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Present-Bias and Time-Inconsistency I: Theory and evidence

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**(with very large debts to David Laibson and Ted O'Donoghue;
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The standard model of intertemporal choice in economics is time-separable utility with exponential discounting (Ramsey 1928 *EJ*, Samuelson, 1937 *REStud*):

$$U_t = u_t + \delta u_{t+1} + \delta^2 u_{t+2} + \delta^3 u_{t+3} + \dots$$

With exponential discounting, your trade-offs between receiving utils today and receiving them with delay are independent of when that delay occurs:

- If you prefer 10 utils now to 15 next week, then you must also prefer 10 utils in 25 weeks to 15 in 26 weeks.

This agreement implies “time-consistent” (or “dynamically consistent”) choice, in that early and later selves agree on the ranking of plans, so that the mere passage of time (with no new information) doesn’t change the ranking.

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(In more general models, receiving new information over time could affect your preferences over plans conditional on the information received.)

But if your preferences are time-consistent, receiving new information over time cannot alter your preferences over plans contingent on resolution of uncertainty.)

Thus, an exponential planner can simply maximize lifetime utility at the start of his life without worrying about later selves overturning his decisions.

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Familiar quotations you would never hear from an exponential discounter:

- Next month, I'll quit smoking.
- Next week, I'll catch up on the required reading.
- Tomorrow morning, I'll wake up early and exercise.
- After Christmas, I'll start eating better.
- Next weekend, I'll send in this rebate form.
- Next month, I'll start saving for retirement.

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These quotations all involve a pattern of deviations from time-consistent exponential discounting called "present bias": Excessively favoring gratification (or avoiding nongratification) now at the expense of future gratification.

"Excessive" means "relative to exponential": Exponential discounters do prefer present to future utils; just not as much as present-biased people.

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Future bias would yield quotations you seldom (never?) hear:

- I plan to watch more TV next year.
- I plan to eat more cookies and doughnuts next year.
- I plan to smoke more cigarettes next year.
- I plan to borrow more on my credit card next year.
- I plan to exercise less next year.
- I plan to wake up later next year.

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(But he was a workaholic, so even this may have been present bias.)

The quotations you actually hear also reflect a time-*in*consistent tension between the preferences of current and future selves:

- Early selves plan to further long-term goals but later selves may sacrifice those goals in favor of instant gratification.
- Present bias often leads people to put off unpleasant tasks that yield future benefits, a deviation from time-consistent planning called procrastination.
- E.g. “Da mihi castitatem et continentiam, sed noli modo.” (“Give me chastity and continence—but not yet.”)—Saint Augustine of Hippo

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PROCRASTINATION

HARD WORK OFTEN PAYS OFF AFTER TIME,
BUT LAZINESS ALWAYS PAYS OFF NOW.

Procrastination is an important problem in modern life (numbers from 2011):

- Among common students' problems, procrastination, with 14,800,000 Google hits, is 33% more important than ennui with 11,100,000, but 47% less important than plagiarism with 27,800,000.
- Procrastination is far less important than pride with 418,000,000; lust 216,000,000; wrath 148,000,000; greed 114,000,000; or envy 110,000,000.
- It runs close behind sloth with 15,900,000 (not counting the sloths with toes), but easily tops gluttony with 5,450,000.
- (For calibration: Obama 723,000,000 Google hits; Lady Gaga 479,000,000; The Simpsons 159,000,000; Emma Watson 92,300,000; Marx (not counting Groucho) 90,900,000; Keynes 51,000,800.)

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- "Preproperation" (456 Google hits and counting).

Discount functions and discount rates

- Discount function: u utils in τ periods are worth $D(\tau)$ utils today.
- The discount rate is the rate of decline in the discount function, the rate at which the value of a util declines with delay:

$$\rho(\tau) \equiv -\frac{dD(\tau)/d\tau}{D(\tau)}$$

- Discounting is a feature of preferences, *not* intertemporal transformation possibilities, which in markets are determined by the interest rate.
- With exponential discounting $D(\tau) \equiv \delta^\tau$, for which, when $\delta \simeq 1$,

$$\rho(\tau) \equiv -\frac{dD(\tau)/d\tau}{D(\tau)} \equiv -\ln\delta \simeq 1 - \delta,$$

so the discount rate does not change with the horizon.

- If discounting is stationary (i.e. if we only care about delay from “now”, whenever “now” is), then exponential discounting is the only discount function that yields time-consistent preferences (Strotz, 1956 *REStud*).

Can real people's discounting actually be exponential?

Many people seem to have at least a 1% preference for gratification today over gratification tomorrow.

But with exponential discounting, that would imply a yearly discount factor of $0.99^{365} \simeq 0.026$.

- So 100 utils in a year would be worth 2.6 utils today.
- So 100 utils in 10 years would be worth 1×10^{-14} utils today.

Even $0.999^{365} \simeq 0.694$; and $0.99^{52} \simeq 0.593$.

With exponential discounting, plausible discounting for a year from now implies implausibly low (virtually zero) discounting for a day or a week from now.

Experimental evidence

Thaler (1981 *Economics Letters*)

Thaler asked subjects to choose between money now versus more money later hypothetically (but many similar experiments used real rewards, similar results):

- What amount makes you indifferent between \$15 today and \$X in 1 month?

Typical response: $X = 20$.

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The baseline exponential discounting model has $Y = \delta^t X$. Recall that to relate a discount factor $\delta = e^{-\rho}$ to a discount rate ρ , $\ln \delta = \ln e^{-\rho} = -\rho$.

So for $X = 20$, $\rho = -\ln \delta = \frac{1}{t} \ln \frac{X}{15} = 12 \ln \frac{20}{15} \simeq 345\%$ per year.

- What amount makes you indifferent between \$15 today and \$X in ten years?

Typical response: $X = 100$. Implied discount rate $\simeq 19\%$ per year.

But “money now vs. money later” has many confounds:

- A person who has access to a perfect capital market should maximize net present value and then trade to the intertemporal optimum; but present value reflects market interest rates, not time preferences; and in practice dynamic choices involving money don't even seem to measure the interest rate
- Unreliability of future rewards (trust)
- Curvature or nonseparability of utility function (Andreoni and Sprenger 2012 *AER*)

McClure, Ericson, Laibson, Loewenstein, and Cohen (2007 *J. Neuroscience*)

Dynamic choices in kind (ice cream at t , or more at $t + \tau$) are also confounded via trust and utility function curvature, but probably better than money experiments.

McClure et al. studied very thirsty subjects:

- 60% of subjects chose juice now over twice the juice in 5 minutes
- 30% of subjects chose juice in 20 minutes over twice the juice in 25 minutes

Estimated 5-minute discount rate is 50% and the “long-run” discount rate is 0%.

Read, Loewenstein & Kalyanaraman (1999 J. *Behavioral Decision Making*)

Read et al.'s subjects chose among 24 movie videos:

- Some were “lowbrow”: e.g. *Four Weddings and a Funeral*
- Some were “highbrow”: e.g. *Schindler’s List*

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Canonical highbrow movie:



(A man seeks answers about life, death, and the existence of God as he plays chess against the Grim Reaper during the Black Death.)

Results:

- Choosing for tonight: 66% of subjects chose lowbrow.
- Choosing for next Tuesday: 37% chose lowbrow.
- Choosing for second Tuesday: 29% chose lowbrow.

Read and van Leeuwen (1998 *Organizational Behavior and Human Decision Processes*)

Inferences from static choices in kind (chocolate at t or orange at t , where committed choice can precede t) are probably still more reliable.

In Read and van Leeuwen's study:

- Overwhelming majority of subjects chose a healthy snack now to eat next week
- Overwhelming majority of subjects chose an unhealthy snack now to eat now

Badger, Bickel, Giordano, Jacobs, Loewenstein, and Marsch (2007 *J. Health Economics*)

Badger et al. studied recovering heroin addicts' monetary equivalent for an extra dose of Bupronorphine (heroin substitute like Methadone; aids withdrawal).

They elicited willingness to pay for a second dose from 13 long-time heroin addicts regularly receiving a single dose (but a second dose is still attractive to addicts): Buy a second dose for \$10? \$20?...\$100?

Subjects were always asked about a second dose, always to be delivered in the satiated state (the exact same circumstances, by experienced addicts).

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Half of the subjects were asked when they were more deprived (2 hours before scheduled dose), half when less deprived (right after scheduled dose).

Half of the subjects were asked for a second dose today, half for a second dose on their next visit (5 days hence).

	Immediate	Delayed (+5)
Satiated	\$50	\$35
Deprived	\$75	\$60

Present bias. Also projection bias.

So, what is the “true” willingness to pay?

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The ideal is often deciding in the same emotional/craving state as the situation in which we are trying to estimate the willingness to pay, but ahead of time to avoid self-control problems.

But note that it was we who got to define “bias” here.

Shapiro (2005 *J. Public Economics*)

Shapiro examined consumption patterns among food-stamp recipients, and found that over the month between food-stamp deliveries, caloric intake declines by about 10-15 percent.

- Survey evidence revealed rising desperation over the month, suggesting that high elasticity of intertemporal substitution is not a likely explanation.
- Households with more short-run impatience (estimated from hypothetical intertemporal choices) were more likely to run out of food.

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To explain this with a standard exponential-discounting model would require calibration with an extreme annual discount factor of 0.23.

A quasi-hyperbolic discounting model gracefully explains it with a plausible daily $\beta = 0.91$ and $\delta = 1$.

The data can reject a number of alternative explanations.

Dellavigna and Malmendier (2006 *AER*)

Dellavigna and Malmendier found, analyzing a field panel data set that tracks health club members' usages over time:

- Average cost of gym membership: \$75 per month
- Average number of monthly visits: 4
- Average cost per visit: \$19
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Present bias is not enough to explain these results by itself, because if you correctly predicted your future decisions, you wouldn't join in the first place.

Explanation may also involve some naiveté about own future behavior.

Ariely and Wertenbroch (2002 *Psychological Science*)

To the extent that people are aware of their own bias, it should create a demand for commitment, which is always valuable from the standpoint of the current self. (That might also help to explain Dellavigna and Malmendier's results.)

Ariely and Wertenbroch ran experiments using course term papers and proofreading tasks.

Term paper subjects were 99 professionals in an executive-education course at MIT, where three short papers were required.

Each paper had a deadline, with a 1% grade penalty per day late for all subjects.

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Two treatments were run between subjects in different sections:

- No-choice: Exogenously imposed, evenly spaced deadlines.
- Free-choice: Each student chose his own deadlines.

Results

In the free-choice group, 37/51 people imposed deadlines on themselves.

- Average deadline for paper 1 was 42 days before end of term on average.
- Average deadline for paper 2 was about 26 days before end of term.
- Average deadline for paper 3 was about 10 days before end of term.

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- People chose to make costly commitments, which is consistent with present bias and some degree of sophistication.
- But their chosen commitments were far from optimal, suggesting some naiveté.

Subjects were also recruited for the proofreading tasks and paid for performance, with a \$1 penalty per day late for all subjects, and a \$0.10 penalty for errors:

“Sexual identity is intrinsically impossible,” says Foucault; however, according to de Selby [1], it is not so much sexual identity that is intrinsically impossible, but rather the dialectic, and some would say the satsis, of sexual identity. Thus, D’Erlette [2] holds that we have to choose between premodern dialectic theory and subcultural feminism imputing the role of the observer as poet.”

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There were three such texts, and three treatments were run between subjects in different groups:

- No-choice: Exogenously imposed, evenly spaced deadlines of seven days for each text.
- End-deadline: All three texts due at the end of 21 days.
- Free-choice: Each student chose his own deadlines for each text, within the 21-day window.

Results

- Subjects in the free-choice group spaced out their deadlines.
- Performance (both freedom from errors and on-time delivery) was highest in the no-choice treatment, followed by free-choice and end-deadline treatments.
- Again subjects chose to make costly commitments, but their chosen commitments were far from optimal.

The results of both studies suggest that subjects had some present bias and some degree of sophistication, but also some naiveté about their own behavior.

Quasi-hyperbolic discounting

What kind of model can explain these patterns?

Consider “quasi-hyperbolic” discounting (a.k.a. “hyperbolic” or “ β, δ ”; Phelps and Pollak 1968 *REStud*, Laibson 1997 *QJE*, O’Donoghue and Rabin 1999 *AER*)

$$D(\tau) \equiv \begin{cases} 1 & \text{if } \tau = 0 \\ \beta \delta^\tau & \text{if } \tau = 1, 2, \dots \end{cases}$$

Tractably approximates (non-quasi-)hyperbolic discounting, $D(x) = 1/(1 + k\tau)$.

Plausibly, $\beta \ll 1$ and $\delta < 1$. E.g. $\beta = 2/3$ (for one day) and $\delta \simeq 0.95$.



Moe:

“This thing can **flash fry** a **Water Buffalo** in **40 seconds.**”



Homer:

“Ohhhhh, **40 seconds!**
But I want mine Now.”

But one issue that is not always discussed is, Where, between 40 seconds from now and the end of your life, does the “present” stop?

(Considered to some extent in early work, e.g. by Ainslie; but now often conflated with length of period in theory or time unit of data collection.)

We can write a quasi-hyperbolic utility function as

$$\begin{aligned}U_t &= u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \dots \\ &= u_t + \beta(\delta u_{t+1} + \delta^2 u_{t+2} + \delta^3 u_{t+3} + \dots)\end{aligned}$$

Quasi-hyperbolic discounters choose more patiently in the long than in short run:

$$\text{Future discount rate} = \frac{D(\tau) - D(\tau + 1)}{D(\tau)} = \frac{\beta\delta^\tau - \beta\delta^{\tau+1}}{\beta\delta^\tau} = 1 - \delta \approx 0.05$$

$$\text{Present discount rate} = \frac{D(0) - D(1)}{D(0)} = \frac{1 - \beta\delta}{1} = 1 - \beta\delta \approx 0.5$$

Time-inconsistency

When $\beta < 1$, the quasi-hyperbolic β, δ model can generate a conflict between earlier and later selves and the time-inconsistency it causes.

The time-inconsistent model predicts “self-control problems” like procrastination.

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For the examples that follow, consider a special case to build intuition: $\beta = 1/2$ and $\delta = 1$:

$$D(\tau) = \{1, \beta\delta, \beta\delta^2, \beta\delta^3, \dots\} = \{1, 1/2, 1/2, 1/2, \dots\}.$$

- Relative to the present period, all future periods are worth less.
- All discounting takes place between present and immediate future.
- In the “long run” we’re relatively patient—utils tomorrow are just as valuable as utils the day after tomorrow.

- Exercise has benefit today of -6. Exercise has delayed benefit of 8.
- Exercise today? No.

$$-6 + \frac{1}{2}8 = -2 < 0$$

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- But tomorrow you'll again want to postpone exercising (Akerlof 1991 *AER*; O'Donoghue and Rabin 1999 *AER*)

The quasi-hyperbolic model implies that decisions are sensitive to present vs. future timing: Only asynchronous benefits and costs bias decisions.

Naifs and Sophisticates

Some of the phenomena discussed above, e.g. Ariely and Wertenbroch's and Dellavigna and Malmendier's, cannot be explained by present bias alone.

People have therefore considered models that combine present bias with less-than-perfect sophistication about the person's own future behavior.

(Odysseus was as remarkable for his sophistication about his own behaviour (e.g. the sirens) as he was for his sophistication about others' (the horse).)

- Naifs falsely believe future selves will maximize today's preferences (Strotz 1957 *REStud*).
- Solution concept: maximization (mispredict future discount rates)
- Prediction (Dellavigna and Malmendier 2006 *AER*): never exercise (but join gym).

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 - Solution concept: maximization (mispredict future discount rates)
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- Sophisticates have rational expectations (Strotz 1957 *REStud*).
 - Solution concept: subgame perfect equilibrium in game among selves.
 - Prediction: never exercise (and don't join gym).
- Partial naiveté (O'Donoghue and Rabin 2001 *AER*)
 - Solution concept: subgame perfect equilibrium, using $\hat{\beta}$ such that $\beta < \hat{\beta} < 1$. (Naifs use $\hat{\beta} = 1$ and sophisticates use $\hat{\beta} = \beta$.)

Illustrative model (O'Donoghue and Rabin 1999 *AER*)

- Must do a project in one of $T < \infty$ periods, at a cost that increases over time.
- In time period t the project costs $(\frac{3}{2})^t$ utils to execute. (No discounting.)
- Each period, must choose to either “do” or “wait”; commitment is impossible.
- If wait until period T , must do it then.

With no discounting, an exponential discounter would do it now, with or without commitment.

Naifs

Naifs are fully unaware of their future self-control problems and therefore expect to behave in future exactly as they currently would like to behave in the future.

- When will a quasi-hyperbolic naïf with $\beta = 1/2$ and $\delta = 1$ so the project?

From the current self's perspective, it's always better to postpone doing the project until next period:

$$\left(\frac{3}{2}\right)^t > \beta\delta\left(\frac{3}{2}\right)^{t+1} = \frac{1}{2}\left(\frac{3}{2}\right)^{t+1} = \frac{3}{4}\left(\frac{3}{2}\right)^t < \left(\frac{3}{2}\right)^t$$

(Even partial naifs can make the same kind of mistake.)

Sophisticates

Sophisticates are quasi-hyperbolic with $\beta < 1$, hence time-inconsistent, but are aware of their self-control problems and so correctly predict their future behavior.

Hence they follow a finite-horizon subgame-perfect equilibrium, treating their own future selves as independent players.

- When will a quasi-hyperbolic sophisticate with $\beta = 1/2$ and $\delta = 1$ do the project?

If T is even, sophisticates will do the project in even but not in odd periods (complete contingent plan, as required by the strategic analysis):

- In the penultimate period, it's better to postpone for the same algebraic reason that it was better for a naïf, because a sophisticate believes (now correctly) that his period- T self will do the project. And so on, working back to the start.

If T is odd, then sophisticates will do the project in odd but not in even periods, for similar reasons.

The model's implications are less bizarre with uncertainty or an infinite horizon.

Another problem with the sophisticated (or partially naive) model is that the model predicts that people will often seek out and pay for commitment.

But:

- We see surprisingly little endogenous commitment for commitment's sake.
- Most commitment is ancillary (e.g. obligatory monthly mortgage payments).
- Very little commitment is gratuitous and advertised as such (e.g. Christmas clubs).