]$ for *L1*—are *basic* in that they logically precede other operations. Like CGCB, CGC assumed that subjects perform basic operations via adjacent look-ups, remembering their results, and otherwise relying on repeated look-ups rather than memory. Basic operations are then represented by adjacent look-ups that can appear in either order but cannot be separated by other look-ups. The look-ups of other operations can appear in any order and are conservatively allowed to be separated. In Table 1 such operations are represented by look-ups within curly brackets or parentheses.[22]

[21] In CGC's design, unlike in CGCB's, *Equilibrium*'s minimal search implications are simpler than any boundedly rational type's implications. This makes it harder to explain deviations from equilibrium by cognitive complexity. But we will see that high-compliance *Equilibrium* robot/trained subjects search more than high-compliance robot/trained subjects of other types, so CGC's *Equilibrium* search implications may not reflect its complexity.

[22] *L1*'s search implications illustrate an important advantage of the automatic adjustment feature of CGC's design. *L1*'s ideal guess depends on its own target but only its partner's limits, while *L2*'s and *D1*'s depend on both players' targets and limits and *Equilibrium*'s depends on both players' targets and a combination of its own and its partner's lower or upper limits. In other designs, such as CGCB's, *L1*'s decisions almost inevitably depend only on its own payoff parameters, and more sophisticated types' decisions depend on both own and other's parameters. Thus the automatic adjustment feature allows CGC to separate solipsism from the strategic naivete of *L1*. CGC's data give no evidence of solipsism, but a great deal of evidence of naivete. CGC's data also show that most subjects understood and relied upon automatic adjustment, which was carefully explained to them.

An *L1* player i, for instance, best responds to the belief that player j's guess is uniformly distributed between his limits. This yields a guess for j that is never adjusted, and that averages $[a^j+b^j]/2$. CGC (2006, Section I.B) shows via a certainty equivalence property of CGC's games (Observation 2) that *L1*'s ideal guess is $p^i[a^j+b^j]/2$, which will be automatically adjusted, if necessary, to $R(a^i,b^i;\, p^i[a^j+b^j]/2) \equiv \min\{b^i, \max\{a^i, p^i[a^j+b^j]/2\}\}$. The only basic operation is $[a^j+b^j]$. An *L1* player i therefore has minimal look-up sequence: $\{[a^j, b^j]$ (to compute j's average guess), p^i (to identify i's ideal guess)$] \equiv \{[4, 6], 2\}$, of which [4, 6] cannot be separated.

An *L2* player i best responds to the belief that player j is *L1*, taking the adjustment of j's guess into account. An *L1* player j's adjusted guess is $R(a^j,b^j;\, p^j[a^i+b^i]/2)$, so an *L2* player i's ideal guess is $p^iR(a^j,b^j;\, p^j[a^i+b^i]/2)$, which will be automatically adjusted to $R(a^i,b^i;\, p^iR(a^j,b^j;p^j[a^i+b^i]/2))$. An *L2* player i therefore has look-up sequence: $\{([a^i, b^i], p^j)$ (to predict j's *L1* ideal guess), a^j, b^j (to predict j's *L1* adjusted guess), p^i (to identify i's ideal guess)$] = \{([1, 3], 5), 4, 6, 2\}$.[23]

In CGC's (2006) econometric analysis of search, not discussed here, search compliance for a given subject, type, and game is measured by the density of the type's complete minimal search sequence in the subject's look-up sequence for the game, allowing for the heterogeneity of search behavior.[24] CGC's measure is a significant advance on CGCB's measure, which is based on the percentages of a type's Occurrence and Adjacency requirements satisfied by the entire sequence.

4c. Sample search data

Table 2 gives a sample of the information search data for CGC's robot/trained subjects and Table 3 gives an analogous sample for baseline subjects of various assigned or apparent types, with Table 1's search implications repeated for convenience. In each case the subjects were chosen for high exact compliance with their types' guesses, not for compliance with any theory of search; subjects' frequencies of exact guesses are in parentheses after their types. Only the orders of look-ups are shown; and only from the first two or three games, but these games are representative.

[23] With automatic adjustment, an *L2* player i does not need to know his own limits to play the game or think about the effects of his own guess being adjusted, only to predict j's *L1* guess. By contrast, an *L1* player i doesn't need to know his own limits, only j's. Because the possible values of the limits are not public knowledge, an *L2* player i *cannot* infer that adjustment of player j's ideal guess can occur only at his upper (lower) limit when $p^j > 1$ ($p^j < 1$). An *L2* subject who incorrectly infers this may omit $a^j = 4$ ($b^j = 6$) when $p^j > 1$ ($p^j < 1$).

[24] As is evident from Tables 2 and 3, subjects' look-up sequences vary widely in what CGC called "style": Most robot/trained and baseline subjects with high exact compliance consistently look first at their type's minimal search sequence and then continue looking, apparently randomly, or stop and enter their guess (for example *L2* robot/trained subject 910, *L3* subject 1008, and *D1* subject 1501 in Table 2; and *L2* baseline subjects 108 and 206 in Table 3). But some such subjects look randomly first and turn to the relevant sequence at the end (*L1* robot/trained subject 904). CGC's (2006 Section II.E) econometric analysis uses a binary nuisance parameter to distinguish these "early" and "late" styles and filter them out to obtain a better measure of search compliance.

Recalling that the theory allows any order of look-ups grouped within square or curly brackets or parentheses, the searches of high-guess-compliance robot/trained or baseline subjects conform closely to CGC's theory, with a subject's assigned or apparent type's minimal sequence unusually dense in his observed sequence.[25] The only exception is the *Equilibrium* subjects, who search far longer and in more complex patterns than CGC's theory suggests, perhaps because its minimal *Equilibrium* search requirements allow more luck than these subjects enjoyed.[26] Baseline *L1*, *L2*, and perhaps *L3* and *Equilibrium* subjects' searches are very close to those of their robot/trained counterparts, suggesting that (unlike in CJ) training had little effect on their search behavior.[27] Perhaps *Equilibrium* search in normal-form games is less unnatural than backward-induction search in CJ's extensive-form games. For the simpler types *L1*, *L2*, and perhaps *L3*, the algorithms subjects use to identify their types' guesses can be directly read from their searches.

CGC's (2006, Section II.E, Table 7) econometric analysis shows that such inferences are usually consistent with estimates based on guesses alone, and that search compliance as measured here is also useful in identifying the types of subjects whose types are not apparent from their searches. For some subjects, econometric estimates based on guesses and search together resolve tensions between guesses-only and search-only estimates in favor of a type other than the guesses-only estimate. Those estimates confirm the presence of significant numbers of subjects of types *L1*, *L2*, *Equilibrium*, or hybrids of *L3* and/or *Equilibrium* in the population, and the absence of significant numbers of subjects of other types. Once again, subjects' deviations from equilibrium can be attributed mostly to non-equilibrium strategic thinking, not preferences or irrationality.

For some subjects search is an important check on type inferences based on guesses. Baseline subject 309, whose 16 exact *L2* guesses seem overwhelming evidence that his type is *L2*, violated

[25] CGC's specification analysis turned up only one clear violation of their proposed characterization of types' search implications, which is instructive. Baseline subject 415 (not shown in Table 3), whose apparent type was *L1* with 9 exact guesses, had zero *L1* search compliance in 9 of the 16 games because he had no adjacent $[a^j,b^j]$ pairs. His look-up sequences, however, were rich in (a^j,p^i,b^j) and (b^j,p^i,a^j) triples, in those orders; but not in such triples with other superscripts. This strongly suggests that 415 was an *L1* who happened to be more comfortable with three numbers in working memory than CGC's characterization of search assumes, or than their other subjects were. But because this violated CGC's assumptions on search, this subject was "officially" estimated to be *D1*.

[26] One of the methods CGC allow for identifying *Equilibrium* guesses is equilibrium checking, which has the least search requirements among all methods. Equilibrium checking can identify the *Equilibrium* guess very quickly if the player has the luck to check the equilibrium first (CGC 2006, Appendix H; CGC 2007). Allowing this is unavoidable without risking incorrectly concluding that a subject has violated *Equilibrium*'s search implications.

[27] CGC's baseline subjects with high compliance for some type are like robot/untrained subjects, which don't usually exist because you can't tell robot subjects how they will be paid without training them in how the robot works. These "naturally occurring" baseline robot subjects provide an unusual opportunity to separate the effects of training and strategic uncertainty, by comparing their behavior with robot/trained subjects' behavior.

L2 Occurrence by missing one of its required look-ups in games 1-5 (Table 3 shows his look-ups for games 1-3). Just as for CJ's subjects who never looked at the second- or third-round pie, in games 1-5 this subject could not have been making *L2* guesses for the reason the theory assumes, and his compliance could not be expected to persist.[28] Fortunately, 309 had a Eureka! moment after game 5, and from then on complied almost perfectly with *L2*'s search requirements.

5. Further Questions Search Analysis Might Answer

To illustrate some of the further possibilities for search analysis, this section discusses two questions raised by CGC's (2006) analysis of guesses that resist analysis via decisions alone. These questions will be addressed in CGC (2007).

5a. What are CGC's baseline apparent *Equilibrium* subjects really doing?

Figure 5 (CGC (2006, Figure 4)) graphs the guesses of CGC's 8 baseline subjects with 7 or more exact *Equilibrium* guesses. The 16 games are ordered by strategic structure as in CGC (2006, Table 3) (not in the randomized order in which subjects played them), with the 8 games with mixed targets (one greater and one less than one) in the right half of the figure. Of these subjects' 128 guesses in the 16 games, 69 (54%) were exact *Equilibrium* guesses. In CGC's (2006) likelihood-based econometrics, given their a priori specification of possible types and the large strategy spaces of CGC's games, this is overwhelming evidence that their types are *Equilibrium*. But as Figure 5 makes clear, their *Equilibrium* compliance was far higher for games without mixed targets (55 out of 64 possible exact *Equilibrium* guesses, or 86%) than for games with mixed targets (14 out of 64, or 22%). Thus it is (nonparametrically) clear that these subjects, despite *Equilibrium* compliance that is off the scale by normal standards, are actually following a rule that only mimics *Equilibrium*, and that only in games without mixed targets.

The puzzle is deepened by noting that all the ways game theorists teach people to identify equilibria (best-response dynamics, equilibrium checking, and iterated dominance) work equally well with and without mixed targets. Further, CGC's *Equilibrium* robot/trained subjects, who were taught these three ways to identify their equilibrium guesses, have roughly the same equilibrium compliance with and without mixed targets (Figure 6; CGC (2007)). Thus whatever the baseline

[28] Subject 309 omitted look-ups 4 and 6 (his partner's lower and upper limits) in game 1 and look-up 4 in games 2-5. This suggests that he did not yet understand that he needed to check his partner's lower limit to be sure of his *L2* guess even when his own target, or the product of targets, was greater than 1; but he omitted look-up 4 even in game 4 where both targets were less than 1, showing that his error was probably more complex. That these omissions did not lead to non-*L2* guesses in games 1-5 is an accident of our design with no greater significance.

apparent *Equilibrium* subjects were doing, it's not one of the first things a game theorist would think of. (Subjects' debriefing questionnaires did not reveal what it was.) Nonetheless, the rule or rules they follow have a decidedly non-random structure: *All* 44 of those subjects' deviations from *Equilibrium* (the solid line in Figure 5) when it is separated from *L3* (dotted line), with or without mixed targets, are in the direction of (and sometimes beyond) their *L3* guesses; though this could reflect the fact that in CGC's games, *L3* guesses are always less extreme than *Equilibrium* guesses.

CGC's (2007) analysis will try to resolve the puzzle by using the search data to answer the following questions:

(1) How do the baseline apparent *Equilibrium* subjects find their equilibrium guesses in the games without mixed targets: best-response dynamics, equilibrium checking, iterated dominance, or something else that doesn't "work" with mixed targets? Refining CGC's (2006) characterization of *Equilibrium* search to separate the three methods and redoing the estimation with the refined compliance measures, separately for games with and without mixed targets, should be revealing. The absence of baseline *Dk* subjects suggests that iterated dominance, even finitely truncated, is unlikely. Best-response dynamics, perhaps truncated after 1-2 rounds, seems more likely.

(2) How do the baseline apparent *Equilibrium* subjects' search patterns differ in games with and without mixed targets? How do the differences compare to the differences for baseline *L1*, *L2*, or *L3* subjects? CGC's 20 apparent baseline *L1* subjects' compliance with *L1* guesses is almost the same with and without mixed targets (CGC, Figure 1), which is unsurprising because whether or not the targets are mixed is irrelevant to subjects who don't try to model others' incentives. But the 12 apparent *L2* (Figure 4; CGC, Figure 2) and 3 apparent *L3* (CGC, Figure 3) subjects' compliance with their types' guesses is much lower with than without mixed targets. This is curious, because for *L2* and *L3*, unlike for *Equilibrium*, games with mixed targets require no deeper understanding.

(3) How do *Equilibrium* robot/trained subjects with high compliance find their equilibrium guesses even in games with mixed targets? How do their searches in those games differ from baseline apparent *Equilibrium* subjects' searches? CGC strove to make *Equilibrium* robot/trained subjects' training as neutral as possible, but something must come first, and they were taught equilibrium checking first, then best-response dynamics, then iterated dominance. To the extent that these subjects used one of those methods, it explains why they

have equal compliance with and without mixed targets. But if some of them used something else that deviates from equilibrium mainly in games with mixed targets, it might provide important clues to what the baseline *Equilibrium* subjects did.

5b. Why are *Lk* the only types other than *Equilibrium* with nonnegligible frequencies?

CGC's (2006) analysis of decisions and search estimated significant numbers of subjects of types *L1*, *L2*, *Equilibrium*, or hybrids of *L3* and/or *Equilibrium*, and nothing else that does better than a random model of guesses for more than one subject. Why do these types predominate, out of the enormous number of possibilities? Why, for instance, are there no significant numbers of *Dk* types, which are closer to what game theorists teach?

CGC's (2007) analysis will try to answer this question by using search and other methods to look more deeply into the following phenomena:

> (1) Most robot/trained subjects could reliably identify their type's guesses, even for types as difficult as *Equilibrium* or *D2*. Individual subjects' exact compliance with their type's guesses was usually bimodal within type, on very high and very low.

Even so, there are several signs of differences in difficulty across types:

> (2) None of CGC's 70 robot/trained *Lk* subjects ever failed their type's Understanding Test, while 1/31 failed the *D1* Test, 1/20 failed the *D2* Test, and 7/36 failed the *Equilibrium* Test.

> (3) For those who passed the Test, compliance was highest for *Lk* types, then *Equilibrium*, then *Dk* types. This suggests that *Dk* is harder than *Equilibrium*, but more analysis is needed to tell if this was an artifact of the more stringent screening of the *Equilibrium* Test.

> (4) Within the *Lk* and *Dk* type hierarchies, compliance was higher for lower *k* as one would expect, except that *L1* compliance was lower than *L2* or *L3* compliance. This may be because *L1* best responds to a random *L0* robot, which some subjects think they can outguess; but *L2* and *L3* best respond to a deterministic *L1* or *L2* robot, which doesn't invite gambling.

> (5) Remarkably, 7 of our 19 robot/trained *D1* subjects who passed the *D1* Understanding

Test, in which *L2* answers are wrong, then “morphed” into *L2*s when making their guesses, significantly reducing their earnings (Figure 7 and subject 804 in Table 2; recall that *L2* and *D1* are cousins, both making 2-rationalizable guesses). This kind of morphing is the only kind that occurred, which seems compelling evidence that *Dk* types are unnatural. But a comparison of *Lk*'s and *Dk-1*'s search and storage requirements may have something to add.

6. A Rational-Choice Model of Optimal Search for Hidden Payoff Information

This section outlines a simple rational-choice analysis in support of the Occurrence and Adjacency assumptions that underlie CGCB’s and CGC’s models of cognition and search. The analysis is general in that it takes the formula that relates a type's decision to the hidden parameters as given. It views search for hidden payoff information as just another kind of rational decision, deriving subjects’ demand for it from the benefits of making better decisions under plausible assumptions about the benefits and costs of search and storing numbers in working memory.

The model rests on two assumptions about cognition and search:

(1) The costs of look-ups are small. There is a great deal of evidence that subjects in experiments with hidden but freely accessible payoff parameters perceive the cost of looking them up as negligible, scarcely larger than the cost of reading them in a printed table. Having to look things up has small effects on their decisions (as shown in CGCB’s and CGC’s (2006) Open Boxes control treatments), subjects usually make many more look-ups than efficient search requires, and they usually make some motivated purely by curiosity.

(2) There is a flow cost of keeping numbers in working memory, which starts small for the first number but even then is larger than the cost of a look-up; and which increases with the number of stored numbers. Total memory cost is the time integral of the flow cost, and is therefore proportional, other things equal, to total storage time, and increasing in the number of stored numbers. (If working memory were free, nothing would prevent the scanning and memorization referred to in my discussions of CJ and CGCB, but this is plainly unrealistic.)

Occurrence follows immediately from assumption (1). A rational player looks up all costlessly available information that might affect his beliefs. When, as in these designs, information comes in discrete quanta with nonnegligible effects on beliefs and the optimal decision, this conclusion extends to information available at low cost.[29]

Given Occurrence, Adjacency (in CGC's sense that the basic (innermost) operations in square brackets in the right-hand side of Table 1 are represented by adjacent look-ups) follows from assumption (2). Under (2), a player minimizes the total memory plus look-up cost by processing the basic operations needed to evaluate the expression for his ideal guess before other operations with whose results they are to be combined, storing the results (meanwhile "forgetting" the parameters they combine), and then combining them. Basic operations take precedence over other operations because "distributing" them increases memory cost.[30] For example, in evaluating the expression $p^i\,[a^j+b^j]/2$ for *L1*'s ideal guess, processing $[a^j+b^j]$ first, storing the result, and then combining it with p^i yield the following sequence of numbers of numbers in working memory: 1, 2, 1, 2, 1. The distributed alternative of processing $p^i\,a^j$, storing the result, then processing $p^i\,b^j$ and combining it with $p^i\,a^j$ yields the sequence: 1, 2, 1, 2, 3, 2, 1, which dominates the first sequence. The first method also saves the cost of looking up p^i a second time, but this is much less important.

Although Occurrence and Adjacency are only necessary conditions for optimal search, I stop with them because they have considerable empirical support, they make the main patterns of subjects' search behavior in CJ's extensive-form and CGCB's and CGC's normal-form games intelligible, and they seem more transparent than other conditions for optimality and thus more likely to be descriptive of subjects' search behavior.

I close by noting that although this model supports CJ's, CGCB's, and CGC's use of Occurrence and Adjacency, it says nothing directly about how to measure search compliance in an econometric analysis. CGC's use of the density of a type's minimal search sequence in the part of the observed sequence where the subject tends to make his relevant look-ups (his search "style," in CGC's terminology) is a judgment call, which seems to be well supported by inspecting the data.

[29] Note that because MouseLab allows a subject to enter a tentative guess without confirming it (the *s in the data in Tables 2 and 3), thereby saving storage cost, the variations in search style noted in Section 4b, footnote 24, are consistent with optimality when look-up costs are negligible even if storage costs are not.

[30] This effect is related to the reason that backward induction is the most efficient way to solve a finite-horizon dynamic programming problem like those subjects faced in CJ's design: other ways are feasible, but wasteful of storage and computational capacity (though the latter is assumed to be freely available here).

7. Conclusion

CJ's, CGCB's, and CGC's analyses of cognition in games via monitoring subjects' searches for hidden but freely accessible payoff information bridge part of the gap between neuroeconomics and conventional economics because they rest on explicit models of cognition but search, unlike neural correlates of cognition, can be viewed as a rational choice. This paper has sought to use those analyses to make two points about the potential uses of neural data in economics.

First, standard assumptions of rational choice and equilibrium have yielded successful explanations of many phenomena, which as GP note can usefully be tested via revealed preference analysis of decision data. But there are other, equally important phenomena that appear to stem from failures of the implicit assumptions about cognition that underlie standard analyses, for which tests that don't take cognition explicitly into account are likely to be biased and misleading.

Second, with unbounded capacity for experimentation it might be possible to discover all we need to know about behavior by observing decisions alone. But this is an arbitrary constraint, and CJ's, CGCB's, and CGC's analyses show that expanding the domain of analysis beyond decisions can yield a clearer view of behavior than is practically achievable by observing only decisions.

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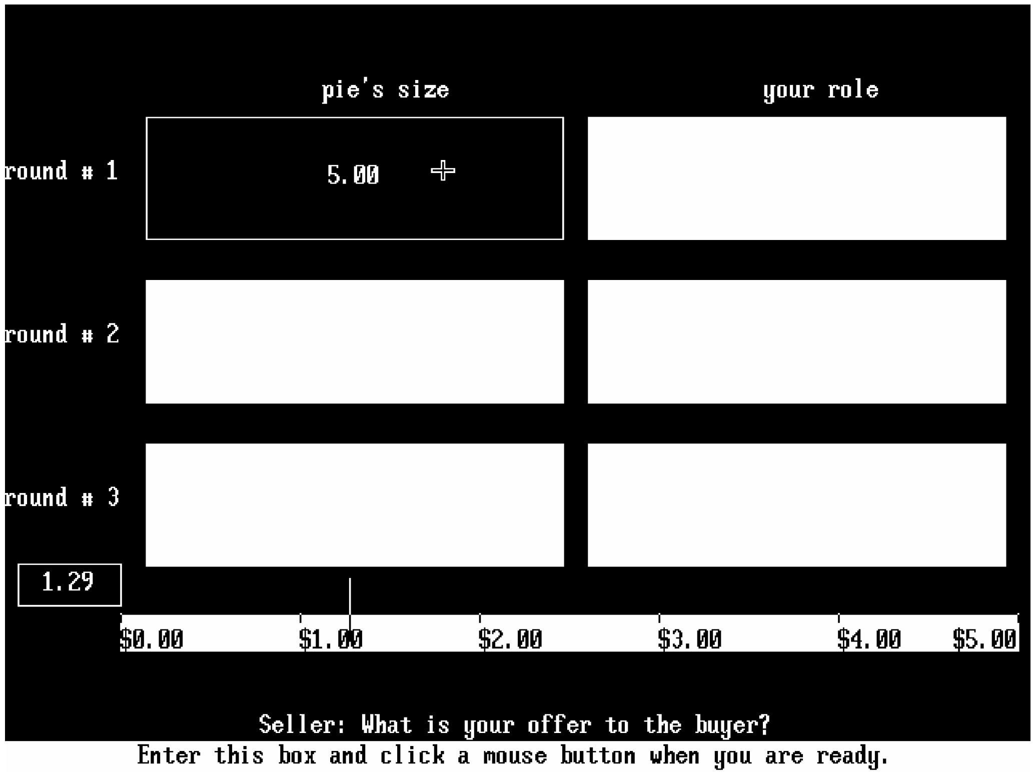

Figure 1. Display for CJ's Alternating-Offers Bargaining Experiments (CJ (2002), Figure 1)

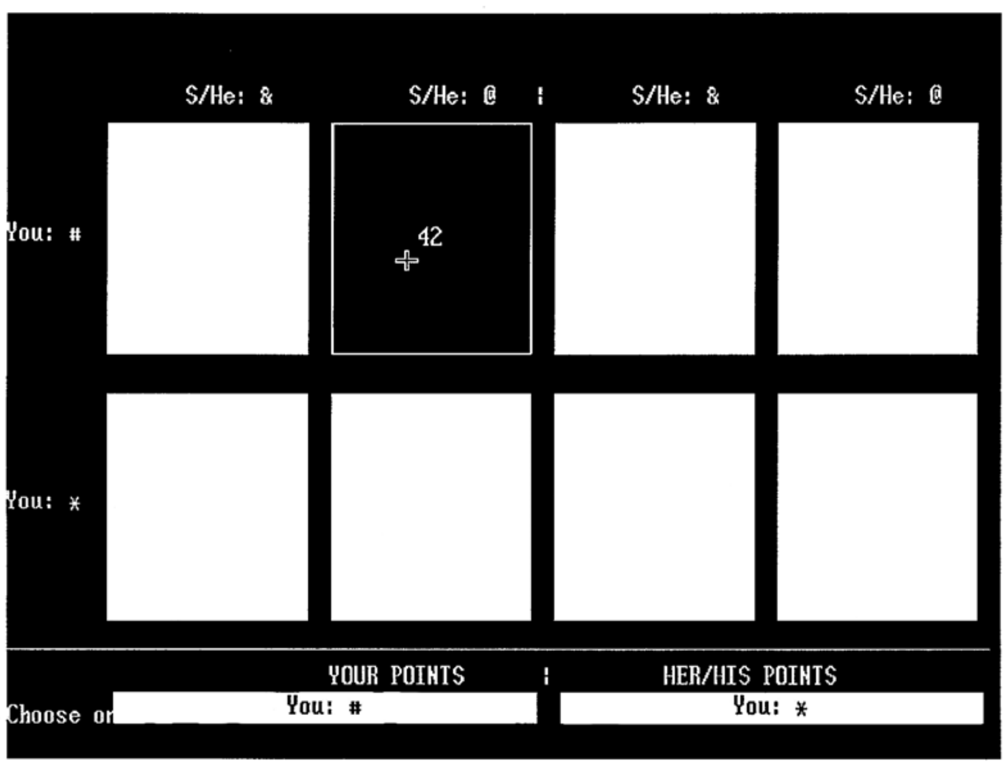

Figure 2. Display for a 2×2 Game in CGCB's Matrix-Game Experiments (CGCB, Figure 1)

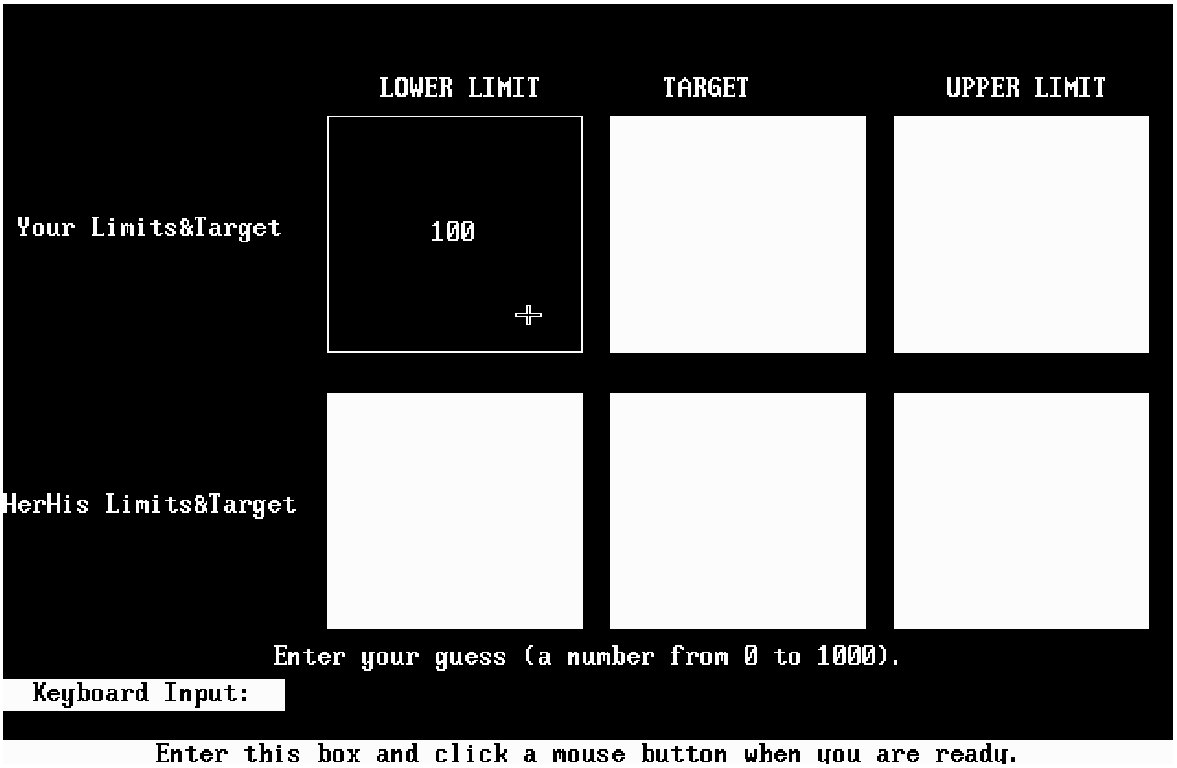

Figure 3. Display for CGC's Two-Person Guessing Games (CGC (2006), Figure 6)

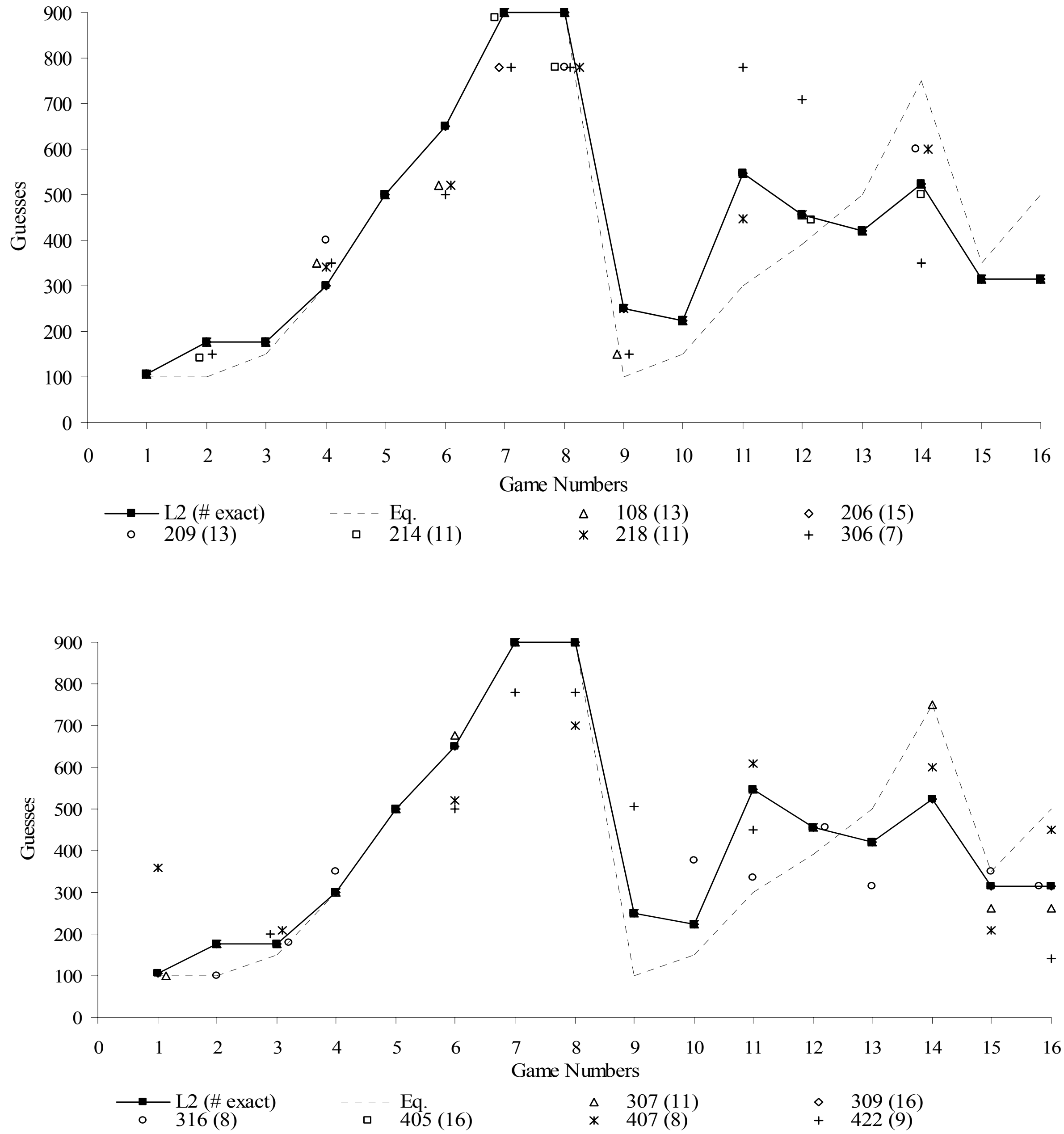

Figure 4. “Fingerprints” of 12 Apparent *L2* Baseline Subjects (CGC (2006), Figure 2)

Note: Only deviations from *L2*'s guesses are shown.

138 (72%) of these subjects' 192 guesses were exact *L2* guesses.

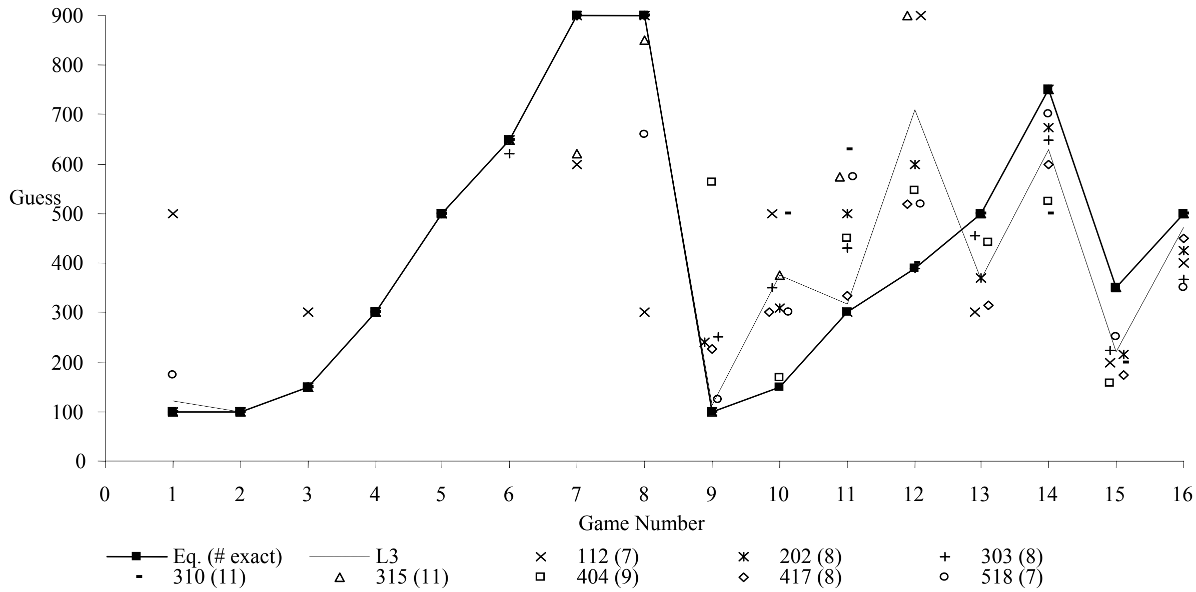

Figure 5. "Fingerprints" of 8 Apparent *Equilibrium* Baseline Subjects (CGC (2006), Figure 4))
Note: Only deviations from Equilibrium's guesses are shown.
69 (54%) of these subjects' 128 guesses were exact *Equilibrium* guesses.

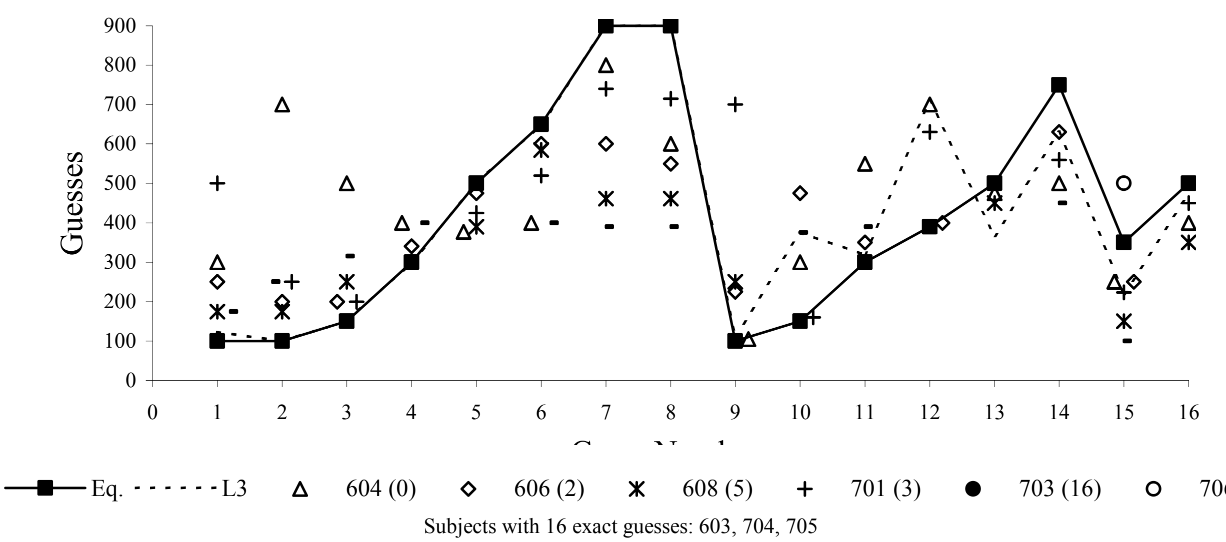

Figure 6. "Fingerprints" of 10 UCSD *Equilibrium* Robot/Trained Subjects ((CGC (2007))
Note: Only deviations from *Equilibrium*'s guesses are shown.
92 (58%) of these subjects' 160 guesses were exact *Equilibrium* guesses.

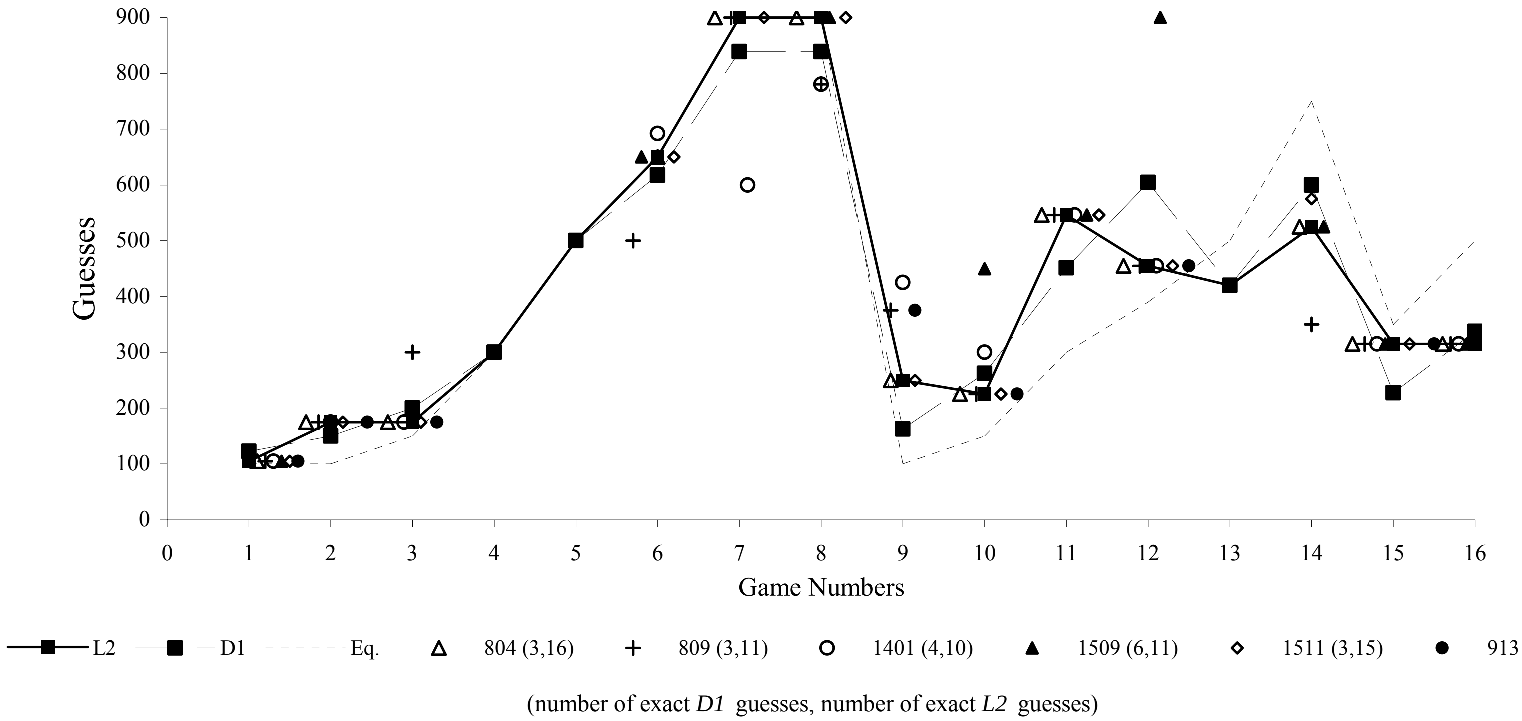

Figure 7. "Fingerprints" of 6 York Robot/Trained Subjects Who "Morphed" from *D1*to *L2* (CGC (2007))
Note: Only deviations from *D1*'s guesses are shown.
28 (29%) of these subjects' 96 guesses were exact *D1* guesses and 72 (75%) were exact *L2* guesses.

Type	Ideal guess	Minimal search sequence
L1	$p^i\,[a^j+b^j]/2$	$\{[a^j,b^j],p^i\} \equiv \{[4, 6], 2\}$
L2	$p^iR(a^j,b^j;\, p^j[a^i+b^i]/2)$	$\{([a^i,b^i],p^j),a^j,b^j,p^i\} \equiv \{([1, 3], 5), 4, 6, 2\}$
L3	$p^iR(a^j,b^j;\, p^jR(a^i,b^i;\, p^i[a^j+b^j]/2))$	$\{([a^j,b^j],p^i),a^i,b^i,p^j\} \equiv \{([4, 6], 2), 1, 3, 5\}$
D1	$p^i(\max\{a^j,p^ja^i\} + \min\{p^jb^i,b^j\})/2$	$\{(a^j,[p^j,a^i]),(b^j,[p^j,b^i]),p^i\} \equiv \{(4,[5,1]),(6,[5,3]),2\}$
D2	$p^i[\max\{\max\{a^j,p^ja^i\},p^j\max\{a^i,p^ia^j\}\}$ $+\min\{p^j\min\{p^ib^j,b^i\},\min\{p^jb^i,b^j\}\}]/2$	$\{(a^i,[p^i,a^j]),(b^i,[p^i, b^j]),(a^j,[p^j,a^i]),(b^j,[p^j,b^i]),p^j,p^i\}$ $\equiv \{(1,[2,4]),(3,[2,6]),(4,[5,1]),(6,[5,3]),5,2\}$
Eq.	p^ia^j if $p^ip^j < 1$ or p^ib^j if $p^ip^j > 1$	$\{[p^i,p^j],a^j\} \equiv \{[2, 5], 4\}$ if $p^ip^j < 1$ or $\{[p^i,p^j],b^j\} \equiv \{[2, 5], 6\}$ if $p^ip^j > 1$
Soph.	[no closed-form expression, but CGC took *Soph.*'s search implications to be the same as *D2*'s]	$\{(a^i,[p^i,a^j]),(b^i,[p^i, b^j]),(a^j,[p^j,a^i]),(b^j,[p^j,b^i]),p^j,p^i\}$ $\equiv \{(1,[2,4]),(3,[2,6]),(4,[5,1]),(6,[5,3]),5,2\}$

Table 1. Types' Ideal Guesses and Minimal Search Sequences (CGC (2006), Table 4)

Note: $R(a,b;x) \equiv \min\{b, \max\{a, x\}\} \equiv \max\{a, \min\{b, x\}\}$ denotes x's adjusted guess with limits a and b.

Table 2. Selected Robot/Trained Subjects' Information Searches and Assigned Types' Search Implications

MouseLab box number

	a	*p*	*b*
You (*i*)	1	2	3
S/he (*j*)	4	5	6

Types' Search Implications

L1	{[4,6],2}
L2	{([1,3],5),4,6,2}
L3	{([4,6],2),1,3,5}
D1	{(4,[5,1], (6,[5,3]),2}
D2	{(1,[2,4]),(3,[2,6]),(4,[5,1],(6,[5,3]),5,2}
Eq	{[2,5],4} if pr. tar.<1,{[2,5],6} if > 1

Subject	**904**	**1716**	**1807**	**1607**	**1811**	**2008**	**1001**	**1412**	**805**	**1601**	**804**	**1110**	**1202**	**704**	**1205**	**1408**	**2002**
Type(#ex.)	L1 (16)	L1 (16)	L1 (16)	L2 (16)	L2 (16)	L2 (16)	L3 (16)	L3 (16)	D1 (16)	D1 (16)	D1 (3)	D2 (14)	D2 (15)	Eq (16)	Eq (16)	Eq (15)	Eq (16)
Alt.(#ex.)											L2 (16)						
Game																	
1	123456 4623	146462 134646 23	462513	135462 1313	134446 5213*4 6	111313 131313 5423	462135 21364* 246231 52	146231 564623 1	154356 423213 2642	254514 36231	154346 5213	135464 2646*1 313	246466 135464 641321 342462 422646 124625 5*1224 654646	123456 363256 565365 626365 652651 452262 6526	123456 424652 562525 6352*4 65	123123 456445 632132 11	142536 125365 253616 361454 613451 213452 63
2	123456 4231	462462 13	462132 25	135461 354621 3	134653 125642 313562 52	131313 566622 333	462135 642562 223146 2562*6 2	462462 546231 546231	514535 615364 23	514653 6213	515135 365462 3	135134 642163 451463 211136 414262 135362 *14654 6	123645 132462 426262 241356 462*13 524242 466135 6462	123456 525123 652625 635256 262365 456	123456 244565 565263 212554 146662 654251 44526* 31	123456 456123 643524 1	143625 361425 142523 625656 3

Notes: The subjects' frequencies of making their assigned types' (and when relevant, alternate types') exact guesses are in parentheses after the assigned type. A * in a subject's look-up sequence means that the subject entered a guess there without immediately confirming it.

Table 3. Selected Baseline Subjects' Information Searches and Apparent Types' Search Implications

	MouseLab box number		
	a	*p*	*b*
You (*i*)	1	2	3
S/he (*j*)	4	5	6

Types' Search Implications

L1	{[4,6],2}
L2	{([1,3],5),4,6,2}
L3	{([4,6],2),1,3,5}
D1	{(4,[5,1], (6,[5,3]),2}
D2	{(1,[2,4]),(3,[2,6]),(4,[5,1],(6,[5,3]),5,2}
Eq	{[2,5],4} if pr. tar.<1,{[2,5],6} if > 1

Subject	**101**	**118**	**413**	**108**	**206**	**309**	**405**	**210**	**302**	**318**	**417**	**404**	**202**	**310**	**315**
Type(#ex.)	L1 (15)	L1 (15)	L1 (14)	L2 (13)	L2 (15)	L2 (16)	L2 (16)	L3 (9)	L3 (7)	L1 (7)	Eq (8)	Eq (9)	Eq (8)	Eq (11)	Eq (11)
Alt.(#ex.)								Eq (9)	Eq (7)	D1 (5)	L3 (7)	L2 (6)	D2 (7)		
Alt.(#ex.)								D2 (8)			L2 (5)		L3 (7)		
Game															
1	146246 213	246134 626241 32*135	123456 545612 3463*	135642	533146 213	1352	144652 313312 546232 12512	123456 123456 213456 254213 654	221135 465645 213213 45456* 541	132456 465252 13242* 1462	252531 464656 446531 641252 462121 3	462135 464655 645515 21354* 135462 426256 356234 131354 645	123456 254613 621342 *525	123126 544121 565421 254362 *21545 4*	213465 624163 564121 325466
2	46213	246262 2131	123564 62213*	135642 3	531462 31	135263 1526*2 *3	132456 253156 456545 463123 156562 62	123456 465562 231654 456*2	213546 566213 545463 21*266 54123	132465 132*46 2	255236 62*365 243563	462461 352524 261315 463562	123456 445613 255462 513565 23	123546 216326 231456 *62	134652 124653 656121 3
3	462*46	246242 466413 *426	264231	135642 53	535164 2231	135263	312456 5231*1 236545 5233** 513	123455 645612 3	265413 232145 563214 563214 523*65 4123	134652 1323*4	521363 641526 5263*6 52	462135 215634 *52	123456 123562 3	123655 463213	132465 544163 *3625