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JOHN NASH AND THE ANALYSIS OF STRATEGIC BEHAVIOR

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Introduction

I take particular pleasure in writing about John Nash's scientific contributions, for he has long been one of my intellectual heroes. I may even have felt the influence of Nash's work as early as April 1950, when I was born under the sign of Nash (1950b), his first great paper on bargaining. The influence became more tangible in the 1970's, when I first read his extraordinary papers on bargaining and noncooperative game theory. This essay describes one economist's view of how Nash's work influenced the development of game theory as a tool for analyzing strategic behavior.

To fully appreciate Nash's contributions, one must understand the state of game theory as he found it. In the late 1940's the field was dominated by von Neumann and Morgenstern's classic *Theory of Games and Economic Behavior* (1944, 1947, 1953). Their book elaborated on von Neumann's (1928) theory of zero-sum two-person games, and on that foundation proposed a generalization to non-zero-sum *n*-person games that is the forerunner of what is now called cooperative game theory. It also set the pattern for subsequent game-theoretic analyses, as follows.

A *game* is defined by the decision-makers, called *players*; the decisions they must make and their information; how their decisions determine the outcome; and their preferences over outcomes, represented by *utility* or *payoff* functions. A player's decisions are summarized by a *strategy*, a complete contingent plan for playing the game, specifying a decision at each point he may be called upon to make one as a function of what he knows. Because strategies are complete contingent plans, they can (in fact must) be thought of as chosen without knowledge of each other's choices at the start of play; and specifying a strategy for each player determines an outcome in the game. Most research in game theory assumes that players are rational in some sense. I assume, following von Neumann and Morgenstern and Nash, that each player has a finite set of *pure* (unrandomized) strategies, and that *mixed* (randomized) strategies are allowed. Most game theorists agree that the notion of rationality should be consistent with payoff maximization, and that uncertainty is adequately handled by assigning payoffs (or *von Neumann-Morgenstern utilities*) to outcomes so that players choose as if to maximize expected payoffs. In games, however, payoff maximization is

ambiguous because players' payoff-maximizing strategies depend on others' strategies, and there is no generally accepted definition of rationality (see however Harsanyi and Selten (1988)).

Nash extended von Neumann and Morgenstern's theory in three directions. He first (1950a) sharpened their cooperative analysis of bargaining, proposing axioms that characterize a unique outcome now called the *Nash bargaining solution*. He then (1950a, 1951) proposed an alternative to von Neumann and Morgenstern's "cooperative" generalization of von Neumann's (1928) zero-sum two-person analysis, the notion of what is now called *Nash equilibrium* in a non-zero-sum *n*-person game, the foundation of modern noncooperative game theory. Finally, he (1953) showed that the idea of equilibrium, used to analyze a suitably detailed model of the bargaining process, can elucidate bargaining, providing a powerful complement to von Neumann and Morgenstern's cooperative analysis and using "noncooperative" methods to analyze cooperative outcomes in the prototype of what is now called the *Nash program*. I now briefly describe these developments.

Cooperative and Noncooperative Game Theory

A *zero-sum two-person game* has two players, whose payoffs sum to zero for all possible strategy combinations, so that what one gains, the other necessarily loses. This opposition of interests makes mutually beneficial coalitions impossible, greatly simplifying the analysis. The theory of zero-sum two-person games yields powerful conclusions, and has exerted strong and lasting influences on how game theorists analyze more general games and on how economists view game theory, even though such games are very special from the point of view of applications.

Von Neumann and Morgenstern began their analysis of zero-sum two-person games by considering each player's *maximin* strategy: his strategy that yields him the highest minimum expected payoff, where the minimum is taken over the other's strategies. Their main result, the minimax theorem, shows (under the above assumptions) that the expected payoffs of players' maximin strategies sum to zero, so the highest minimum a player can guarantee himself coincides with the lowest maximum the other can hold him to. They also showed that any combination of players' maximin strategies has the property that each player's maximin strategy maximizes his own expected payoff, given the other's. Thus, any combination of maximin strategies is what is now called a Nash equilibrium (or just an equilibrium), and the existence of maximin strategies implies the existence of a Nash equilibrium in mixed strategies for finite zero-sum two-person games.

These results have several strong implications. Because a player's set of maximin strategies is determined by his own payoffs, the equivalence of maximin and equilibrium strategies implies

that even if the equilibrium strategies are not unique, they are interchangeable, in that any combination of players' maximin strategies is in equilibrium; and equivalent, in that all such combinations yield both players the same payoffs. Thus, as long as players play maximin strategies, it does not matter whether, or how, they coordinate their strategy choices. Nor does it matter if a player observes his opponent's maximin mixed strategy before choosing his own strategy, or if players can discuss the game in advance or make binding agreements about how to play it.

These strong implications suggested to von Neumann and Morgenstern that generalizing their zero-sum theory to non-zero-sum n-person games would capture something of the richness of real strategic interactions. To understand the choices they faced in deciding how to generalize their theory, note that their analysis of zero-sum two-person games expresses the idea of rationality in two ways, which are reinforcing in such games but different in general: A player in a zero-sum two-person game who wishes to maximize his expected payoff, and who expects his opponent to anticipate his strategy, plainly cannot do better than to play his maximin strategy. And if all players predict the same combination of strategies, those strategies are consistent with expected payoff maximization if and only if they are in equilibrium. An equilibrium is thus an example of what economists call a *rational-expectations equilibrium*, but one in which players form expectations about others' strategies rather than about the exogenous uncertainty usually studied in economics.

Von Neumann and Morgenstern chose to generalize their zero-sum theory by focusing on the first notion of rationality. They summarized players' strategic possibilities by each possible coalition's maximin total payoff, seeking generality by suppressing all other aspects of the structure of the game. They then proposed a characterization of the set of outcomes that might emerge when rational players are free to communicate, form coalitions, and make agreements. Their theory is a true generalization of their zero-sum two-person theory, the forerunner of the modern theory of cooperative games; but its conclusions are far less specific and less compelling.

Von Neumann and Morgenstern's restriction to coalitions' maximin total payoffs is without loss of generality in zero-sum two-person games, where single players are the only effective coalitions and their maximin payoffs are the only aspect of the structure of the game that matters in a theory based on rationality. But maximin strategies are plainly too conservative in non-zero-sum games, where maximizing one's own payoff is no longer equivalent to minimizing the other player's. Recognizing this point, Nash (1950a, 1951) chose to generalize von Neumann and Morgenstern's zero-sum theory by focusing instead on the notion of rational expectations about

others' strategies, which remains equally plausible in non-zero-sum *n*-person games. Using simple fixed-point arguments, Nash proved the existence of Nash equilibrium for a wide class of non-zero-sum *n*-person games. His work lay the foundation of noncooperative game theory, now the predominant mode of analysis of strategic interactions in economics, political science, and biology. This predominance is no accident, because formalizing the notion of rational expectations of others' strategies in non-zero-sum games makes it necessary to retain details of the structure that von Neumann and Morgenstern discarded, including players' strategies and how they determine outcomes and payoffs. Such details are often relevant to predicting how institutions influence outcomes, and noncooperative game theory is a rich language for describing this influence. (Cooperative game theory still plays an important role in applications, in studying environments whose structure is difficult to specify with confidence. The literature may be approaching a synthesis, but any such synthesis will include a very large measure of noncooperative game theory.)

Bargaining

The first demonstration of the potential of noncooperative game theory in applications was Nash's (1953) analysis of the bargaining problem. Nash's (1953) analysis grew out of his (1950a) analysis, in which, as in von Neumann and Morgenstern's cooperative analysis of bargaining, he considered the situation of two players who can make a binding agreement about how to play a game. Nash (1950a) sought to sharpen von Neumann and Morgenstern's conclusions by expressing the payoffs bargainers can "rationally anticipate" as a function—his analysis is distinguished by its goal of identifying a unique bargaining outcome—of the data of the bargaining problem. (Applying von Neumann and Morgenstern's cooperative theory to bargaining problems simply replicates Edgeworth's (1881) conclusion that the outcome must lie on the *contract curve*: the set of outcomes such that no feasible outcome yields higher payoffs for both bargainers and each bargainer's payoff is at least his minimax payoff.) He showed that one such function, the Nash bargaining solution, is uniquely characterized by plausible axioms. These axioms generalize the widely accepted principle of sharing of the gains from agreement equally to bargaining problems with nonlinear utilities and sets of feasible outcomes, in which the meaning of equal-sharing is not readily apparent.

In Nash's (1953) analysis of bargaining, the innovation was to use the notion of equilibrium to characterize players' rational strategies in an explicit, noncooperative model of the bargaining process. In his model, bargainers make simultaneous, once-and-for-all demands, expressed as utilities. If their demands are feasible, taken together, bargaining ends with a binding agreement

that yields them the utilities they demanded; otherwise the process ends in disagreement. Nash defended this *demand game* specification as follows: "Of course, one cannot represent all possible bargaining devices as moves in the non-cooperative game. The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utilize all the essential strengths of his position." Schelling (1960, Appendix B) and Harsanyi and Selten (1988, pp. 23-26)) outlined more detailed, dynamic models in its support.

In the demand game, any pair of demands on the contract curve is in equilibrium. A player who reduced his demand, starting from such a pair, would lower his payoff with no compensating benefit; and a player who increased his demand would cause a disagreement, again lowering his payoff. Nash's analysis thus far suggests the following view of bargaining: There are many efficient agreements that are better than disagreement for both bargainers; but all are consistent with equilibrium, which is therefore no help in choosing among them. Thus, bargaining may generate a great deal of uncertainty about how players will respond to its multiplicity of equilibria, even when there is no other uncertainty in the bargaining environment. Unless the bargainers find a way to resolve this uncertainty, they may not realize any of the gains from reaching an agreement: At the heart of the bargaining problem lies a coordination problem. The dynamic give-and-take of real bargaining is surely, in part, a robust response to this coordination problem. Nash went on to suggest two complementary resolutions of this coordination problem: He suggested that the normative force of his (1950a) bargaining solution might focus players' expectations on the associated demand-game equilibrium. And he outlined a "smoothing" argument (the forerunner of the modern theory of equilibrium refinements) that yields the same conclusion.

Nash's (1953) noncooperative analysis of bargaining is important because it explains, *within the theory*, why bargaining is a problem, and thus provides a framework in which the influence of the environment on bargaining outcomes can be evaluated. The Nash program—the analysis of noncooperative models of cooperation—has since greatly improved our understanding of efficient and inefficient outcomes in bargaining and many other aspects of economic life.

Conclusion

In this essay I have tried to illustrate John Nash's contributions by discussing the work on equilibrium and bargaining that forms the core of his theory of strategic behavior. This only scratches the surface of the advances in applications his work in game theory made possible.

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