Nonparametric Analysis of Reference-Dependence

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Reference-dependence: a meaningful theorem?

$$\max_{\mathbf{q}} u\left(\mathbf{q}, \mathbf{q} - \mathbf{r}_t\right) \text{ subject to } \mathbf{p}_t'\mathbf{q} = x_t$$

"By a meaningful theorem I mean simply a hypothesis about empirical data which could conceivably be refuted."

P. Samuelson, *Foundations of Economic Analysis*, p.4, (1947).

Afriat's Theorem[1]. The following statements are equivalent:

1. There exists a continuous, non-satiated utility function which rationalises the data.

2. There exists a continuous, concave, monotonic utility function which rationalises the data.

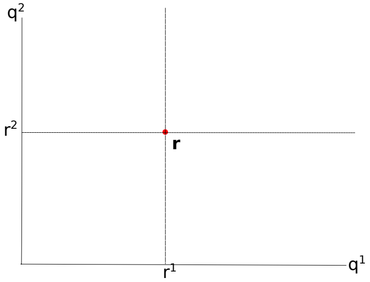
3. The data satisfy GARP.

4. There exists real numbers $\{U_t, \lambda_t > 0\}_{t=1,\ldots,T}$ such that $U_s \leq U_t + \lambda_t \mathbf{p}_t' (\mathbf{q}_s - \mathbf{q}_t)$

[1] Afriat (1967), Diewert (1973), Varian (1982)

1. Reference-dependent preferences.
2. An *Afriat's Theorem* for reference-dependent preferences?
3. Another look at the NYC taxi drivers.
4. Which theory is best?

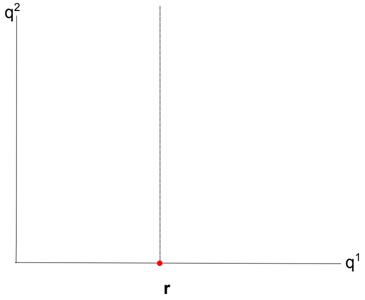
- We index observations by $t = 1, ..., T$.
- We observe prices $\mathbf{p}_t \in \mathbb{R}^K_{++}$ and demand choices $\mathbf{q}_t \in \mathbb{R}^K_+$.
- Reference points are denoted $\mathbf{r}_t \in \mathbb{R}^K$.



- Reference points have the same dimensionality as the choice variable.
- This is a "two-dimensional reference point" (there is reference-dependence in both dimensions).
- This creates four regimes (defined by gains/losses) relative to $\boldsymbol{r}$.

Reference-dependence

Reference points



- If the reference point for a good is zero there is "one-dimensional reference point".
- Here there is reference-dependence wrt good 1, but not good two.
- This creates two regimes (defined by gains/losses with respect to good 1) relative to ***r*** .

Reference-dependence

Preferences are such that people care about differences wrt to $\mathbf{r}$ as well as levels $(\mathbf{q})$ of consumption

$$u(\mathbf{q}, \mathbf{q} - \mathbf{r})$$

Through suitable choice of u you get :

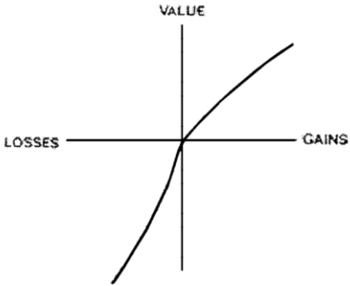
The standard model:	$u(\mathbf{q}, \mathbf{q} - \mathbf{r}) = v(\mathbf{q})$
Tversky & Kahneman (1991):	$u(\mathbf{q}, \mathbf{q} - \mathbf{r}) = w(\mathbf{q} - \mathbf{r})$
Koszegi & Rabin (2006):	$u(\mathbf{q}, \mathbf{q} - \mathbf{r}) = m(\mathbf{q}) + n(\mathbf{q} - \mathbf{r})$

Reference-dependence

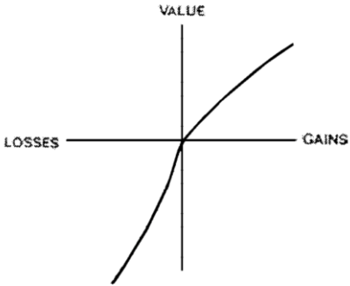
The main ingredients usually involve:

- *Reference-dependence*: “carriers of value are gains and losses relative to a reference point".
- *Loss aversion*: "the [utility] function is steeper in the negative than the positive domain".
- *Diminishing sensitivity*: "the marginal values of both gains and losses decreases with their size"

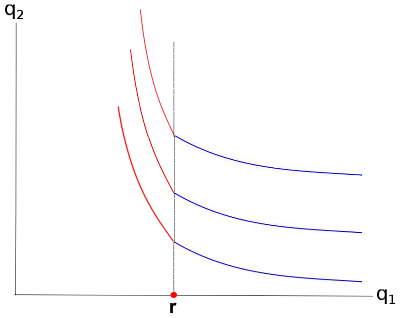
Reference-dependence
Reference points



- People are endowed with a “book” of utility functions.
- The utility function changes from one form to another at the reference point.
- This creates multiple regimes defined by gains/losses with respect to $\boldsymbol{r}$.



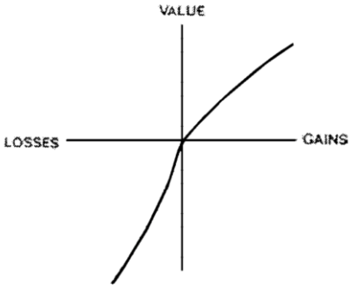
- "Loss aversion" occurs when the slope of the utility function is locally steeper below r than above.
- The marginal utility of gains are smaller than loses (locally).



- The drop in marginal utility at the reference point associated with "loss aversion" changes the MRS discretely.
- Hard to distinguish from regular concavity given finite data.

Reference-dependence

Constant sensitivity/sign-dependence

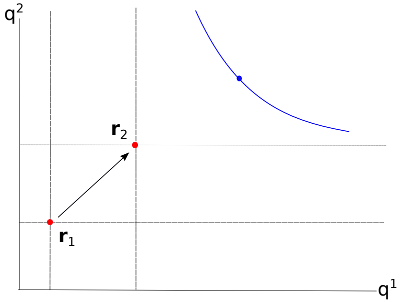


- Diminishing sensitivity is more than "concave above, convex below".
- It relates to the way the local slope of the utility function changes as the reference point changes and gets nearer/further away.

Movements in the reference points do two things

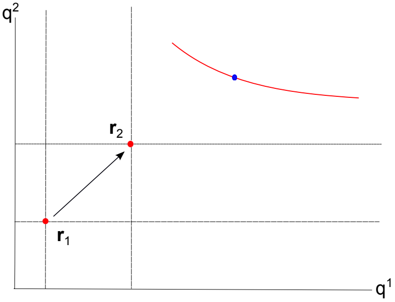
1. Local maps changes their area of application.
2. The local maps themselves may alter.

The consequence of this is that global maps will alter iff the reference point moves.

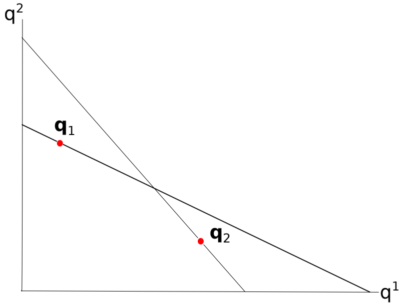


- Constant sensitivity means that the MRS through the point of interest does not change even though the reference point has moved.
- There is no *local* change in the indifference curve.
- The global map does change though, as the local regimes shift.

Reference-dependence
Non-constant sensitivity/sign-independence

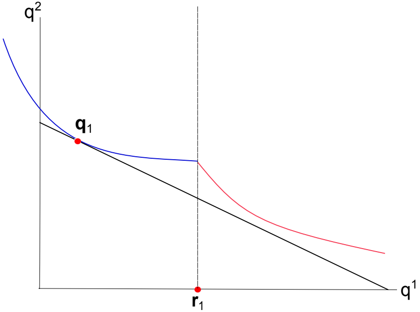


- With diminshing sensitivity the reference point is closer, and both the marginal utilities and hence the MRS change.
- Since we are closer in both dimensions the overall effect on the MRS is not restricted.
- There is a local change in the indifference curve remote from the reference point.
- Both global and local maps change.



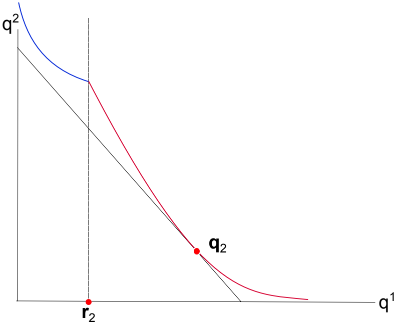
- Here we have a violation of WARP.
- There is no neoclassical utility function which can explain these observations.

Reference-dependence
Rationalising the un-rationalisable

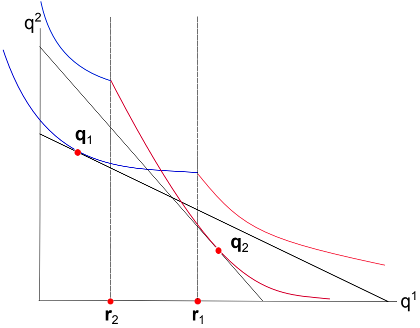


- But suppose we have reference dependence wrt good 1.
- There is a book of two indifference curves maps defined wrt loss-gain regimes relative to $\mathbf{r}_1$.
- $\mathbf{q}_1$ was selected in the loss regime using loss-side preferences.

Reference-dependence
Rationalising the un-rationalisable



- Now the reference point changes from $\mathbf{r}_1$ to $\mathbf{r}_2$.
- $\mathbf{q}_2$ was chosen using different gain-side preferences.

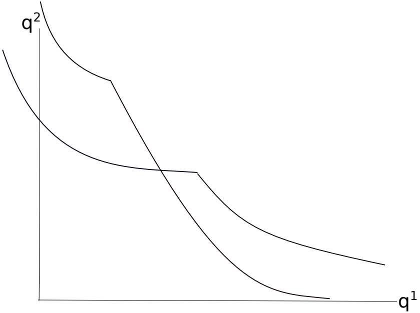


- As long as
  - the global indifference curves do not cross the budget constraints
  - each within-regime indifference curve maps is well-behaved

we can rationalise these choices.

Reference-dependence

Rationalising the un-rationalisable



- The global indifference curves can look severely non-standard (although with loss-aversion they may merely become more concave at the boundary).
- Nonetheless, for a given reference point they are (fairly) well-behaved.
- But they change with the reference point and therefore global indifference curves are tangled as the reference point moves.

Reference-dependence

Data: $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ repeated observations on an individual consumer.
The data may include $\{\mathbf{r}_t\}_{t=1,\dots,T}$

Are these data (in)consistent with this ...

$$\max_{\mathbf{q}} u\left(\mathbf{q}, \mathbf{q} - \mathbf{r}_t\right) \text{ subject to } \mathbf{p}_t'\mathbf{q} = x_t$$

...?

Reference-dependence

Overview of theoretical results

	Non-constant sensitivity	Constant sensitivity
Unobserved $\{\mathbf{r}_t\}$	Prop. 1	Prop. 3
Observed $\{\mathbf{r}_t\}$	Prop. 2	Prop. 4

Reference-dependence

"Rationalise"

Definition: "Rationalise"

A reference dependent utility function $u\left(\mathbf{q}, \mathbf{q}-\mathbf{r}\right)$ and a set of reference points $\left\{\mathbf{r}_t\right\}_{t=1,\dots,T}$ rationalise the data $\left\{\mathbf{p}_t, \mathbf{q}_t\right\}_{t=1,\dots,T}$ if $u\left(\mathbf{q}_t, \mathbf{q}_t-\mathbf{r}_t\right) \geq u\left(\mathbf{q}, \mathbf{q}-\mathbf{r}_t\right)$ for all $\mathbf{q}$ such that $\mathbf{p}_t\mathbf{q} \leq \mathbf{p}_t'\mathbf{q}_t$.

Proposition 1 (non-constant sensitivity, unobserved $\mathbf{r}_t$)

For any dataset $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ there exists a set of reference points $\{\mathbf{r}_t\}_{t=1,\dots,T}$ and a utility function $u(\mathbf{q}, \mathbf{q}-\mathbf{r})$ which is continuous, non-satiated and non-decreasing with respect to $\mathbf{q}$ for a given $\mathbf{r}$ which rationalises those data.

Proposition 2 (non-constant sensitivity, observed $\mathbf{r}_t$)

The following conditions are equivalent:

1. There exists a reference-dependent utility function that rationalizes the data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1,\ldots,T}$ such that $u(\mathbf{q}, \mathbf{q} - \mathbf{r})$ is continuous, non-satiated, and non-decreasing with respect to $\mathbf{q}$.
2. The observations within each subset of the data defined a common reference point satisfy GARP.

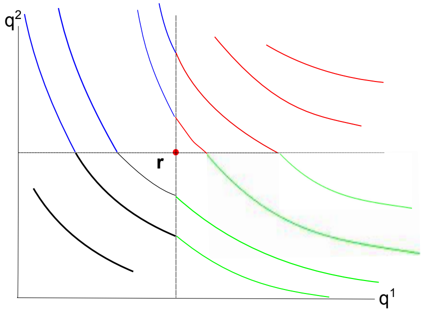
Proposition 3 (constant sensitivity, unobserved $\mathbf{r}_t$)

The following conditions are equivalent:

1. There exist a set of reference points $\{\mathbf{r}_t\}_{t=1,\dots,T}$ such that $r_t^k \neq q_t^k$ for some k and a utility function $u(\mathbf{q}, \mathbf{q} - \mathbf{r})$ which is continuous, non-satiated and non-decreasing with constant sensitivity which rationalise the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$.
2. There exists a partition of the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ into 2^K subsets or fewer which satisfy GARP.

Reference-dependence

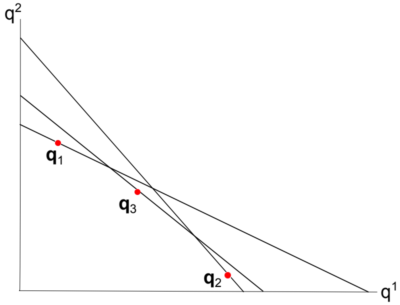
Constant sensitivity, unobserved $\mathbf{r}_t$



- The global indifference curves are composed of at most four local maps.
- These maps are locally invariant to changes in the reference point.
- The set of observations must be rationalisable by not more than four maps.

Reference-dependence

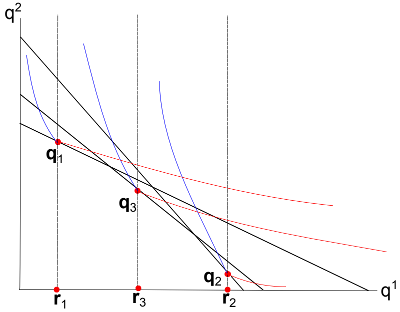
Constant sensitivity, unobserved $\mathbf{r}_t$



- This requires 3 maps.
- Not rationalisable with a 1-dimensional reference point.
- Except ....

Reference-dependence

Constant sensitivity, unobserved $\mathbf{r}_t$



- If (at least one of the) bundles lie on the boundaries then this is not the case.
- You can rationalise with fewer than three maps.

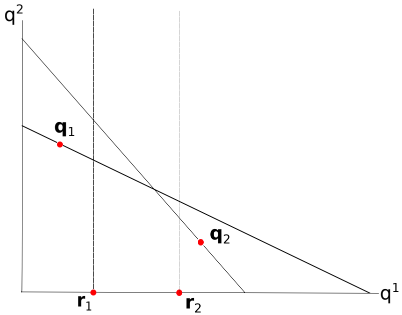
Proposition 4 (constant sensitivity, observed $\mathbf{r}_t$)

The following conditions are equivalent:

1. There exists a reference-dependent utility function that rationalizes the data such that $u(\mathbf{q}, \mathbf{q} - \mathbf{r})$ exhibits constant sensitivity and is continuous, non-satiated, and non-decreasing with respect to $\mathbf{q}$ for any given $\mathbf{r}_t$.
2. The data within each regime defined by the reference points satisfy GARP and the implied global revealed preferred and revealed worse sets for each observation are disjoint.

Reference-dependence

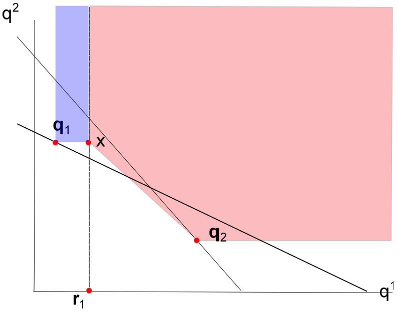
Constant sensitivity, unobserved $\mathbf{r}_t$



- This satisfies the necessary condition (2 maps).
- But it is not rationalisable.
- This is because no global map, composed of the local ones, can rationalise the observations.

Reference-dependence

Constant sensitivity, unobserved $\mathbf{r}_t$



- x is (globally) preferred to both $\mathbf{q}_1$ and $\mathbf{q}_2$
- Given $\mathbf{r}_1$, the gain side revealed preference is $\mathbf{q}_2$ is preferred to x.
- Hence there must be affordable convex combinations of x and $\mathbf{q}_2$ which are preferred to $\mathbf{q}_1$.

1. If the reference point does not move, or $\mathbf{r}_t \leq 0 \ \forall t$ then the model collapses to the standard neoclassical model.
2. You can also test for a less than $K-$dimensional reference point model (special case).
3. You need at least $2^K + 1$ observations to falsify the model with a $K-$dimensional reference point.

In the absence of large income effects, a neoclassical model of labor supply predicts a positive wage elasticity of hours worked.

Camerer et al. (*QJE*, 1997) found a strongly negative elasticity of hours with respect to their closest analog of a wage, realised earnings per hour.

They interpret their finding as evidence of **target earnings** behaviour by taxi drivers.

Target earnings behaviour can be an example of reference-dependent preferences - there is a sharp kink in the utility function at the reference/target level so that workers quit when income reaches the reference/target level.

Empirical Application
NYC taxi drivers

Farber *AER*, 2008 collected his own NYC taxidriver data.

He estimated a non-structural model which allowed for an earnings reference point.

He found:

- There was evidence in favour of an earning reference point.
- But reference points varied unpredictably from day to day - they were too unstable and imprecisely estimated to allow a useful reference-dependent model of labour supply.

Empirical Application

NYC taxi drivers

V. Crawford and Meng *AER*, 2011 re-examined Farber's data.

They using a structural model based on Koszegi & Rabin's (*QJE*,2006) theory model which conceptualises reference points as rational expectations. They assume constant sensitivity. They also allowed for hours targets.

They found:

- Reference points are an important element of the story.
- Using Koszegi & Rabin's model as a guide they were able more successfully to fit the variation in reference points.

Empirical Application

NYC taxi drivers

Farber collected 538 "trip sheets" for 15 drivers between June 1999 and May 2001.

Each trip sheet records the driver's name, hack number, and date and the details of each fare.

For each fare the data record the start time, start location,end time, end location, and fare.

Empirical Application

NYC taxi drivers

TABLE 1—SHIFT LEVEL SUMMARY STATISTICS, BY DRIVER

Driver	Number of shifts	Average trips	Working hours	Driving hours	Waiting hours	Break hours	Total income	Average wage
Driver 1	39	23.56	6.85	4.32	2.53	0.90	157.58	23.16
Driver 2	14	12.29	3.89	2.78	1.11	2.41	97.10	25.11
Driver 3	40	22.10	6.28	4.52	1.76	0.39	147.51	23.89
Driver 4	23	16.52	6.46	3.98	2.48	2.11	144.96	23.65
Driver 5	24	22.29	6.47	4.42	2.05	0.74	160.71	25.59
Driver 6	37	25.32	7.78	5.13	2.64	0.86	172.44	22.54
Driver 7	19	25.58	7.17	5.47	1.70	0.54	162.02	23.23
Driver 8	45	20.27	6.35	3.90	2.45	1.65	133.19	21.46
Driver 9	13	19.46	6.15	4.03	2.13	0.55	157.95	25.78
Driver 10	17	21.29	7.06	4.49	2.57	0.64	165.84	23.59
Driver 11	70	25.06	6.84	4.56	2.28	0.93	172.01	25.62
Driver 12	72	28.10	8.53	5.84	2.69	0.60	203.05	24.01
Driver 13	33	17.06	6.91	4.63	2.29	0.97	163.51	23.91
Driver 14	46	24.46	7.10	4.80	2.30	0.67	156.23	22.00
Driver 15	46	19.17	5.32	3.66	1.66	0.24	128.97	24.72
All	538	22.65	6.84	4.58	2.26	0.87	160.03	23.80

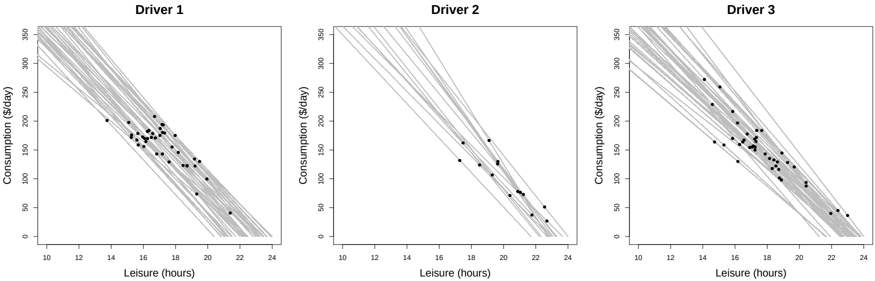
Our data: $\left\{\mathbf{p}_t^i, \mathbf{q}_t^i\right\}_{t=1,\ldots,T_i}^{i=1,\ldots,15}$ where

$$\mathbf{q}_t^i = \begin{bmatrix} l_t^i \\ c_t^i \end{bmatrix} = \begin{bmatrix} 24 - \text{work hours} - \text{breaks} - \text{waiting} \\ \text{consumption} \end{bmatrix}$$

$$\mathbf{p}_t^i = \begin{bmatrix} w_t^i \\ 1 \end{bmatrix} = \begin{bmatrix} \text{hourly earnings} \\ \text{numeraire} \end{bmatrix}$$

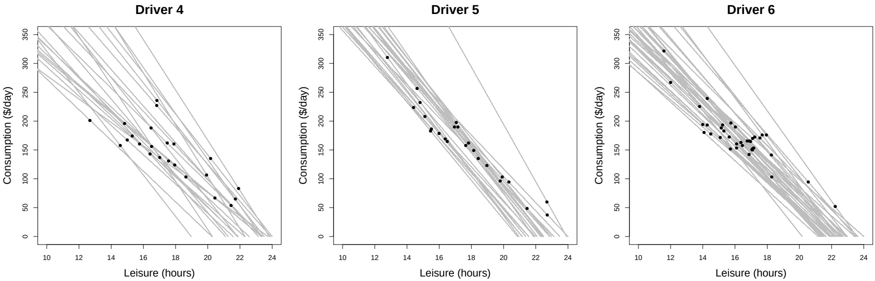
We adjust hourly earnings for time spent waiting.

Empirical Application
NYC taxi drivers

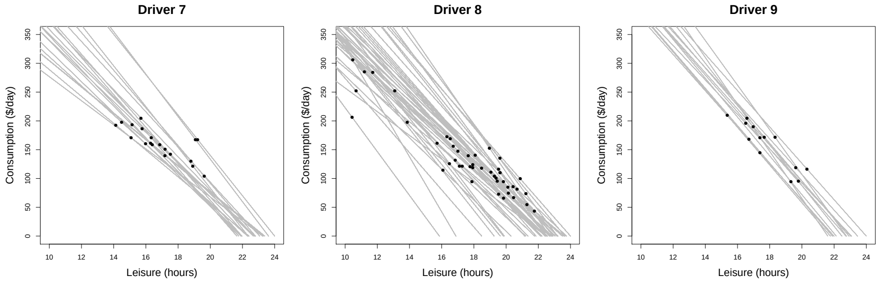


Empirical Application

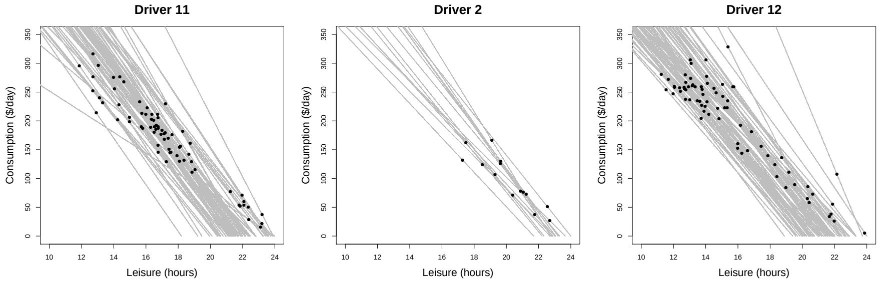
NYC taxi drivers



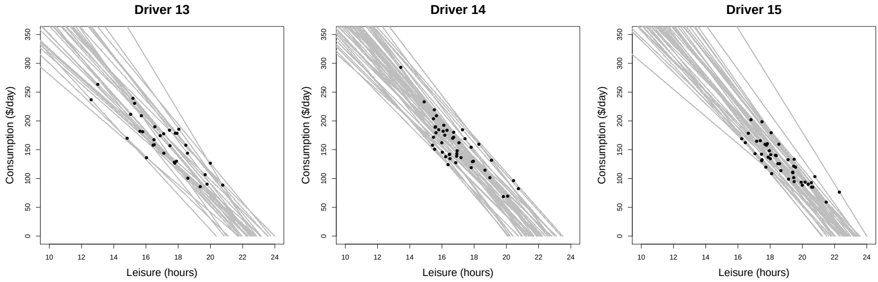
Empirical Application
NYC taxi drivers



Empirical Application
NYC taxi drivers



Empirical Application
NYC taxi drivers



For each individual driver we apply the various tests of reference-dependence with constant sensitivity.

1. Unobserved preference points
    1. reference-dependence in one dimension
    2. reference-dependence in both dimensions
2. Alternative models of observed reference points
    1. reference-dependence in either dimension
    2. reference-dependence in both dimensions

- It's an attempt to apply Occam's Razor empirically.
- It trades-off

1. The pass rate (proportion of the data consistent with the model), denoted by $r \in [0, 1]$.
2. The size of the set of theory-consistent actions relative to the size of the set of all feasible actions, denoted by $a \in [0, 1]$

- A good theory combines
  - a small a (theory which constrains predicted behaviour)
  - with a high r (behaviour with a high level of theory-consistency).
- A bad theory combines
  - a big a (almost anything goes)
  - with a low r (behaviour cannot even be consistent with almost anything goes).
- Selten argues $m(r, a)$ and

$$
\begin{aligned}
m(1,0) &> m(0,1) \\
m(0,0) &= m(1,1) \\
m(\bar{\mathbf{r}}, \bar{\mathbf{a}}) &= \bar{\mathbf{m}}
\end{aligned}
$$

- Selten's measure of predictive success:

$$m(r, a) = r - a$$

- $m \in [-1, 1]$
- $m \to 1$: a good theory
- $m \to -1$: a pathologically bad theory

- Another interesting way of combining r and a is to think about how informative the empirical exercise is relative to the permissiveness of a model given by a.
- This is an attempt to answer the question "given how flexible the model is, what did we learn from the data?"
- We use the Kullback-Leibler information gain

$$KL(r, a) = r \log_2 \frac{r}{a} + (1 - r) \log_2 \frac{(1 - r)}{(1 - a)}$$

- What did we learn?
  - if the model is very flexible $(a \to 1)$ and the data accorded with it $(r \to 1)$, then "not much"
  - if the model is very restrictive $(a \to 0)$ and the data failed to fit $(r \to 0)$, then "not much either"

$$KL(0,0) = KL(1,1) = KL(c,c) = 0$$

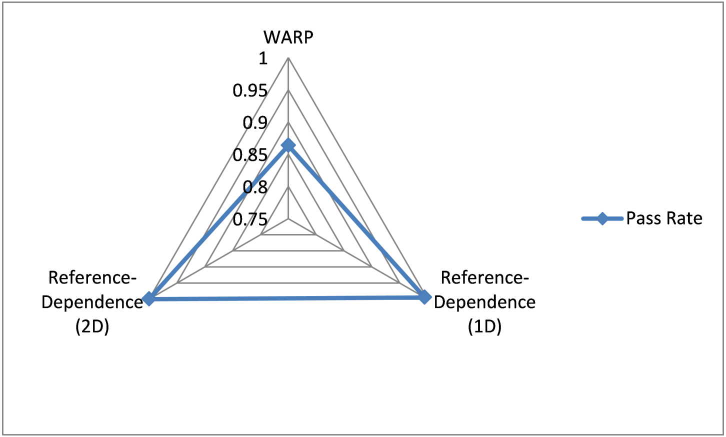
  - but if the model is very restrictive $(a \to 0)$ and the data fits well $(r \to 1)$, or the model is very flexible $(a \to 1)$ and the data fits poorly $(r \to 0)$, then "quite a bit" (either way).

$$KL(0,1) \to \infty, \; KL(1,0) \to \infty$$

- The two measures try to address two different questions
  - Kullback Leibler Divergence: the expected amount of information gained by examining the data, relative to the flexibility of the model.
  - Selten Index: how well does the model fit the data given its flexibility

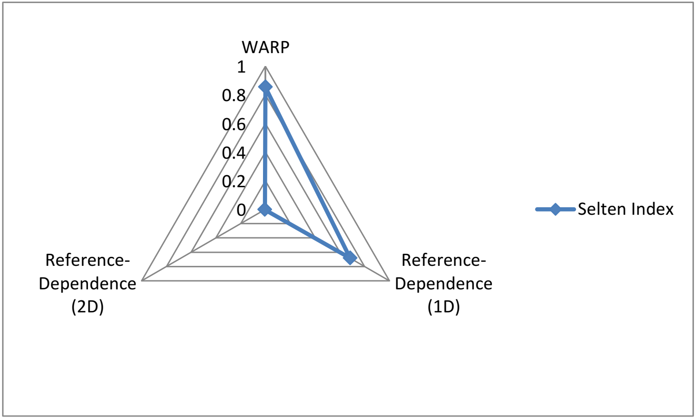
Empirical Application

Unobserved Reference points - Pass Rate



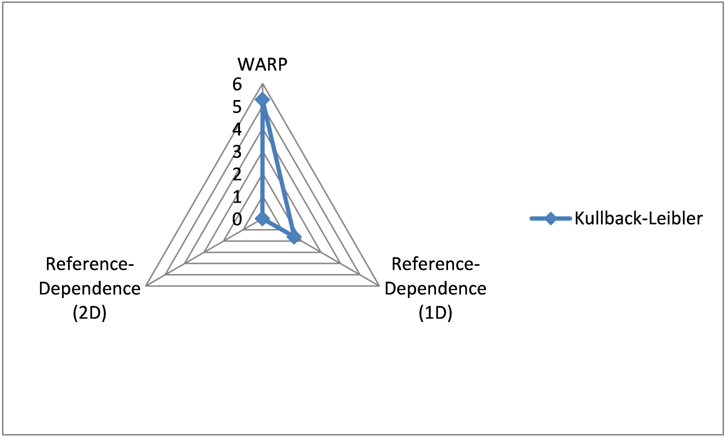
Empirical Application

Unobserved Reference points - Selten Index of Predictive Success



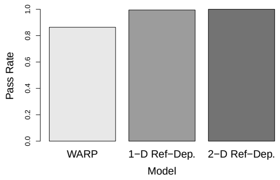
Empirical Application

Unobserved Reference points - KLIC

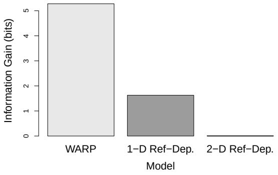


Empirical Application
Unobserved Reference points

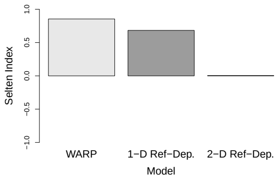
Pass Rate



Information Gain



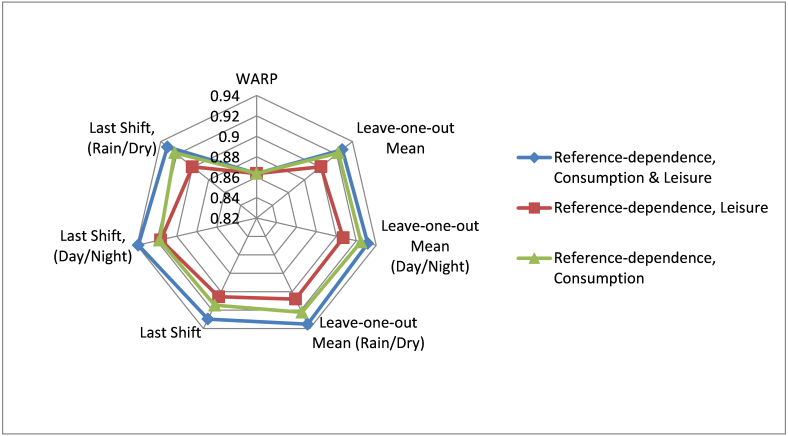
Predictive Success



- We consider 18 models of the reference point.
- 1D and 2D reference-dependence in which we model the reference point as a known function of the data:
  - the leave-me-out-mean
  - the leave-me-out-mean conditional on day/night shift
  - the leave-me-out-mean conditional on raining/not raining
  - the last shift
  - the last shift conditional on day/night
  - the last shift conditional on rain

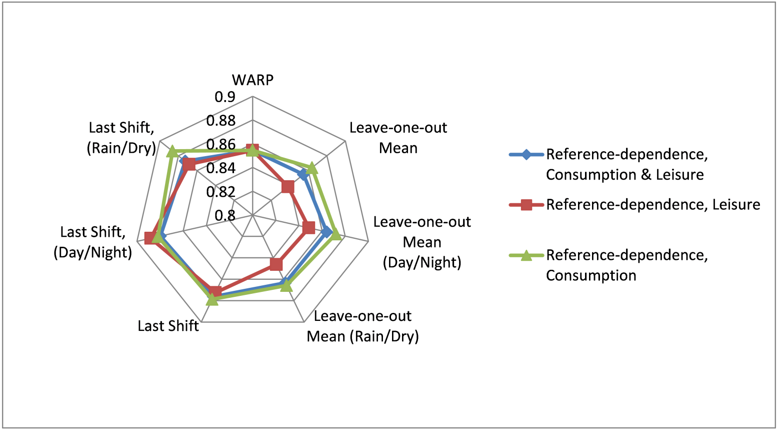
Empirical Application

Observed Reference points - Pass Rates

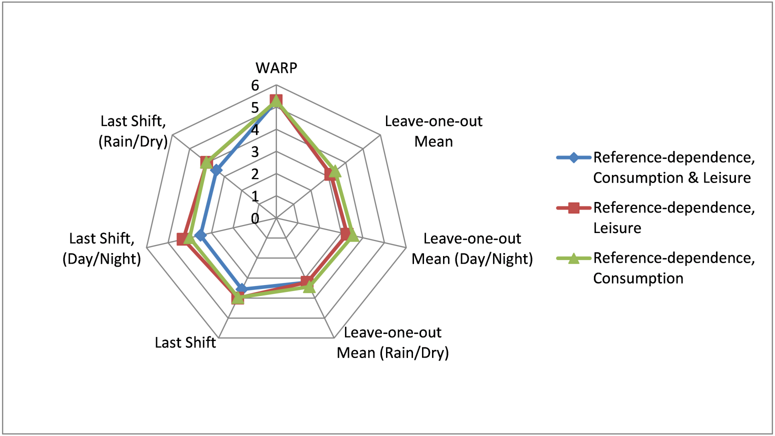


Empirical Application

Observed Reference points - Selten Index of Predictive Success

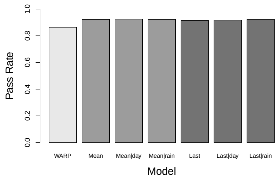


Empirical Application
Observed Reference points - KLIC

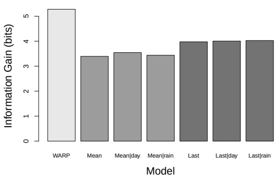


Empirical Application
Observed Reference points - Earnings

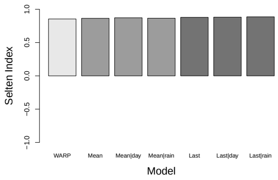
Pass Rate



Information Gain

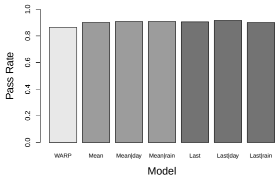


Predictive Success

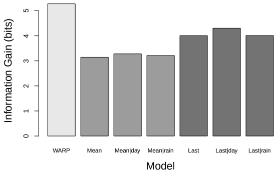


Empirical Application
Observed Reference points - Hours

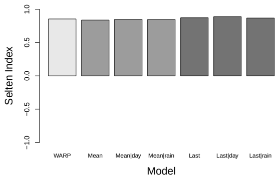
Pass Rate



Information Gain



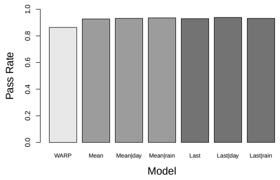
Predictive Success



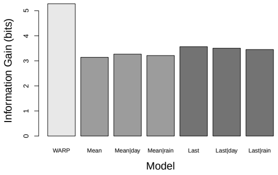
Empirical Application

Observed Reference points - Earnings & hours

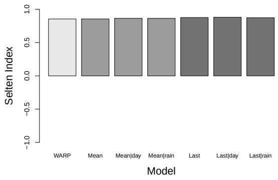
Pass Rate



Information Gain



Predictive Success



Conclusions

- Reference-dependence is a meaningful theorem.
- We will refrain from making population level statements on the basis of a sample of 15 individuals. But we might tentatively suggest:
  - The neoclassical cannot fit as well as the behavioural alternative. Yet once you penalise models for flexibility it seems that it is hard to beat.
  - Nonetheless, some reasonably plausible observed reference-points provide a better account of behaviour than the neoclassical model.
- It is probably not always necessary (or advisable) to reach for the behavioural explanation if WARP will do.